Scattering of on-axis polarized Gaussian light beam by spheroidal water coating aerosol particle

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Small particle light scattering can produce light with polarization characteristics different from those of the incident beam. An analytical solution to the scattering by a spheroid with inclusion for an on-axis polarized Gaussian beam incidence is provided within the generalized Lorenz-Mie theory framework. The shapes of the inclusion can be spherical, confocal spheroid, or non-confocal spheroid. The Muller scattering matrix elements are computed for plane wave incidence or Gaussian light beam incidence. The effect of the size and shape of the inclusion or the coating on the polarized Gaussian light scattering characteristics by a spheroidal water coating aerosol particle are computed and analyzed.

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Natural particles often exhibit nonspherical and inhomogeneous overall shapes and complex morphologies. Some of these particles can be modeled by core-shell particles, such as the nucleated blood cells, cloud particles, and hydrometeors with condensation nuclei. The spheroidal particle model, one of the possible analytical models for the scatterer, has also been applied in numerous cases, e.g., spheroids describe well the shape of plant and animal cells, bacteria, spores, or microalgae, microcavities, aerosols in the atmosphere, and so on.

To date, the light scattering method is effective in the retrieval of the microphysics characteristics of small particles. The scattering characteristics of an inhomogeneous particle should be accurately simulated to apply this light scattering technique in analyzing particles containing inclusion. Light scattering produces light with polarization characteristics different from those of the incident beam. When the incident beam is unpolarized, the scattered light has at least one nonzero Stokes parameter other than intensity, and this phenomenon is often called "polarization". Although light scattering intensity is the most commonly measured quantity, polarization can be measured with significantly higher accuracy than light intensity.

An accurate theory method is required when laser is used as light source to study the scattering properties of the spheroidal particles. The generalized Lorenz-Mie theory (GLMT) developed by Gouesbet *et al.* can effectively describe the interaction of a shaped beam with a spherical particle by relying on the separability of the variables[1,2]. This theory has been expanded in numerous studies to multilayered spheres[3], spheroids[4], and infinite cylinders[5]. Han *et al.*[6] studied in detail the scattering of Gaussian beam by multi-layer spheroid, and the equations for calculating the normalized differential scattering cross sections are given. In this letter, we analyze the polarization of Gaussian light scattered by a single spheroidal particle with an inclusion.

In Fig. 1, a Gaussian beam propagates in free space and from the negative z' to the positive z' axis of the Cartesian coordinate system $O'x'y'z'$, with the middle of its beam waist located at the origin O', and the time-dependent part of the electromagnetic fields is assumed to be $\exp(-\mathrm{i}\omega t)$. An accessory system $Oxyz$ parallel to $O'x'y'z'$ is introduced to define the location of a coated spheroidal particle.The center of the coated spheroid is located at origin O and has the Cartesian coordinates $(0,\ 0,\ z_0)$ in $O'x'y'z'$. The major axis of the coated spheroid is along the z axis. The semifocal distance and the semimajor and semiminor axes are denoted by f_1, a_1, and b_1 for the inclusion spheroidal particle surface, and by f_2, a_2, and b_2 for the outer surface of the dielectric coating.

Within the GLMT framework,we have obtained an expansion in Ref. [7] of the electromagnetic fields of an incident Gaussian beam in terms of the spheroidal vector wave functions $\mathbf{M}^{r(1)}_{^e_omn}(c,\zeta,\eta,\phi)$ and $\mathbf{N}^{r(1)}_{^e_omn}(c,\zeta,\eta,\phi)$, with respect to the system $Oxyz$ as

$$\begin{aligned}\mathbf{E}^i =&E_0\sum_{m=0}^{\infty}\sum_{n=m}^{\infty} i^n[G^m_{n,\mathrm{TE}}\mathbf{M}^{r(1)}_{emn}(c,\zeta,\eta,\phi)\\ &+iG^m_{n,\mathrm{TM}}\mathbf{N}^{r(1)}_{omn}(c,\zeta,\eta,\phi)],\end{aligned} \tag{1}$$

$$\begin{aligned}\mathbf{H}^i =&E_0\frac{k}{\omega\mu}\sum_{m=0}^{\infty}\sum_{n=m}^{\infty} i^n[G^m_{n,\mathrm{TM}}\mathrm{M}^{r(1)}_{omn}(c,\zeta,\eta,\phi)\\ &-iG^m_{n,\mathrm{TE}}\mathbf{N}^{r(1)}_{emn}(c,\zeta,\eta,\phi)],\end{aligned} \tag{2}$$

where $c = kf_2$ and $G^m_{n,\mathrm{TE}}$ and $G^m_{n,\mathrm{TM}}$ are the BSCs.

When the Gaussian beam expansion is obtained, the

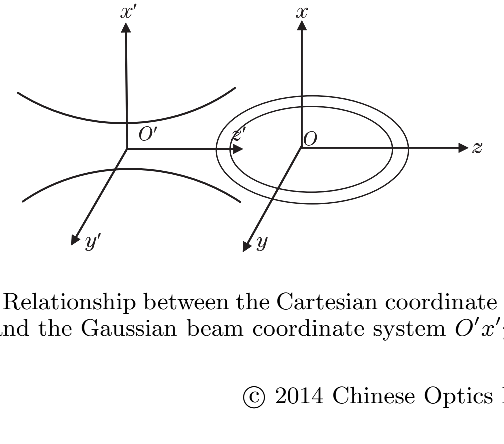

Fig. 1. Relationship between the Cartesian coordinate system $Oxyz$ and the Gaussian beam coordinate system $O'x'y'z'$.

scattered fields of the spheroidal particle can be expanded in terms of the appropriate spheroidal vector wave functions as[7]

$$\mathbf{E}^s = E_0 \sum_{m=0}^{\infty}\sum_{n=m}^{\infty} i^n[\beta_{mn}\mathbf{M}_{emn}^{r(3)}(c,\zeta,\eta,\phi) + i\alpha_{mn}\mathbf{N}_{omn}^{r(3)}(c,\zeta,\eta,\phi)]. \quad (3)$$

The electromagnetic fields within the spheroidal particle can be represented by

$$\mathbf{E}^{w(1)} = E_0 \sum_{m=0}^{\infty}\sum_{n=m}^{\infty} i^n[\delta_{mn}^{(1)}\mathbf{M}_{emn}^{r(1)}(c_1,\zeta,\eta,\phi) + i\gamma_{mn}^{r(1)}\mathbf{N}_{omn}^{r(1)}(c_1,\zeta,\eta,\phi)], \quad (4)$$

where a_{mn}, β_{mn}, δ_{mn}, and γ_{mn} are the unknown expansion coefficients to be determined using the boundary conditions, $c_1 = f_1k_1$, $k_1 = k\widetilde{n}_1$, and $\widetilde{n}_1$ is the refractive index of the material of the spheroidal particle relative to that of free space.

To overcome the difficulty of applying the boundary conditions on the inclusion particle and coating surfaces that are concentric and non-confocal, the electromagnetic fields within the dielectric coating are expanded in terms of the spheroidal vector wave functions attached to the spheroid and coating surfaces, respectively, as[8]

$$\begin{aligned}\mathbf{E}^w =& E_0 \sum_{m=0}^{\infty}\sum_{n=m}^{\infty} i^n[\delta_{mn}\mathbf{M}_{emn}^{r(1)}(c_2,\zeta,\eta,\phi) \\ &+ \chi_{mn}\mathrm{M}_{emn}^{r(3)}(c_2,\zeta,\eta,\phi) + i\gamma_{mn}\mathbf{N}_{omn}^{r(1)}(c_2,\zeta,\eta,\phi) \\ &+ i\tau_{mn}\mathrm{N}_{omn}^{r(3)}(c_2,\zeta,\eta,\phi)], \end{aligned} \quad (5)$$

$$\begin{aligned}\mathbf{E}^w =& E_0 \sum_{m=0}^{\infty}\sum_{n=m}^{\infty} i^n[\delta'_{mn}\mathbf{M}_{emn}^{r(1)}(c'_2,\zeta,\eta,\phi) \\ &+ \chi'_{mn}\mathrm{M}_{emn}^{r(3)}(c'_2,\zeta,\eta,\phi) + i\gamma'_{mn}\mathbf{N}_{omn}^{r(1)}(c'_2,\zeta,\eta,\phi) \\ &+ i\tau'_{mn}\mathrm{N}_{omn}^{r(3)}(c'_2,\zeta,\eta,\phi)], \end{aligned} \quad (6)$$

where $c_2 = k_2f_1$, $c'_2 = k_2f_2$, $k_2 = k\widetilde{n}_2$, and $\widetilde{n}_2$ is the refractive index of the dielectric coating material relative to that of free space.

With ζ_1 and ζ_2 as the radial coordinates of the boundary surfaces of the spheroid and coating, respectively, the boundary conditions on the surface $\zeta = \zeta_2$ are described by

$$\left.\begin{aligned} E_\eta^i + E_\eta^s = E_\eta^w, \quad E_\phi^i + E_\phi^s = E_\phi^w \\ H_\eta^i + H_\eta^s = H_\eta^w, \quad H_\phi^i + H_\phi^s = H_\phi^w \end{aligned}\right\} \text{at} \quad \zeta = \zeta_2. \quad (7)$$

And the surface $\zeta = \zeta_1$ is described by

$$\left.\begin{aligned} E_\eta^{w(1)} = E_\eta^w, \quad E_\phi^{w(1)} = E_\phi^w \\ H_\eta^{w(1)} = H_\eta^w, \quad H_\phi^{w(1)} = H_\phi^w \end{aligned}\right\} \text{at} \quad \zeta = \zeta_1. \quad (8)$$

The relation between the scattered fields and the incident fields can be written as[9,10]

$$\begin{aligned}\begin{bmatrix} E_\vartheta^{\mathrm{sca}} \\ E_\varphi^{\mathrm{sca}} \end{bmatrix} &= \frac{\exp(ikR)}{R}\mathbf{S}^{\mathrm{L}}(\widehat{n}_{\mathrm{sca}},\widehat{n}_{\mathrm{inc}})\begin{bmatrix} E_\vartheta^{\mathrm{inc}} \\ E_\varphi^{\mathrm{inc}} \end{bmatrix} \\ &= \frac{\exp(ikR)}{R}\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}\begin{bmatrix} E_\vartheta^{\mathrm{inc}} \\ E_\varphi^{\mathrm{inc}} \end{bmatrix}, \end{aligned} \quad (9)$$

where $\widehat{n}_{\mathrm{inc}}$ denotes the incident direction, $\widehat{n}_{\mathrm{sca}}$ denotes the scattering direction, and $\mathbf{S}^{\mathrm{L}}$ is a 2×2 amplitude matrix that transforms the electric field vector components of the incident wave into the electric field vector components of the scattered waves.

The polarization state of a light beam is traditionally described by a vector $I = (I,Q,U,V)^{\mathrm{T}}$ composed of four Stokes parameters (T means transpose). The first Stokes parameter, I, is the intensity, whereas the other three parameters describe the polarization state of the beam. The Stokes parameters are always defined with respect to a reference plane, e.g., the meridional plane of the beam in a spherical coordinate system. The scattering of a light beam by a single particle is fully characterized by a 4×4 Mueller matrix $\mathbf{F}$ that describes the transformation of the Stokes vector of the incident beam into the Stokes vector of the scattered beam.

$$\begin{pmatrix} I_{\mathrm{s}} \\ Q_{\mathrm{s}} \\ U_{\mathrm{s}} \\ V_{\mathrm{s}} \end{pmatrix} = \begin{bmatrix} F_{11} & F_{12} & F_{13} & F_{14} \\ F_{21} & F_{22} & F_{23} & F_{24} \\ F_{31} & F_{32} & F_{33} & F_{34} \\ F_{41} & F_{42} & F_{43} & F_{44} \end{bmatrix}\begin{bmatrix} I_{\mathrm{i}} \\ Q_{\mathrm{i}} \\ U_{\mathrm{i}} \\ V_{\mathrm{i}} \end{bmatrix}. \quad (10)$$

When a particle has a mirror symmetry relative to the principal plane, the signs of amplitude matrix elements S_{12} and S_{21} change for the mirror particle. The complete particle can be considered to consist of two halves that are touching. When the scattering amplitude matrix in this case is summed up, only the S_{11} and S_{22} survive. Consequently, the corresponding Muller matrix elements F_{13}, F_{14}, F_{23}, F_{24}, F_{31}, F_{32}, F_{41}, and F_{42} are zero, and F_{22}/F_{11} unites. Notably, the deviation of F_{22}/F_{11} from unity has normally been regarded as an indication of the nonsphericity of a scattering particle[11,12], and $F_{12} = F_{21}$, $F_{33} = F_{44}$, and $F_{43} = -F_{34}$ for the light beam on-axis incidence on a mirror symmetry particle. F_{11}, F_{12}/F_{11}, F_{33}/F_{11}, and F_{34}/F_{11} will be discussed as follows.

The results of the GLMT computations for light scattering by a spheroidal water coating aerosol particle is presented in this section. The Muller matrix elements F_{11}, F_{12}, F_{33}, and F_{34} are computed for normal incidence of an on-axis Gaussian beam with a wavelength of λ=0.66328 μm, in which case the refractive index of the water coating is $\widetilde{n} = 1.33$ and that of the inclusion particle is $\widetilde{n}_1 = 1.5$.

The Muller matrix elements for a two-layer spherical particle illuminated by the on-axis Gaussian beam with the waist radius of $w_0 = \infty$ (plane wave), $\lambda/2$, λ, and 2λ are illustrated in Fig. 2. The size parameter of the inclusion and the water coating are defined as $ka_1 = 4$ and $ka_2 = 10$, respectively.

The Muller matrix elements for a two-layer confocal spheroidal particle illuminated by the on-axis Gaussian beam with the waist radius $w_0 = \infty$ (plane wave), $\lambda/2$,

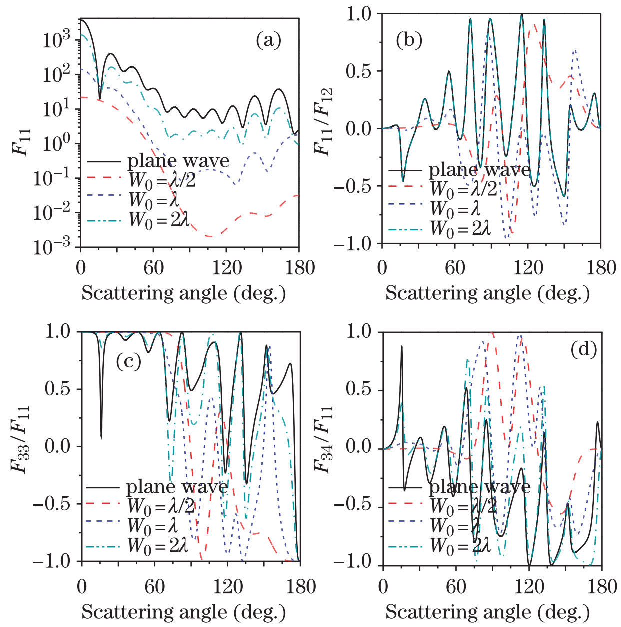

Fig. 2. Muller matrix elements for a single spherical particle for the Gaussian light beam scattering with different beam waist radii.

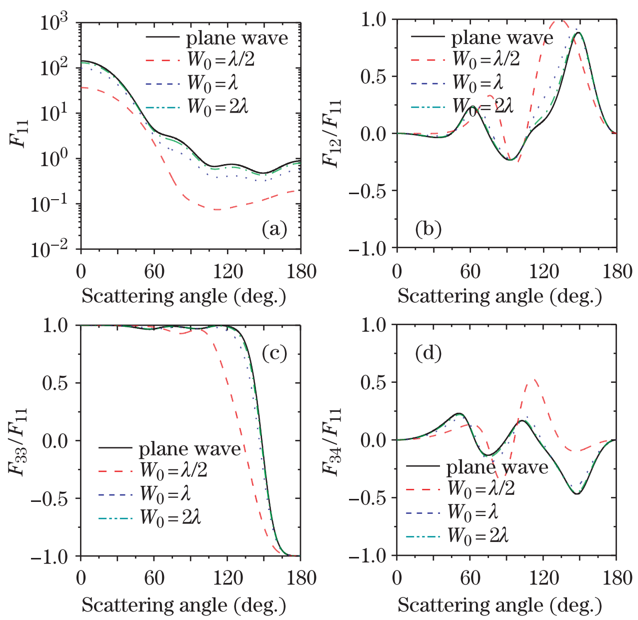

Fig. 3. Muller matrix elements for a single confocal spheroidal particle for Gaussian light beam scattering with different beam waist radii.

λ, and 2λ, respectively, are shown in Fig. 3. The semimajor and semiminor axes are denoted by f_1, a_1, and b_1. The size parameter of the inclusion and the water coating are defined as $ka_1 = 3$ and $ka_2 = 4$, respectively, and the semifocal distanceis $kf = 2$. The scattering properties of a spheroid with plane wave incidence may be significantly different from those of the Gaussian beam (Fig. 3). The first element of the Muller matrix (which is associated with the single scattering phase function) for plane wave incidence are larger than the others. The scattering characteristics of a confocal spheroid for Gaussian beam incidence approaches the plane wave incidence case with increasing beam waist radius.

Similar to Fig. 3, Fig. 4 also presents the elements of the Muller matrix element of a water-coated non-confocal spheroid aerosol, with aspect ratio values of $\varepsilon = 2.0$ for both inclusion and coating. The incidence wavelength, size parameter, and refractive index of the spheroid are similar to those in Fig. 3. Remarkable differences are found between the curves of the confocal and non-confocal spheroidal water coating aerosol particle, although they have similar size parameters (Figs. 3 and 4).

The elements of the Muller matrix F_{11} and F_{12}/F_{11} varying with the aspect ratio of the inclusion for a two-layer spheroid are shown in Fig. 5. The size parameters of the shell and the inclusion are $ka_2 = 10$ and $ka_1 = 4$, respectively. The waist radius of the incidence Gaussian beam is $w_0 = \lambda$, and the aspect ratio of the shell is $a_2/b_2 = 1.5$. The elements of the Muller matrix F_{11} and F_{12}/F_{11} varying with the aspect ratio of the inclusion for a sphere with a spheroid inclusion are shown in Fig. 6. The waist radius of the incidence Gaussian beam is $w_0 = \lambda$. The size parameters of the shell and the inclusion are similar to those in Fig. 5. The elements of the Muller matrix F_{11} and F_{12}/F_{11} varying with the aspect

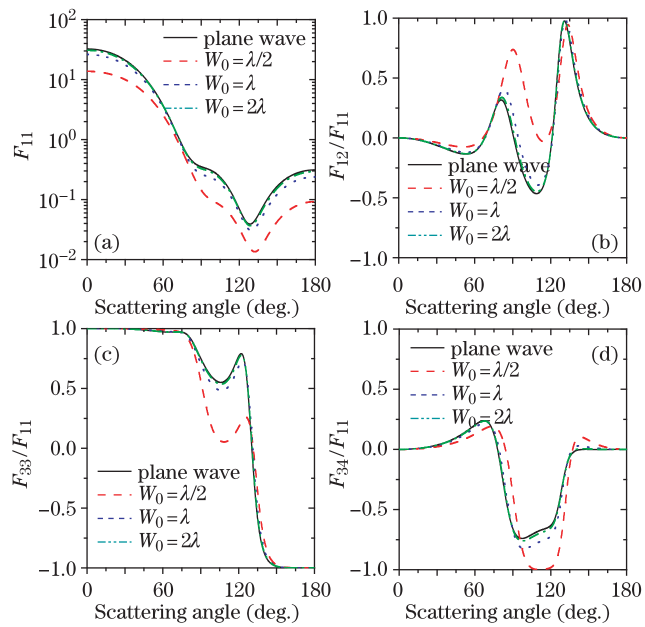

Fig. 4. Muller matrix elements for a single non-confocal spheroidal particle for the Gaussian light beam scattering with different beam waist radii.

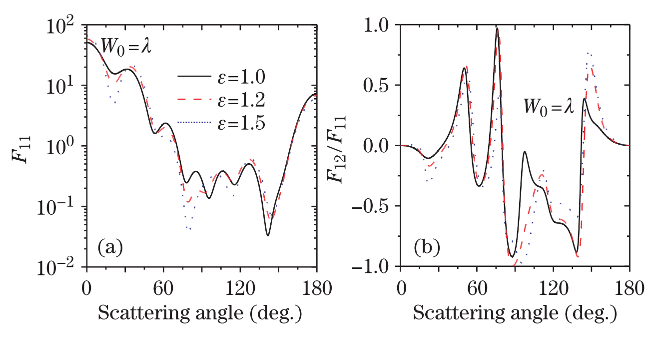

Fig. 5. Muller matrix elements (a) F_{11} and (b) F_{12}/F_{11} for a single non-confocal spheroidal particle for the Gaussian light beam scattering with different aspect ratios of the inclusion.

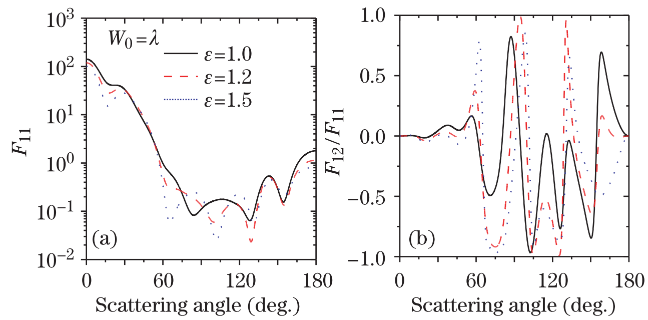

Fig. 6. Muller matrix elements (a) F_{11} and (b) F_{12}/F_{11} variation with the aspect ratio of the inclusion for a sphere with a spheroid inclusion.

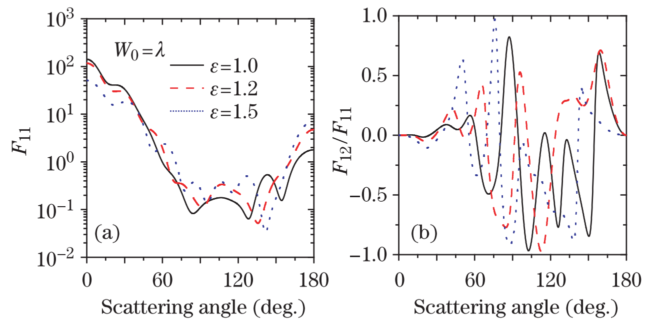

Fig. 7. Muller matrix elements (a) F_{11} and (b) F_{12}/F_{11} variation with the aspect ratio of the inclusion for a spheroid with a sphere inclusion.

ratio of the shell for a spheroid with a sphere inclusion are illustrated in Fig. 7. The waist radius of the incidence Gaussian beam is $w_0 = \lambda$. The size parameters of the shell and the inclusion are similar to those in Fig. 5.

Both the shape of the shell and the inclusion significantly affect the scattering characteristics of the particle, and the light scattering polarization degree is very sensitive to the shape of the inclusion and shell (Figs. 5–7). This sensitivity makes the accurate measurement of the Muller matrix a valuable technique for particle characterization.

In conclusion, we provide the general formulas that can be used to compute the amplitude matrix for Gaussian light scattering by a spheroidal particle with a sphere or spheroid inclusion within the GLMT framework. The numerical results for the Muller matrix are presented for the polarized Gaussian light scattering using a spheroidal water coating aerosol particle, and the results are compared with the case of the plane wave at normal incidence. Significant differences are found for the Gaussian light scattering with different waist beam radii, and the Muller scattering matrix are very sensitive to the shape and size parameters of the shell and the inclusion for Gaussian light beam scattering. The values of the scattering matrix element F_{11} increase with increasing waist beam radius, but the values of F_{11} slightly change with varying aspect ratio of the shell and the inclusion. The other three Muller scattering matrix elements, especially F_{12}/F_{11}, are more sensitive to the inclusion than F_{11}. The formulas and the computed results are useful for the determination of the characteristics of spheroidal particles.

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References

1. G. Gouesbet, B. Maheu, and G. Gréhan, J. Opt. Soc. Am. A **5,** 1427 (1988).
2. J. Yu, X. Tong, C. Li, Y. Huang, and A. Ye, Chin. Opt. Lett. **11,** 091701 (2013).
3. Z. Wu, L. Guo, K. Ren, G. Gouesbet, and G. Gréhan, Appl. Opt. **36,** 5188 (1997).
4. Y. Han, H. Zhang, and G. Han, Opt. Express **15,** 735 (2007).
5. K. Ren, G. Gréhan, and G. Gouesbet, J. Opt. Soc. Am. A **14,** 3014 (1997).
6. H. Zhang, Y. Han, and G. Han, J. Opt. Soc. Am. B **24,** 1383 (2007).
7. H. Zhang and Y. Sun, J. Opt. Soc. Am. B **27,** 883 (2010).
8. H. Zhang and Y. Han, J. Opt. Soc. Am. B **11,** 255 (2008).
9. M. I. Mishchenko, Appl. Opt. **39,** 1026 (2000).
10. X. Huang, Y. Bu, and X. Wang, Chin. Opt. Lett. **8,** 546 (2010).
11. C. F. Bohren and D. R. Huffman, *Absorption and Scattering of Light by Small Particles* (Wiley, New York, 1983).
12. M. I. Mishchenko, L. D. Travis, and A. A. Lacis, *Scattering, Absorption, and Emission of Light by Small Particles* (Cambridge University Press, Cambridge, 2002).