Supermode analysis of seven-core photonic quasi-crystal fibers

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We design custom-shaped modes for a sixfold symmetric photonic quasi-crystal fiber (PQF), an optical fiber with a sixfold symmetric quasi-periodic array of air holes in the cladding region. The supermodes of the PQF are calculated by the finite element method, and the coupling of an in-phase supermode for the quasi-periodic optical fiber is numerically optimized to obtain identical values. The optimization is guaranteed by the selection of appropriate PQF design parameters. The eigenvalue equation associated with a seven-core PQF is derived from a coupled mode equation. We realize mode shaping and provide the far-field distribution mode of PQFs. The results are beneficial for the structural design and uniform distribution of the in-phase supermodes of PQFs.

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Optical fibers are the backbone of modern telecommunication networks, prompting researchers to conduct significant studies on the design and performance of these innovations. Novel types of fibers push forward the frontiers of optical science and technologies[1]. Multicore fibers (MCFs) have attracted considerable attention because of their potential applications in optical switching, mode locking, phase-locked high-power lasers[2,3], and other related fields[4]. Scholars have extensively explored photonic crystal fibers (PCFs) to elucidate and exploit their wide range of potential benefits[5−8]. Specifically designed PCFs support the single-mode guidance of light beams, thereby substantially increasing the output power of fiber–laser systems[9].

Multicore PCFs (MCPCFs) are used to modify supermode shapes through the adjustment of the diameter of air holes among guiding cores. Mafi *et al* indicate that mode shaping in MCPCFs is intended to obtain roughly equal field amplitudes in all active cores by fiber engineering[10]. The strength of core-to-core coupling is controlled by varying air hole diameter. Several promising PCF design strategies for a spatially flat in-phase supermode in MCFs have been proposed. For instance, a principle for constructing custom-shaped modes was formulated on the basis of the hexagonal lattice of air holes in a hybrid PCF structure, where the cores were separated by high-index solid rods and microstructure cladding[11]. For a square lattice MCPCF, in-phase supermode shapes that had the same amplitude in each core were obtained[12]. For the two structures of MCPCFs, the core array is axially symmetric and distributed in a ring pattern. In the two symmetric structures, even though the wavelength or air filling fraction varies, the relative field intensity remains almost unchanged even without varying the diameters of air holes among the cores.

Studies on the shaped-mode properties of photonic quasi-crystal fibers (PQFs) with a quasi-periodic array of air holes in the cladding region are limited[13]. Quasi-periodic fibers cause unusual phenomena and properties that contrast with those of their periodic counterparts. In quasi-crystals fibers, quasi-periodic structures are constructed according to long-range sequences, with no periodicity applied[14]. The air holes in PQFs can be flexibly adjusted, and supermode shape is sensitive to air hole size. The air holes in the outer cladding decrease the average refractive index of the cladding region and confine light to the silica cores, thereby decreasing confinement loss.

We consider the index-guiding sixfold symmetric PQF and propose a simple procedure for modifying and customizing mode shapes for multicore photonic quasi-crystal fibers (MCPQFs). Theoretical work based on coupled mode theory and the finite element method (FEM) is presented. We study a seven-core PQF and evaluate core-to-core coupling strength as a function of air hole diameter. The procedure for constructing custom-shaped modes for the seven-core PQF is also presented.

The cross-section of a sixfold symmetric PQF is shown in Fig. 1. The refractive index of silica is $n = 1.45$, and the diameters of the double cladding air holes are $d_1 = 1$ μm and $d_2 = 1.8$ μm, respectively. The air holes, which run along the fiber, are located on a hybrid triangular and square lattice with a pitch $\Lambda = 2$ μm. Six air holes in the inner cladding are removed to form the seven guiding cores. Quasi-periodic fibers can be designed as being on endless single mode for all wavelengths when d/Λ is less than 0.525[13]. Coupling strength directly affects mode shape, and can be adjusted by varying the diameters of air holes among guiding cores.

The general case of a MCPCF with N cores labeled by index m is considered; ψ_i is the single mode guided in the ith core without other cores. The supermode ψ of the MCPCF can be approximated as a weighted superposition of the individual modes:

$$\psi(x, y, z) = \sum_m A_m(z)\psi_m(x, y), m = 1, 2, \cdots N. \quad (1)$$

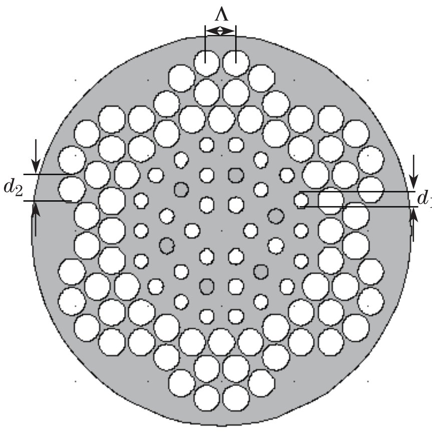

Fig. 1. Cross-section of the seven-core hybrid PQF.

We define a propagating supermode as a field vector

$$\psi(z) = \psi(0)\exp(\mathrm{i}\beta z), \tag{2}$$

where β is the propagation constant of ψ in the z direction. According to coupled mode theory, we obtain

$$\widetilde{C}\,\psi(z) = \mathrm{i}\beta\psi, \tag{3}$$

where $\widetilde{C}$ is a $N \times N$ matrix that describes the mode field coupling for each core[10]. $\widetilde{C}$ can be written as

$$\widetilde{C} = \begin{bmatrix} \mathrm{i}s_1 & \mathrm{i}k_{12} & \mathrm{i}k_{13} & \mathrm{i}k_{14} & \mathrm{i}k_{15} & \mathrm{i}k_{16} & \mathrm{i}k_{17} \\ \mathrm{i}k_{21} & \mathrm{i}s_2 & \mathrm{i}k_{23} & & & & \\ \mathrm{i}k_{31} & \mathrm{i}k_{32} & \mathrm{i}s_3 & \mathrm{i}k_{34} & & & \\ \mathrm{i}k_{41} & & \mathrm{i}k_{43} & \mathrm{i}s_4 & \mathrm{i}k_{45} & & \\ \mathrm{i}k_{51} & & & \mathrm{i}k_{54} & \mathrm{i}s_5 & \mathrm{i}k_{56} & \\ \mathrm{i}k_{61} & & & & \mathrm{i}k_{65} & \mathrm{i}s_6 & \mathrm{i}k_{67} \\ \mathrm{i}k_{71} & \mathrm{i}k_{72} & & & & \mathrm{i}k_{76} & \mathrm{i}s_7 \end{bmatrix}, \tag{4}$$

where $s_m(m = 1, 2, \cdots 7)$ is the propagation constant of the mth individual core, and $k_{mn}(n = 1, 2, \cdots 7)$ is the coefficient of coupling between the mth and nth cores. Eigenvector $\psi(z)$ denotes the modal profile and eigenvalue β stands for the corresponding propagation constant[11].

The eigenvector that corresponds to the in-phase mode is $[E_1, 1, 1, 1, 1, 1, 1]$. We use FEM to calculate the in-phase supermode distributions at different frequencies and the ratios of the inner air holes to pitch. The relative electric field distribution of core-to-core coupling is shown in this letter. Under such distribution, the seven-core PQF is adjusted by varying the size of air holes among guiding cores, enabling us to obtain identical modes in the seven cores. Thus, the eigenvector of the in-phase supermode is expressed as [1,1,1,1,1,1,1].

For the proposed PQF structure, the white circles denote air holes and the shadowed ones denote the silica cores. In the seven-core PQF, the modes in individual cores affect one another. The optical fields that propagate in these cores are coupled, thereby resulting in the formation of supermodes. If each core is of single-mode and neighboring cores are optically coupled, then the total number of nondegenerate supermodes equals the number of cores. Each supermode has its own field distribution and optical properties, but only the in-phase supermode in Fig. 2 has a Gaussian-shaped far-field distribution (also see Fig. 3). The far-field beam profile is provided by the in-phase supermode of the seven-core PQF, and is calculated by FDTD.

We use FEM to design custom-shaped modes for MCPQFs. Obtaining a flat in-phase supermode necessitates appropriately adjusting the coefficients of coupling among various cores. Supermode shape is sensitive to air

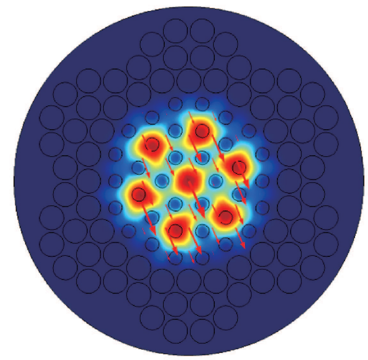

Fig. 2. Electric field distribution of the in-phase supermode of the PQF simulated by FEM.

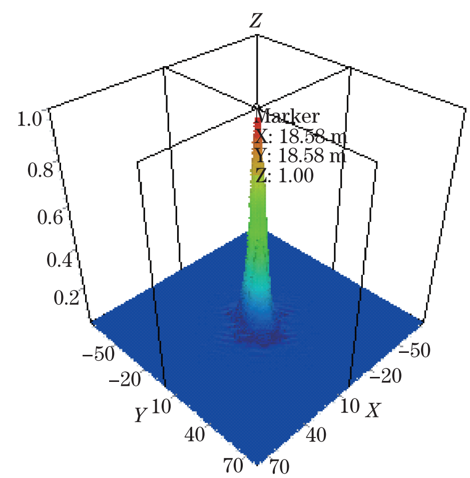

Fig. 3. Far-field beam profile provided by the in-phase supermode of the seven-core PQF.

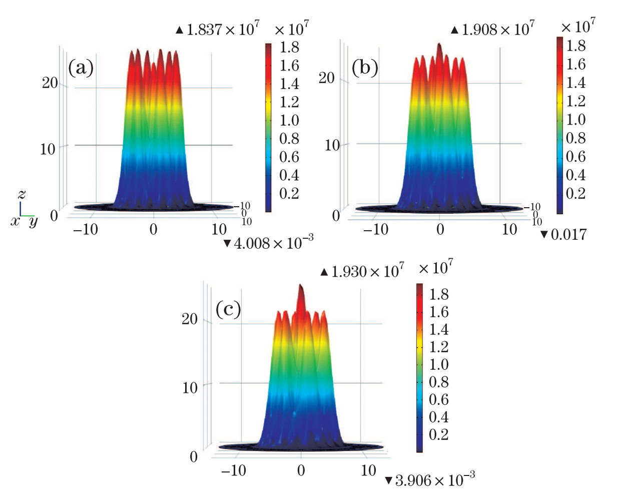

Fig. 4. In-phase supermodes at different frequencies with $\Lambda = 2$ μm, $d_1/\Lambda = 0.5$, $d_2/\Lambda = 0.9$ under the operating wavelengths of (a) 1.3, (b) 1.55, and (c) 1.8 μm, respectively.

hole size, and FEM is used to qualitatively analyze how relative coupling strength affects the relative electric field strength among the seven cores. Figure 4 shows the in-phase supermodes at different frequencies for the FEM calculations with $\Lambda = 2\ \mu$m, $d_1/\Lambda = 0.5$, and $d_2/\Lambda = 0.9$. In Fig. 4, the operating wavelengths are 1.3, 1.55, and 1.8 μm, respectively.

The intensity profiles of the in-phase supermode of the central core increases with increasing wavelength, whereas the intensity profiles of the in-phase supermodes of the other six cores vary in the opposite direction. The absolute values of the amplitude difference between the central and the other six cores are non-identical–a result that differs from those observed in orthohexagonal or square structures. In a hexagonal or square lattice, the relative field intensity among the seven cores imperceptibly changes with wavelength.

The supermodes in lasers compete with one another by sharing population inversion; thus, a necessary requirement is to choose appropriate structural parameters in PQFs to suppress other modes and build one in-phase mode. For a PQF characterized by a quasi-periodic arrangement of air holes, the ratio of air hole diameter $d$to pitch Λ must be smaller than $\sim$0.525. The satisfaction of this requirement guarantees single-mode operation for all the wavelengths in one missing air hole core. Previously reported d/Λvalues are 0.406 and 0.442, which were applied as the cutoff ratios for a triangular and square lattice PCF, respectively[13]. The field distribution of mode depends on the air hole diameters and pitch for a given operating wavelength.

Figure 5 illustrates the intensity profiles of the in-phase supermodes with different inner air hole sizes at λ = 1 550 nm. Meanwhile, the air hole pitch is fixed at Λ =2 μm, the large outer diameter is 1.8 μm, and the inner diameters are 0.6, 0.8, and 1.0 μm, respectively. In single-mode operation, increasing the inner air hole diameters reduces the discrepancy in amplitudes between the central core and the other six cores. The difference in field intensity across the cores is the minimum (Fig. 5(c)), and no mode leak from the inner to the outer cladding occurs. The outer cladding mode of the PQF is more tightly confined because of the circular quasi-periodic air hole arrangement in the large outer cladding. In the square lattice PCF, the field can expand further into the outer cladding. The relative field intensity moderately decreases in both the hexagonal and square lattice PCFs with varying d/Λ.

In both the hexagonal and square lattice PCFs, the central core is identical to the other cores with fixed air holes among them, and the relative field intensity between the central core and the other six cores is almost independent of the coupling strengths among the cores. Under this premise for the quasi-periodic structure, however, the central core differs from the other cores because of the quasi-periodic air hole arrays. The relative field intensity between the central core and the other six cores also decreases with changing strength of coupling among the cores. These results indicate that such a PQF is preferable for mode shaping.

The manner by which the quasi-periodic arrays of air holes among the seven cores affect the relative field intensity among the cores should be quantitatively analyzed. The inner cladding consists of three layers of air holes with diameter d_1. The diameter of the outer air hole is d_2= 1.8 μm. The diameters of the air holes in the inner cladding for the first, second, and third layers are denoted by d_{11}, d_{12}, andd_{13}, respectively. FEM is extensively used to calculate the supermodes in the PQF with fixed parameters, where Λ =2 μm and $d_2/\Lambda = 0.9$.

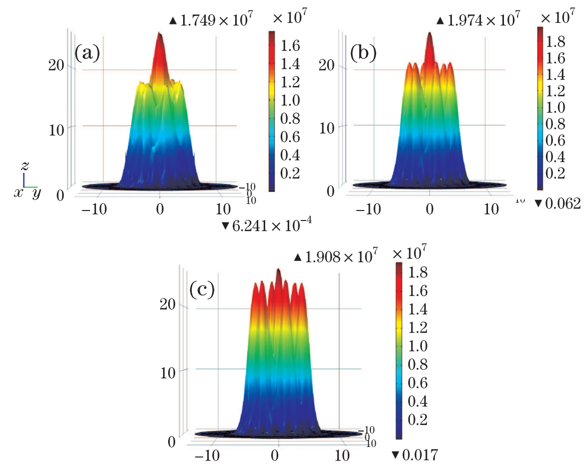

Fig. 5. In-phase supermodes of the seven-core PQF with different inner air holes of (a) 0.6 , (b) 0.8 , and (c) 1.0 μm, respectively.

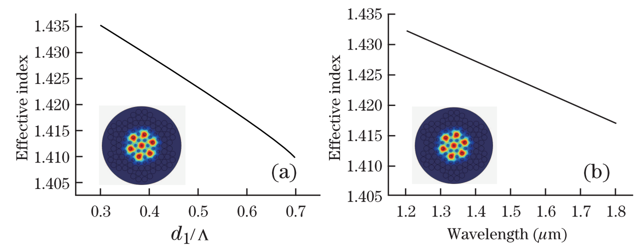

Fig. 6. Effective index of fundamental in-phase supermode as a function of (a) d_1/Λ at λ = 1 550 nm and (b) wavelength with d_1 = 1 μm, and d_2= 1.8 μm. Insets: Distribution of fundamental in-phase mode intensity.

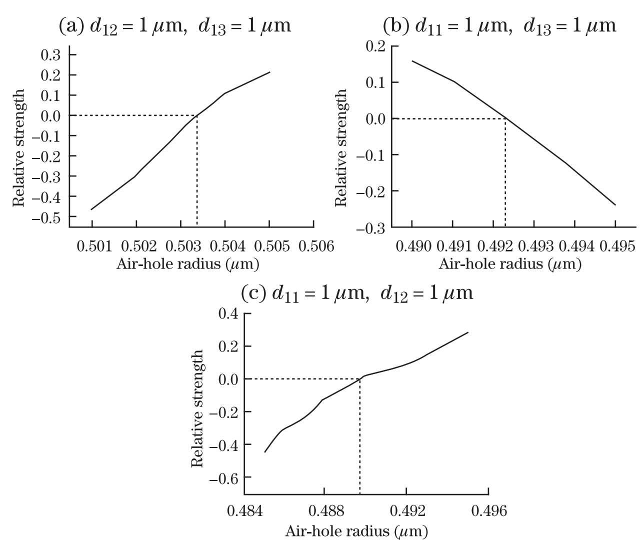

Fig. 7. Relative field strength of the central core compared with the changes in the field strengths of the six cores with the air hole diameter of three layers in the inner cladding. The short dotted lines represent the radius of the air holes under the same strengths of the seven cores.

Figure 6(a) presents the effective index of the fundamental in-phase mode as a function of d_1/Λ at a wavelength of $\lambda = 1\,550$ nm. Figure 6(b) illustrates the effective indexes at different wavelengths with $d_1/\Lambda = 0.5$. The intensity profiles of the in-phase supermodes are shown in the insets in Figs 6(a) and (b). The effective index of the fundamental in-phase supermode decreases with increasing d_1/Λ and wavelength value. The strength of coupling among the seven cores is controlled by varying the diameters of the air holes. For an in-phase supermode, the mode field of the seven cores can be adjusted to the same amplitude by optimizing the mode, which depends strongly on the air hole size of the inner cladding.

As previously noted, for our designed structure, the amplitudes of the central core and the other six cores differ to a relatively small extent; thus, air hole size does not require significant alteration. The relative field intensity of the core-to-core coupling in the seven-core PQF is plotted against the inner air hole radius. We investigate the dependence of the relative field intensity distribution of the in-phase supermodes on the diameters of inner air holes d_{11}, d_{12}, and d_{13}. Figure 7 shows the dependence of the relative strength of mode field on the radius of inner air holes. For a given parameter $d_1 = 1\ \mu$m, the optimum values of d_{11}, d_{12}, and d_{13} can be obtained by calculating the field intensity distribution of the in-phase supermode.

The FEM simulations with the adjusted diameters that correspond to a particular solution are depicted in figures. We test the solutions with different diameters of d_{11}, d_{12}, and d_{13}, which result in a uniformly distributed mode. We can increase d_{11} or decrease d_{12} and d_{13} while keeping other parameters constant to obtain in-phase supermodes with the same amplitude. The mode field distribution in the seven-core PQF is highly sensitive to d_{11}, making it difficult to control. The decrease in d_{12} and d_{13} can contribute to enlarging the effective mode area of the MCPQF and core-to-core coupling strength. Thus, if d_{12} or d_{13} is reduced, a more suitable result for fiber design can be achieved; operation will remain in endless single mode.

The variations in air hole size for the inner cladding modify core-to-core coupling strength. The relative field intensity of core-to-core coupling strength exhibits monotonicity, and the uniformly distributed modes can be easily achieved by modifying the diameter of the air holes in any layer of the inner cladding; i.e., d_{11}, d_{12}, or d_{13}. In addition, the large outer diameter causes the fiber to stiffen and reduces coupling loss because the core mode is tightly confined, thereby avoiding penetration into cladding layers. These features are beneficial to fiber design and fabrication.

In conclusion, MCFs with uniformly distributed modes are particularly useful in applications involving phase-locked MCPCF lasers, in which each of the cores equally contributes to output power. We propose a novel seven-core PQF with sixfold symmetry, and develop a method for designing custom-shaped modes for MCPQFs. Coupled mode theory and FEM enable the uniform optimization of distributed modes by adjusting the air hole radius in the seven-core PQF. The seven-core PQF is analyzed in detail, and the results are applicable to high-power multicore fiber lasers and amplifiers.

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