Propagation and interaction of two-component vector dark solitons in the defocusing nonlinear media

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Dynamics of two-component vector dark solitons are investigated by the variational approach in the defocusing nonlinear media, and effects of the weak nonlocality on the soliton propagation and interaction are analyzed. The nonlocality degree determines the intensity distribution of the dark solitons in the stationary states, enhances the intensity transfer between two vector solitons, and affects the propagation and interaction. The numerical results confirm the theoretical findings.

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A spatial optical soliton is a beam which propagates in a nonlinear medium without changing its structure, and exists as a robust object withstanding even strong perturbations. Nonlocality plays an important role in many areas of nonlinear physics, acts to spread out the effects of localized excitations, and can suppress modulation instabilities of homogeneous states[1,2]. The weak nonlocal nonlinear media can support solitons, and solitons have been typically considered in the context of so-called local nonlinear media, where the refractive index change induced by an optical beam in a particular point depends solely on the beam intensity[3,4]. Depending on the type of nonlinearity, nonlinear media may support either bright or dark solitons, and dark solitons are more complex objects, as they represent an intensity dip in an otherwise constant background with nontrivial phase profile. Spatial dark solitons have been observed and studied in media with a negative or self-defocusing nonlinearity[5,6]. In this letter, the effects of the weak nonlocality is investigated on the propagation and interaction of the two-component vector dark solitons in self-defocusing nonlinear media, and some novel results are obtained.

Two mutually incoherent wave-packets are considered propagating along the z axis within the defocusing media, and the governing equations of the vectorial Manakov system which consists of two vector components can be described by

$$j\frac{\partial u_1}{\partial z} + \frac{1}{2}\frac{\partial^2 u_1}{\partial x^2} - \int R(x-x')I(x')u_1 = 0,$$

$$j\frac{\partial u_2}{\partial z} + \frac{1}{2}\frac{\partial^2 u_2}{\partial x^2} - \int R(x-x')I(x')u_2 = 0, \quad (1)$$

where u_1 and u_2 are normalized vector components, respectively; z and x are the distance and transverse coordinate, respectively; $I(x) = |u_1|^2 + |u_2|^2$ is the total intensity; $R(x)$ is the nonlocal response function, and $R(x) = \delta(x)$ in a local Kerr medium. The actual form of the nonlocal response is determined by the details of the physical process responsible for the nonlocality, and the response function may be Gaussian-shaped response and exponential-decay response, such as $R(x) =$

$(2\sigma)^{-1}\exp(-|x|/\sigma)$ originating from a Lorentzian in the Fourier domain. It is well known that the nonlinear contribution to refractive index $n(x)$ can be given by

$$n(x) = \int R(x-x')I(x'),$$

$$n(x) - \rho\frac{\partial^2}{\partial x^2}n(x) = |u_1|^2 + |u_2|^2, \quad (2)$$

where $\rho(\geqslant 0)$ is the nonlocality degree, which governs the diffusion strength of the refractive index in the nonlocal nonlinear media. The weak nonlocality can be calculated through following expansion of the nonlinear refractive index

$$n(x) \approx |u_1|^2 + |u_2|^2 + \rho\frac{\partial^2}{\partial x^2}(|u_1|^2 + |u_2|^2). \quad (3)$$

Substituting Eq. (3) into Eq. (1) leads to the following equations:

$$j\frac{\partial u_1}{\partial Z} + \frac{1}{2}\frac{\partial^2 u_1}{\partial x^2} - \left[|u_1|^2 + |u_2|^2 + \rho\frac{\partial^2}{\partial x^2}(|u_1|^2 + |u_2|^2)\right]u_1 = 0,$$

$$j\frac{\partial u_2}{\partial Z} + \frac{1}{2}\frac{\partial^2 u_2}{\partial x^2} - \left[|u_1|^2 + |u_2|^2 + \rho\frac{\partial^2}{\partial x^2}(|u_1|^2 + |u_2|^2)\right]u_2 = 0. \quad (4)$$

The effects of the weak nonlocality on the vector dark solitons can be investigated by variation approach based on the renormalized integrals of motion. To adopt Lagrangian variational approach, the dark soliton wavefunction $u_i(i=1,2)$ is rescaled $u_i(i=1,2) \rightarrow u_i \exp(2jz)$ to remove the background wave, and the renormalized equations can be expressed as

$$j\frac{\partial u_1}{\partial z} + \frac{1}{2}\frac{\partial^2 u_1}{\partial x^2} - [(|u_1|^2 - 1) + (|u_2|^2 - 1)]$$

$$+ \rho\frac{\partial^2}{\partial x^2}(|u_1|^2 + |u_2|^2)]u_1 = 0,$$

$$j\frac{\partial u_2}{\partial z} + \frac{1}{2}\frac{\partial^2 u_2}{\partial x^2} - [(|u_1|^2 - 1) + (|u_2|^2 - 1)]$$

$$+ \rho\frac{\partial^2}{\partial x^2}(|u_1|^2 + |u_2|^2)]u_2 = 0. \quad (5)$$

We adopt trial dark soliton wave-functions below as the solutions to Eq. (4)

$$u_1 = \mathrm{j}\cos\theta + \sin\theta \tanh\left[\sin\theta\left(x+\frac{\Delta}{2}\right)\right],$$

$$u_2 = \mathrm{j}\cos\theta - \sin\theta \tanh\left[\sin\theta\left(x-\frac{\Delta}{2}\right)\right], \qquad (6)$$

where $\Delta(z)$ is the spatial interval between two vector solitons; $\theta(z)$ is the distributing angle; $\cos\theta$ is the effective soliton speed; $\sin\theta$ is the dark soliton depth. These parameters above are the functions of the distance z.

We can obtain the averaged Lagrangian by Eqs. (4), (5), and (6) based on the renormalized integrals of motion, and

$$\overline{L}(z) = \int_{-\infty}^{\infty} \left\{ \sum_{i=1}^{2} \left[\frac{\mathrm{j}}{2}\left(u_i^*\frac{\partial u_i}{\partial z} - u_i\frac{\partial u_i^*}{\partial z}\right)\left(1-\frac{1}{|u_i|^2}\right) \right. \right.$$
$$\left. + \frac{1}{2}\left|\frac{\partial u_i}{\partial x}\right|^2 + \frac{1}{2}(|u_i|^2-1)^2 - \rho\left(\frac{\partial |u_i|^2}{\partial x}\right)^2 \right]$$
$$\left. - \rho\left(\frac{\partial |u_1|^2}{\partial x} \times \frac{\partial |u_2|^2}{\partial x}\right) + (|u_1|^2-1)(|u_2|^2-1) \right\},$$

$$\mathrm{d}x = \frac{\mathrm{d}\Delta}{\mathrm{d}z}(\sin 2\theta - 2\theta) + \frac{4\sin^3\theta}{3} - \frac{8\rho}{15}\sin^5\theta$$

$$[1 + \exp(-\Delta^2\sin^2\theta)] + \frac{4\sin^3\theta}{3}\exp(-\Delta^2\sin^2\theta). \qquad (7)$$

The motion equations for the distributing angle and the interval are obtained from the averaged Lagrangian by using $\mathrm{d}\overline{L}(z)/\mathrm{d}\sigma - \mathrm{d}[\mathrm{d}\overline{L}(z)/\mathrm{d}(\mathrm{d}\sigma/\mathrm{d}z)]/\mathrm{d}z = 0$ $(\sigma = \theta, \Delta)$

$$\frac{\mathrm{d}\theta}{\mathrm{d}z} = -\frac{4\rho\Delta}{15}\sin^5\theta\exp(-\Delta^2\sin^2\theta)$$
$$+ \frac{2\Delta\sin^3\theta}{3}\exp(-\Delta^2\sin^2\theta),$$

$$\frac{\mathrm{d}\Delta}{\mathrm{d}z} = \cos\theta - \frac{2\rho}{3}\sin^2\theta\cos\theta$$
$$- \left(\frac{2\rho}{3}\sin^2\theta\cos\theta - \frac{2\rho\Delta^2}{3}\sin^4\theta\sin 2\theta\right)$$
$$\cdot \exp(-\Delta^2\sin^2\theta) + \left(\cos\theta - \frac{\Delta^2}{3}\sin^2\theta\sin 2\theta\right)$$
$$\cdot \exp(-\Delta^2\sin^2\theta). \qquad (8)$$

From the motion Eq. (8), we can see the motion equation of the interval is $\frac{\mathrm{d}\Delta}{\mathrm{d}z} = 2\cos\theta$ for $\Delta \to 0$ when there are no the interaction and the nonlocality ($\rho = 0$). The special case coincides with the general vector dark solitons under the framework of Eq. (4) without the perturbations (the nonlocality and the interaction)[7]. The distance coordinate derivative for the interval is modulated by the nonlocality and the interaction.

By setting the distance coordinate derivatives in Eq. (8) to zero, and the stationary states are

$$\theta_{s1} = \frac{\pi}{2} \quad \text{and} \quad \Delta_{s1} = 0, \qquad (9)$$

$$\theta_{s2} = \arctan\left(\frac{3}{4\rho}\right)^{1/2} \quad \text{and} \quad \Delta_{s2} = 0. \qquad (10)$$

Equation (9) corresponds to $u_1(z,x) = \tanh(x)$ and $u_2(z,x) = -\tanh(x)$, namely there are two black solitons of inverse phase and the same maximum depth. The feature shows that the stationary state may be two states when two solitons are black solitons[7].

Equation (10) corresponds to $u_1 = \mathrm{j}\cos\theta_{s2} + \sin\theta_{s2}\tanh(\sin\theta_{s2}\tau)$ and $u_2 = \mathrm{j}\cos\theta_{s2} - \sin\theta_{s2}\tanh(\sin\theta_{s2}\tau)$, and this situation shows the vector soliton has the other stationary states, whose intensity distribution relates to the nonlocality degree. The feature shows the existence of stable dark soliton solutions for the weak nonlocality, and Eq. (10) becomes Eq. (9) for the nonlocality degree of $\rho \to 0$. The depth for the dark solitons become shallow as the nonlocality degree becomes large in the stationary state. The very shallow dark solitons easily change their structure while propagating in the nonlinear media, and are easily submerged into the background wave.

Equation (10) is stable against perturbations by performing a standard linear stability analysis[8,9] based on Eqs. (8) and (10). Figure 1 is the intensity of the vector dark soliton in Eq. (10) versus the transverse coordinate x for the different nonlocality degrees of $\rho = 0.05$, 0.10, and 0.20, we can see that the soliton intensity distribution of the stationary state depends on the nonlocality, and the depth for the dark solitons in the stationary state becomes shallow as the nonlocality degree becomes large. It is known that larger values of the depth for the dark solitons are physically more interesting because the very deep dark solitons are easily distinguishable from the background wave, and hardly submerged by the background wave in their propagation and interaction. The intensity transfer between two vector solitons easily occurs for the dark solitons with small values of the depth. The features show that the nonlocality may affect propagation stability of the shallow dark solitons in their propagation and interaction, and enhance the intensity transfer between two-component vector solitons.

Equation (8) reduce to the very simple form by linearizing the motion equations in Δ only around the second stationary state of $\theta_{s2} = \arctan(\frac{3}{4\rho})^{1/2}$, and

$$\frac{\mathrm{d}^2\Delta}{\mathrm{d}z^2} = \left(2\sin\theta_{s2} - \frac{4\rho}{3}\sin^3\theta_{s2} + \frac{8\rho}{3}\sin\theta_{s2}\cos^2\theta_{s2}\right)$$
$$\times \left(\frac{4\rho\Delta\sin^3\theta_{s2}}{15} - \frac{2\Delta\sin\theta_{s2}}{3}\right)\exp(-\Delta^2\sin^2\theta_{s2}). \qquad (11)$$

The effective potential corresponding to the spatial interval can be obtained by the formulae $\frac{\mathrm{d}^2\Delta}{\mathrm{d}z^2} = -\frac{\partial V_{\mathrm{eff}}}{\partial\Delta}$, and

$$V_{\mathrm{eff}}(\Delta) = 2\left(\sin\theta_{s2} - \frac{2\rho}{3}\sin^3\theta_{s2} + \frac{4\rho}{3}\sin\theta_{s2}\cos^2\theta_{s2}\right)$$
$$\times \left(\frac{2\rho\sin^3\theta_{s2}}{15} - \frac{\sin\theta_{s2}}{3}\right)\exp(-\Delta^2\sin^2\theta_{s2}). \qquad (12)$$

This suggests a mechanical analogy in which the dark solitons as particles of the spatial coordinate (interval) Δ move in the effective potential. Figure 2 shows the effective potential versus the interval for the different nonlocality degrees of $\rho = 0.05$, 0.10, and 0.20, we see that the effective potentials $V_{\mathrm{eff}}(\Delta)$ are parabolic curves

with the bottom $\Delta = 0$, where the particles can oscillate periodically around $\Delta = 0$. The depth of the effective potential depends on the nonlocality degree, and the potential is shallow as the nonlocality degree is large. In the case, the particles oscillate in very large range around $\Delta = 0$, and the nonlocality enhances the fluctuation of the interval, and the particles become instable because of the margin decrease of the effective potential when the nonlocality degree becomes large. The feature means that the vector dark solitons become instability in the presence of the weak nonlocality. In nature, the nonlocal nonlinearity may affect the vector dark solitons against the modulational instability due to reduction of the effective nonlinearity, and enhance the intensity transfer between two vector solitons which may be explained by the effective potential.

We can perform a series of direct numerical simulations for the nonlinear Schrödinger equations (4) to discuss the effects of the weak nonlocality on the propagation and the interaction of the two dark solitons in the nonlocal nonlinear media. Figure 3 is the normalized soliton intensity versus the propagation distance with the initial interval $\Delta(z = 0) = 0$. The incident dark soliton pulses are $u_1(x, z = 0) = \tanh(x)$ and $u_2(x, z = 0) = -\tanh(x)$, where the initial distributing angle is $\theta(z = 0) = \pi/2$. We can see that the nonlocality plays an important role in the evolution of each dark soliton and interaction between the two vector dark solitons. For example, the nonlocality reduces the stability of the dark soliton if the nonlocality is large enough, and enhances the interaction. The intensity can be transferred from one soliton to the other at the domain wall if the dark solitons are shallow. Meantime the effective soliton speed $(\cos\theta)$ is large
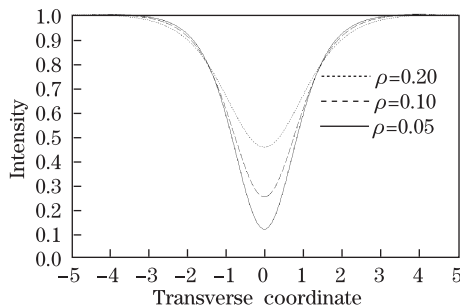

Fig. 1. Intensity of dark solitons in Eq. (10) versus the transverse coordinate for the different nonlocality degrees.
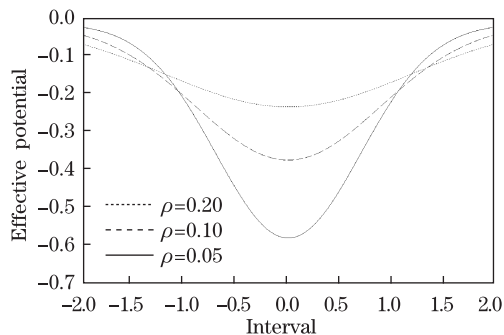

Fig. 2. Effective potential versus the interval for the different nonlocality degrees.
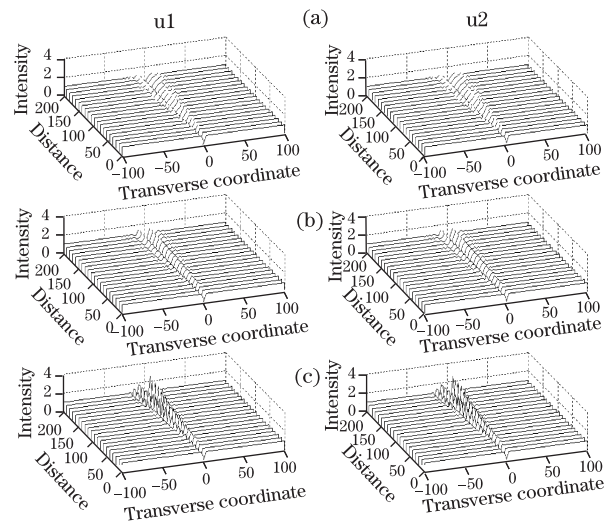

Fig. 3. Normalized dark soliton intensity versus the propagation distance for different nonlocality degrees. (a) $\rho = 0.05$; (b) $\rho = 0.10$; (c) $\rho = 0.20$.

and the soliton depth $(\sin\theta)$ is small, and its stability is reduced. The effecting reason is that the nonlocality enhances the fluctuation of the interval, which fluctuates the linear wave with large amplitude. Subsequently the fluctuated linear wave correlates the two dark solitons by their sideband, then enhances their interaction and reduces the stability of the vector dark solitons.

In conclusion, the dynamics of the two-component vector dark solitons are investigated by the variational approach in nonlocal nonlinear media, and effects of the weak nonlocality on the soliton propagation and interaction between the two vector dark solitons are discussed. The effective potential is obtained by performing a linear analysis, and the mechanical analogy is discussed that the dark solitons as the particles of the spatial coordinate (interval) move in the effective potential. The nonlocality affects the propagation stability of the two-component vector dark solitons if the nonlocality degree is large enough, and enhances the interaction. Finally, the dynamics of the two-component vector dark solitons are investigated by the numerical simulations, and the numerical results confirm these theoretical findings.

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