Design of double-weight code for synchronous OCDMA systems with power control

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Double-weight optical code division multiple access (OCDMA) systems are proposed for studying differentiated quality-of-service transmission. Based on quadratic congruence code (QCC), we construct a one-dimensional double-weight code family, which can be well utilized in incoherent synchronous double-weight OCDMA networks. By introducing algebraic transformation to code sequences of QCC in level 1, we obtain multiple double-weight codes with cross-correlation 1. Under the same-bit-power assumption, the performance of low-weight codes can be significantly improved and is always superior to that of high-weight codes in double-weight OCDMA systems with power control. This property is contrary to previous conclusions under the same-chip-power assumption.

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As a competitive alternative for the future optical communication, differentiated quality-of-service (QoS) optical code division multiple access (OCDMA) has been considered as a promising technology to support rapid growing popularity of multi-media services[1–4]. For the differentiated QoS requirements, many variable-weight codes have been proposed, such as 2D variable-weight optical orthogonal codes[5,6] and variable-weight prime codes by padding or removing pulses[7]. Besides, power control in physical layer is another important way of providing differentiated QoS in OCDMA systems[8].

Unlike the conventional schemes for differentiated QoS, double-weight OCDMA systems with power control have been investigated recently[2,9]. Chen *et al.* found that code-weight and power play different roles in the QoS transmission[2]. They concluded that high-weight codes do not always perform better than low-weight codes under same-bit-power assumption[9]. Although the performance-analytical approximation models as described earlier are fit for 1D and 2D codes, the 1D double-weight code is simpler and more accessible than 2D case for studying differentiated QoS OCDMA systems.

In this letter, based on quadratic congruence code (QCC)[10,11], we construct a 1D double-weight code family for incoherent synchronous double-weight OCDMA systems, named as "double-weight QCC" (DWQCC). Under both same-chip-power and same-bit-power assumptions, the mutual effect of high- and low-weight codes are analyzed. The results show that the proposed codes with low-weight can perform better than those with high-weight in double-weight OCDMA with power control.

QCC starts with Galois field $\mathrm{GF}(p) = \{0, 1, 2,..., p-1\}$ of a prime number $p \geq 3$. Firstly, a set of prime sequence $S_{i,j,k} = \left(s_{i,j,k}(0), s_{i,j,k}(1), ..., s_{i,j,k}(m), ..., s_{i,j,k}(p-1)\right)$ is obtained by a quadratic congruence function $s_{i,j,k}(m) = i \otimes_p m^2 \oplus_p j \otimes_p m \oplus_p k$, where i, j, k, $m \in \mathrm{GF}(p)$ and $i \neq 0$. $\otimes_p$ and $\oplus_p$ denote modulo-p multiplication and modulo-p addition, respectively. Then the prime sequence $S_{i,j,k}$ is mapped into the binary sequence $C_{i,j,k} = \left(c_{i,j,k}(0), c_{i,j,k}(1), ..., c_{i,j,k}(m), ..., c_{i,j,k}(p^2-1)\right)$ of length p^2. The mapping function is given by

$$h(\Psi) = \begin{cases} 1, \text{ for } \ \Psi = s_{i,j,k}(m) + mp \ \text{ with } \Psi \in \mathrm{GF}(p^2), \\ 0, \text{ otherwise}, \end{cases} \tag{1}$$

where Ψ denotes the position of the mth 1 in the binary sequence $C_{i,j,k}$. For instance, when $p = 3$, $i = 2$, $j = 1$, $k = 0$, we can conclude $S_{2,1,0} = (0, 0, 1)$, and thereby $C_{2,1,0} = (100\ 100\ 010)$.

QCC has the properties of multi-level, symmetry, and reasonably correlation[10,11]. Therefore, the code sets of QCC can be divided into $p-1$ groups $\{G_1, G_2, ..., G_i, ...G_{p-1}\}$ in level 2 and each group G_1 is separated into p different partitions $\{P_{i,0}, P_{i,1}, ..., P_{i,j}, ..., P_{i,p-1}\}$ in level 1, where any partition $P_{i,j}$ contains p code sequences $\{C_{i,j,0}, C_{i,j,1}, ..., C_{i,j,k}, ..., C_{i,j,p-1}\}$.

Time shifting is allowed in a synchronous OCDMA system. So in $\{C_{i,j,0}, C_{i,j,1}, ..., C_{i,j,k}, ..., C_{i,j,p-1}\}$, any code sequence $C_{i,j,k}$ can generate a loop subset $\{C_{i,j,k_0}, C_{i,j,k_1}, ..., C_{i,j,k_f}, ..., C_{i,j,k_{p-1}}\}$ by cyclic shift. Shifted code sequence C_{i,j,k_f} is denoted as

$$C_{i,j,k_f} = (c_{i,j,k_f}(0), c_{i,j,k_f}(1), ..., c_{i,j,k_f}(m), ..., c_{i,j,k_f}(p-1)), \tag{2}$$

where $f \in \mathrm{GF}(p)$ denotes the number of cyclic shift and $k_f \in \mathrm{GF}(p^2)$ is the serial number of the fth code sequence of the kth subset. The obtained code sequences of p loop subsets can compose a $p^2 \times p^2$ matrix. Then we insert subsequence at the back of each original subsequence $c_{i,j,k_f}(m)$ by the same length. So, the number of subsequences increases from p to $2p$. Accordingly, the corresponding original subsequence is changed as $c_{i,j,k_f}(2m)$. When $s_{i,j,k_f}(2m)$ (known by Eq. (1), $s_{i,j,k_f}(2m)$ map to $c_{i,j,k_f}(2m)$) satisfies $[s_{i,j,k_f}(2m) \neq k] \bigcup [s_{i,j,k_f}(2m) = k \bigcap s_{i,j,k_f}(2m') = k \bigcap m > m']$, the interleaving subsequence, denoted as $x_{i,j,k_f}(2m+1)$, is defined as null. Here $m' \in \mathrm{GF}(p-1)$ means that, in $S_{i,j,k_f}(2m)$, there exists another element $s_{i,j,k_f}(2m')$ equal to $s_{i,j,k_f}(2m)$. Otherwise, $x_{i,j,k_f}(2m+1)$ is equal to $c_{i,j,k_f}(2m)$.

Secondly, when $k \ominus_p s_{i,j,k_f}(2m) \leq \frac{p-1}{2}$ (where $\ominus_p$ denotes the modulo-p subtraction), we exchange the original subsequences with interleaving subsequences. Otherwise, they remain unchanged. The interleaved and exchanged code sequences should be a $p^2 \times 2p^2$ matrix.

Finally, we transpose the exchanged code sequences. The transposed code sequences are a $2p^2 \times p^2$ matrix which consist of $2p^2$ code sequences of length p^2 and correspond to the original and interleaving subsequences of the exchanged code sequences in column, respectively. Hence, the obtained code sequences can be divided into even and odd subsets (denoting as $E_{i,j,2m}(k_f)$ and $O_{i,j,2m+1}(k_f)$), which are given by

$$\begin{cases} \mathrm{E}_{i,j,2m}(k_f) = \left(\left(c_{i,j,0_0}(2m)^{\mathrm{T}}, \ldots, c_{i,j,0_f}(2m)^{\mathrm{T}}, \ldots, c_{i,j,0_{p-1}}(2m)^{\mathrm{T}}\right), \ldots, \left(c_{i,j,(p-1)_0}(2m)^{\mathrm{T}}, \ldots, c_{i,j,(p-1)_f}(2m)^{\mathrm{T}}, \ldots, c_{i,j,(p-1)_{p-1}}(2m)^{\mathrm{T}}\right)\right), \\ \mathrm{O}_{i,j,2m+1}(k_f) = \left(\left(x_{i,j,0_0}(2m+1)^{\mathrm{T}}, \ldots, x_{i,j,0_f}(2m+1)^{\mathrm{T}}, \ldots, x_{i,j,0_{p-1}}(2m+1)^{\mathrm{T}}\right), \ldots, \left(x_{i,j,(p-1)_0}(2m+1)^{\mathrm{T}}, \ldots, x_{i,j,(p-1)_f}(2m+1)^{\mathrm{T}}, \ldots, x_{i,j,(p-1)_{p-1}}(2m+1)^{\mathrm{T}}\right)\right). \end{cases} \tag{3}$$

null. In this case, at most one interleaving subsequence is non-null. Combining the above two cases we can deduce that one and only one interleaving subsequence is non-null. In other words, the weight of interleaving subsequences is 1. In addition, the weight of original subsequences is p. Hence, the sum of code weights of the original and interleaving subsequences is $p + 1$.

According to the above, each code set of QCC can be divided into p loop subsets. Every loop subset $(C_{i,j,0_f}, C_{i,j,1_f}, \ldots, C_{i,j,k_f}, \ldots, C_{i,j,(p-1)_f})$ has the same properties due to adding 1 shift. Similarly, rows and columns of any subset have the same number characteristics due to cyclic shift. Therefore, under given conditions, the number of interchange subsequences (denoting as w_c) of any two adjacent sequences is the same in columns. That is, the weight of original sequences becomes $p - w_c$ and the weight of corresponding interleaving sequences is $1 + w_c$. Hence, the transposed code sequences contain two weights (w_{ch}, w_{cl}) and w_{ch}, $w_{cl} = p + 1$ as the even and odd subsets derive from the original and interleaving sequences of interchanged sequences, respectively.

Since any interleaving sequence contains only one non-null element and is identical to the original subsequences, the interchange between original and corresponding interleaving subsequences does not deteriorate the cross-correlation but gets two variable weights in columns. In addition, the transposition operation does not worsen the cross-correlation of codes as well. The reasons are as follows. In $S_{i,j,k}$, only one element is single

Since QCC has $(p-1)$ groups in level 2 and each group has p partitions in level 1[10,11], based on partitions we can get $p(p-1)$ multiple code sets and each code set consists of $2p^2$ codes of length p^2. For sake of illustration, let w_{ch} and w_{cl} denote high- and low-weights ($w_{ch} \geq w_{cl}$), respectively. For any code set of DWQCC in level 1, it contains two weights (w_{ch}, w_{cl}) and the sum of high- and low-weights is $p + 1$ ($w_{ch} + w_{cl} = p + 1$). Furthermore, the maximum cross-correlation of any two code sequences is $\lambda_c = 1$ and the auto-correlation constraint of any code sequence is $\lambda_c = 2$.

Proof: When $m=0$, by $s_{i,j,k}(m) = i \otimes_p m^2 \oplus_p j \otimes_p m \oplus_p k$ we can conclude $s_{i,j,k}(0) = k$. Thus, in $S_{i,j,k}$ there is at least one value $s_{i,j,k}(m)$that satisfies $s_{i,j,k}(m) = k$, where $m \in \{0,1,\ldots,p-1\}$, that is, at least one interleaving subsequence will be not null. On the other hand, when $s_{i,j,k_f}(2m)$ satisfies Eq. (3), $x_{i,j,k_f}(2m+1)$ will be

and the others will appear in pairs due to the symmetry of QCC[11]. For any number appearing twice in a row, the two numbers of other loop subsets, locating in the same position, will exactly come in pairs. However, the two numbers of different loop subsets that locate in the same position are not equal due to adding 1 shift. According to the mapping relationship, different columns of interchanged code sequences will map to different subsets of the transposed code sequences. Similarly, different numbers in each column will map to different code sequences of the corresponding transposed subset. Hence, the maximum cross-correlation of codes is still 1. However, the numbers coming in pairs will make the auto-correlation equal to 2 according to the auto-correlation definition.

By relaxing the auto-correlation constraint, we double the code cardinality and get multiple double-weight code sets for a given prime number p. The weight

Table 1. Weight Distribution of DWQCC for Different p

p	Weight Distribution (w_{ch}, w_{cl})	Θ	L
3	(1,3) (2,2)	18	9
5	(1,5) (2,4) (3,3)	50	25
7	(2,6) (3,5) (4,4)	98	49
11	(3,9) (4,8) (5,7) (6,6)	242	121
13	(4,10) (5,9) (6,8) (7,7)	338	169
17	(5,13) (6,12) (7,11) (8,10) (9,9)	578	289
19	(7,13) (8,12) (9,11) (10,10)	762	381
23	(8,16) (9,15) (10,14) (11,13) (12,12)	1058	529
29	(9,21) (10,20) (11,19) (12,18) (13,17) (14,16) (15,15)	1682	841

distribution, code cardinality, and code length of DWQCC for different p are shown in Table 1, where Θ denotes the code cardinality of DWQCC in level 1 and L represents the length of code sequence.

It is necessary to evaluate the hit probability of the code sequences of DWQCC. The possibility of getting one hit of the desired code with high-weight w_{ch} being hit by an interfering code with high-weight w_{ch}, denoted as $q_{ch,ch,1}$, is given by

$$q_{ch,ch,1}(w_{ch}) = \frac{1}{2}\times\frac{w_{ch}\left(w_{ch}-1\right)}{p^2-1}. \tag{4}$$

The factor 1/2 comes from the assumption that data bit ones and zeros are transmitted with equal probability. In the numerator, $w_{ch}(w_{ch}-1)$ means that any two code sequences of DWQCC will cause $w_{ch}(w_{ch}-1)$ hits. And in the denominator, p^2-1 represents the possible number of interfering code sequences, out of a total of p^2 code sequences. Similarly, the hit probability among low-weight codes, denoted as $q_{cl,cl,1}$, is given by

$$q_{cl,cl,1}(w_{cl}) = \frac{w_{cl}(w_{cl}-1)}{2(p^2-1)}. \tag{5}$$

The possibility of getting one hit of the desired code with w_{ch} (or w_{cl}) being hit by an interfering code with w_{cl} (or w_{ch}), denoted as $q_{ch,cl,1}$(or $q_{cl,ch,1}$), is simply given by

$$q_{ch,cl,1}(w_{ch},w_{cl}) = q_{cl,ch,1}(w_{ch},w_{cl}) = \frac{w_{ch}w_{cl}}{2p^2}. \tag{6}$$

Multiple access interference (MAI) is the dominant noise in any on–off keying OCDMA systems. To focus on the effect of double-weight codes, we ignore the background noise, shot noise, and thermal noise, and assume that all optical codes have the same lengths representing the same transmission rate. Under the same-bit-power assumption, the chip powers ($\mathbf{\Phi}_{w_{ch}}$ and $\mathbf{\Phi}_{w_{cl}}$) of the codes with w_{ch} and w_{cl} are related by $w_{ch}\times\mathbf{\Phi}_{w_{ch}} = w_{cl}\times\mathbf{\Phi}_{w_{cl}}$. We can conclude $\mathbf{\Phi}_{w_{ch}} \le \mathbf{\Phi}_{w_{cl}}$ as $w_{ch} \ge w_{cl}$. In other words, the low-weight codes always carry more chip power per bit duration than that of high-weight codes. So the amount of MAI caused by interfering low-weight codes onto a mark chip of the desired high- and low-weight codes always constitutes one complete hit. Hence, the error probability P_{ch} of the desired high-weight code is the same under both same-chip-power and same-bit-power assumptions and is given by[9]

$$P_{eh}(w_{ch}) = \frac{1}{2}\sum_{r=o}^{w_{ch}}(-1)^r\binom{w_{ch}}{r}\left(1-\frac{rq_{ch,ch,1}}{w_{ch}}\right)^{M_{ch}-1} \times\left(1-\frac{rq_{ch,cl,1}}{w_{ch}}\right)^{M_{cl}}, \tag{7}$$

where M_{cl} and M_{ch} denote the numbers of the low- and high-weight users, respectively. However, the amount of MAI caused by interfering high-weight codes onto a mark chip of the desired low-weight code is different from that caused by interfering low-weight codes. Each hit generated by interfering high-weight codes can only constitute w_{cl}/w_{ch} of one complete hit onto a mark chip of the desired low-weight code. Hence, the error probability P_{el} of the desired low-weight code is given by[9]

$$P_{el}(w_{cl}) = \frac{1}{2}\sum_{u=o}^{w_{cl}}\binom{w_{cl}}{u}\left[\sum_{r=o}^{u}(-1)^r\binom{u}{r}\left(1-\frac{rq_{cl,cl,1}}{u}\right)^{M_{cl}-1}\times\sum_{v=0}^{c(w_{cl}-u)}\binom{c(w_{cl}-u)}{v}\times\left(1-\frac{vq_{cl,ch,1}}{c(w_{cl}-u)}\right)^{M_{ch}}\right], \tag{8}$$

Table 2. Hit Probabilities of Multiple Double-weight Code Sets for a Given p = 29

(w_{ch}, w_{cl})	(9,21)	(10,20)	(11,19)	(12,18)	(13,17)	(14,16)	(15,15)
$q_{cl,cl,l}$	3/70	3/56	11/148	11/140	13/140	13/120	1/8
$q_{ch,ch,l}$	1/4	19/84	27/140	51/280	17/105	15/105	1/8
$q_{ch,cl,l}$	189/1682	100/841	209/1682	108/841	221/1682	112/841	225/1682

where $c = \frac{w_{ch}}{w_{cl}}$ and [.] is the ceiling function. Particularly, when $w_{ch} = w_{cl} = (p + 1)/2$ (as $w_{ch} + w_{cl} = (p + 1)$, the DWQCC becomes conventional variable-weight code. The error probability of the double-weight codes are equal under both same-bit-power and same-chip-power assumptions (denoting as P_{e0}), and is given by[12]

$$P_{e0}(w_{ch(cl)}) = \frac{1}{2}\sum_{r=0}^{w_{ch(cl)}}(-1)^r\binom{w_{ch(cl)}}{r}\left(1-\frac{rq_{ch(cl),ch(cl),1}}{w_{ch(cl)}}\right)^{(M_{ch}+M_{cl})-1}. \quad (9)$$

According to Eqs. (4)–(6), the hit probabilities $q_{ch,ch,l}$, $q_{cl,cl,l}$, and $q_{ch,cl,l}$ ($q_{ch,cl,l}$) of multiple double-weight code sets for a given p are shown in Table 2 (where $q_{ch,cl,l} = q_{cl,ch,l}$).

Figure 1 shows the hard-limiting error probabilities, P_{el} and P_{eh} (p = 29, $w_{ch} + w_{cl}$ = 30, $M_{ch} = M_{cl}$ = 30), of codes versus the variation of weights (w_{cl} and w_{ch}) under both same-bit-power and same-chip-power assumptions. As shown in Fig. 1, P_{eh} of the high-weight code and P_{el} of the low-weight code vary as the high and low weights change. Under the same-chip-power assumption, P_{eh} is always lower than P_{el} (curves surrounded by the upper ellipse in Fig. 1). Moreover, P_{eh} ascends as the high weight decreases while P_{el} declines as the low weight increases. The reason is that the high-weight codes always carry higher power under the same-chip-power assumption. However, P_{el} of the low-weight code is observably superior to P_{eh} of the high-weight code under the same-bit-power assumption (curves surrounded by the lower ellipse in Fig. 1), which is contrary to the conclusion under the same-chip-power assumption. This is because the low-weight codes carry higher power under the same-bit-power assumption and, simultaneously, the proposed codes have the lower hit probability which further improves the bit-error rate performance of the low-weight codes. Hence, the performance of double-weight codes can be well tuned by varying code weight and power. It is helpful for power-sensitive applications in optical networks and sensor identification in fiber-sensor systems with the use of optical codes.

Figure 2 shows the hard-limiting error probabilities, P_{eh} and P_{el}(p = 29), of codes with high weight and low weight versus the number of active users (M_{cl} or M_{ch}) under the same-bit-power assumption. As shown in Fig. 2 (solid lines), when the low-weight users M_{cl} increase and the high-weight users M_{ch} are fixed to 100, P_{el} is superior to P_{eh}. Although a large number of active

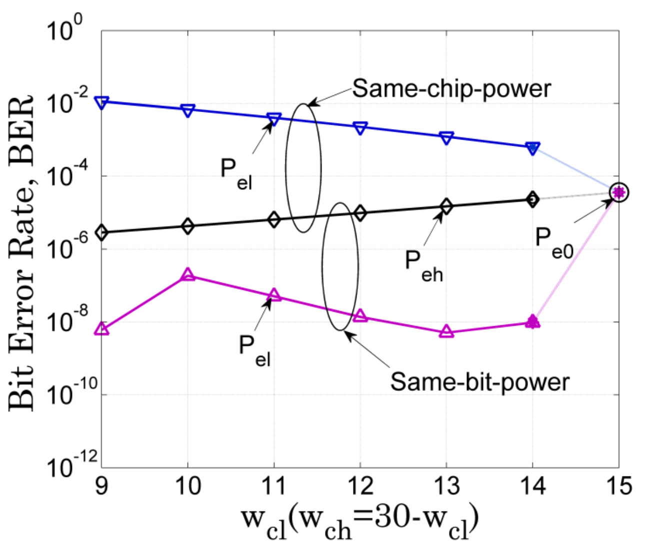

Fig. 1. Hard-limiting error probabilities, P_{eh} and P_{el}(p = 29, $w_{ch} + w_{cl}$ = 30, $M_{ch} = M_{cl}$ = 30), of codes versus variation of weights (w_{cl} and w_{ch}) under both same-chip-power and same-bit-power assumptions.

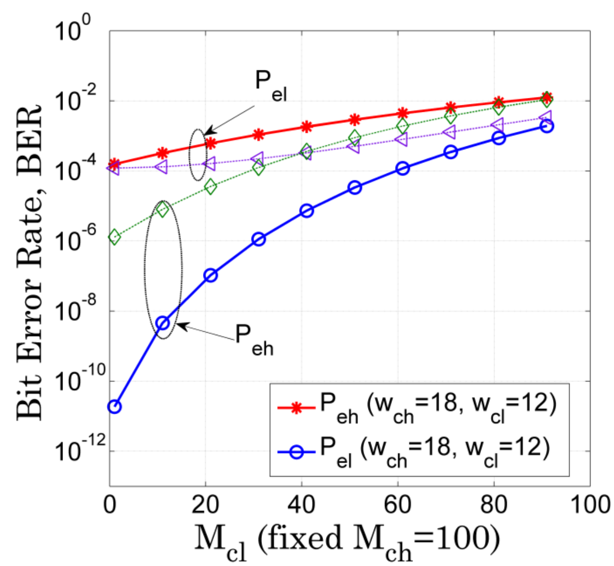

Fig. 2. Hard-limiting error probabilities, P_{eh} and P_{el}, of codes high-weight and low-weight versus the number of active users M_{cl} (or M_{ch}) for a given p = 29.

users always bring a greater MAI, an appropriate amount of low-weight users can be transmitted by high quality in OCDMA systems with power control as many high-weight users exist in advance. It is because the low-weight codes have the higher power and lower hit probability that the defense against MAI is stronger. The case can be well applied to high-quality communication services under strong noise corruption. As illustrated in Fig. 2 (dashed lines), P_{el} is lower than P_{eh} when M_{ch} is small and M_{cl}=100 is fixed. However, P_{el} deteriorates faster than P_{eh} as M_{ch} increases, and P_{el} becomes worse than P_{eh} as M_{ch} increases beyond a certain value. It is because the high-weight codes have more number of pulses, the probabilities of all mark positions being hit are more than those of the low-weight codes.

In conclusion, the 1D double-weight codes are simpler and more viable than 2D case in implementation of OCDMA with QoS requirements. The proposed DWQCC not only doubles the code cardinality of QCC by interleaving subsequences but also provides multiple double-weight codes with cross-correlation 1. We analyze that the performance of low-weight codes is observably improved and superior to those of high-weight codes in double-weight OCDMA systems with power control. Moreover, the results indicate that, when there are many high-weight users in the network, double-weight OCDMA systems still allow an appropriate amount of low-weight users to transmit with high quality. Because the low-weight codes have the higher power and lower hit probability, the defense against MAI is stronger.

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