Optical bistability and multistability via amplitude and phase control in a Duplicated two-level system

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We investigate the optical bistability (OB) in a duplicated two-level system contained in a ring cavity. The atoms are driven by two orthogonally polarized fields with a relative phase. The OB behavior of such a system can be controlled by the amplitude and the relative phase of the coupling field, and it is possible to switch between bistability and multistability by adjusting the relative phase.

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It is well known that quantum coherence and interference can give rise to some interesting phenomena, such as electromagnetically induced transparency (EIT)[1], slow light[2], giant Kerr nonlinearity[3,4], four wave mixing[5], and so on. In the past few decades, optical bistability (OB) has been extensively studied both experimentally and theoretically[6−8]. The role of atomic coherence in OB has been investigated, and it is found that the bistable hysteresis cycle becomes smaller as the Rabi frequency of the control field increases[9]. Different mechanisms to realize OB utilizing atomic coherence have been reported, for example, via the initial coherence of atoms[10], via spontaneous emission induced coherence[11−13], via the phase and amplitude of the driven field[14], via a squeezed vacuum input[15,16], or via multi-Raman-channel interference[17], etc. Double-cavity optical bistability of a three-level ladder system[18] has been reported lately. Experimentally, Joshi *et al.* have demonstrated that the enhanced nonlinearity induced by atomic coherence effects in Λ-type atomic systems can produce the optical bistability and multistability, moreover, the shape and direction of the hysteresis loop can be controlled by the parameters of the coupling field[19,20]. Recently, optical bistability in many other quantum systems has been investigated as well, such as in photonic crystal (PC) nanocavities[21], in PT-symmetric periodic structures[22], in photonic-crystal one-atom laser[23], in a hybrid metal-semiconductor nanodimer[24], in a doubly resonant $\chi^{(2)}$-nonlinear plasmonic nanocavity[25], etc.

Optical bistability has been wildly investigated due to its significant applications in photon control devices[6]. Xiao and coworkers have realized optical switching between two steady states of optical bistability generated in a system with three-level atoms inside an optical cavity[26]. Later, Chang *et al.* reported all-optical flip-flop and storage of optical pulse signals with a low peak power of several tens of microwatts[27]. Optical switch between different delays by exploiting the optical bistability of molecular aggregates arranged in nanofilms is investiged[28]. Very recently, it was reported that the enhancement of "logical" responses by noise in a bistable optical system[29]. What is more, in a bistable cavity polariton system[30], pulsed acoustic excitation can lead to ultrafast switching of the optic response.

A duplicated atomic system has attracted considerable attention recently due to its unique quantum properties. Bouchene and coworkers[31−33] investigated the coherent control of the medium gain for the probe pulse and the effective susceptibility, as well as slow light caused by coherent Zeeman oscillations. A scheme was proposed to double the precision of a two-beam interferometer, where the direct detection of the beat signal is replaced with the monitoring of the fluorescence of a twofold degenerate atomic system resonant with the laser[34]. The propagation effect of elliptical polarized short pulses in such kind of atomic medium was investigated too[35]. Recently, some of us proposed a scheme to control the spatial interference of resonance fluorescence from two duplicated two-level atoms via the relative phase of two orthogonally polarized fields[36].

In this letter, we propose a method of amplitude and phase control of optical bistability in a duplicated two-level system. We find that it is possible to switch between bistability and multistability by adjusting the relative phase between the probe field and coupling field. The relative phase, when increased in some range, may increase the width of bistable hysteresis loop; when the coupling field amplitude is increased, the OB threshold could be reduced, however the width of the hysteresis cycle would decrease. OB can be optimized by properly choosing the relative phase and the amplitude.

The atoms used here are modeled as duplicated two-level atoms (see Fig. 1(a)). The system could be realized in ^6Li atom. The $F = 1/2 \leftrightarrow F = 1/2$ transitions (energy $\hbar\omega_0$) are excited by two orthogonally polarized fields of the same frequency ω. The two lower (upper) states $\{|1\rangle, |2\rangle\}$ ($\{|3\rangle, |4\rangle\}$) with energies $E_1 = E_2$ ($E_3 = E_4$) correspond to the degenerate states of the level $^2S_{1/2}F = 1/2$ ($^2P_{1/2}F = 1/2$) with $m_F = \pm 1/2$. A $\pi-$polarized field is applied to couple the transitions with identical m_F (i.e., $|1\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |4\rangle$), and a $\sigma-$polarized field is applied to couple the transitions with different m_F (i.e., $|2\rangle \leftrightarrow |3\rangle$ and $|1\rangle \leftrightarrow |4\rangle$). We assume that both excited states have the same decay rate γ to the each lower level. In this situation, a closed-loop system is formed, and we define ϕ is the relative phase between these two driving fields. To investigate the optical bistability, we put the ensemble of duplicated two-level

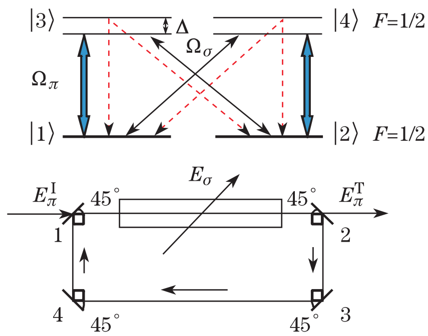

Fig. 1. (Color online) (a) Energy level structure for consideration. (b) Unidirectional ring cavity with a atomic sample of length L. Mirrors 3 and 4 have 100% reflectivity and mirrors 1 and 2 have the intensity reflection and transmission coefficient R and T $(R+T=1)$, respectively. $E_\pi^{\rm I}$ and $E_\pi^{\rm T}$ are the incident and the transmitted $\pi-$polarized fields, respectively. The coupling field E_σ dose not circulate in the cavity.

atoms in a unidirectional ring cavity (shown in Fig. 1(b)). Mirrors 3 and 4 have 100% reflectivity and mirrors 1 and 2 have the intensity reflection and transmission coefficient R and T $(R+T=1)$, respectively. We consider that only the $\pi-$polarized field is the probe field and circulates in the cavity, and the coupling field E_σ is the coupling field and dose not circulate in the cavity.

In the interaction picture, the Hamiltonian of the system in an appropriate rotating frame can be written as

$$\mathrm{H}=\hbar\begin{pmatrix} 0 & 0 & \Omega_\pi & -\Omega_\sigma \mathrm{e}^{-\mathrm{i}\phi} \\ 0 & 0 & -\Omega_\sigma^* \mathrm{e}^{-\mathrm{i}\phi} & -\Omega_\pi \\ \Omega_\pi & -\Omega_\sigma \mathrm{e}^{\mathrm{i}\phi} & \Delta & 0 \\ -\Omega_\sigma^* \mathrm{e}^{\mathrm{i}\phi} & -\Omega_\pi & 0 & \Delta \end{pmatrix}, \tag{1}$$

where $\Delta=\omega_0-\omega$ is the detuning, and the Rabi frequencies are defined as $\Omega_\pi=\mu E_\pi/2\hbar$ and $\Omega_\sigma=\mu E_\sigma/2\hbar$ (μ is the dipole moment). The dynamics of the system can be described using density-matrix approach as

$$\dot{\rho}=-\frac{\mathrm{i}}{\hbar}[\mathrm{H},\rho]+L[\rho(t)]. \tag{2}$$

The Liouvillian matrix $L[\rho(t)]$, which describes relaxation by spontaneous decay, is given by

$$L[\rho(t)]=$$
$$\begin{pmatrix} \gamma(\rho_{33}+\rho_{44}) & 0 & -\gamma\rho_{13} & -\gamma\rho_{14} \\ 0 & \gamma(\rho_{33}+\rho_{44}) & -\gamma\rho_{23} & -\gamma\rho_{24} \\ -\gamma\rho_{31} & -\gamma\rho_{32} & -2\gamma\rho_{33} & -2\gamma\rho_{34} \\ -\gamma\rho_{41} & -\gamma\rho_{42} & -2\gamma\rho_{43} & -2\gamma\rho_{44} \end{pmatrix}. \tag{3}$$

We define the coherences $\rho_\pi=\rho_{42}-\rho_{31}$, $\rho_\sigma=\rho_{32}+\rho_{41}$ responsible for the π- and σ-polarized radiated fields, respectively. We solve the density-matrix Eq. (2) in the steady state and we have

$$\rho_\pi=\frac{(\mathrm{i}\gamma+\Delta)\Omega_\pi(\Omega_\pi^2+\Omega_\sigma^2\mathrm{e}^{2i\phi})}{2|\Omega_\pi^2+\Omega_\sigma^2\mathrm{e}^{2i\phi}|^2+(\Delta^2+\gamma^2)(\Omega_\pi^2+\Omega_\sigma^2)}, \tag{4a}$$

$$\rho_\sigma=\frac{(\mathrm{i}\gamma+\Delta)\Omega_\sigma\mathrm{e}^{-\mathrm{i}\phi}(\Omega_\pi^2+\Omega_\sigma^2\mathrm{e}^{2i\phi})}{2|\Omega_\pi^2+\Omega_\sigma^2\mathrm{e}^{2i\phi}|^2+(\Delta^2+\gamma^2)(\Omega_\pi^2+\Omega_\sigma^2)}. \tag{4b}$$

The Maxwell's equation under slowly varying envelope approximation is

$$\frac{\partial E_\pi}{\partial t}+c\frac{\partial E_\pi}{\partial z}=\mathrm{i}\frac{\omega_\pi}{2\varepsilon_0}P(\omega_\pi), \tag{5}$$

where $P(\omega_\pi)$ is the slowly oscillating term of the induced polarization in $\pi-$transition and is given by $P(\omega_\pi)=N\mu\rho_\pi$. For a perfectly tuned cavity, the boundary conditions in the steady-state are[6]

$$E_\pi(L)=E_\pi^T/\sqrt{T}, \tag{6a}$$

$$E_\pi(0)=\sqrt{T}E_\pi^I+RE_\pi(L), \tag{6b}$$

where E_π^I and E_π^T are the incident and the transmitted $\pi-$polarized field, respectively. L is the length of the atomic sample. The second term on the right-hand side of Eq. (6b) describes a feedback mechanism due to the mirror, which is essential to give rise to bistability, namely, there will be no bistability if $R=0$.

For the steady-state, in the mean-field limit[6], time derivative in Eq. (5) is equal to zero. Using the boundary conditions Eq. (6), we obtain the input-output relationship:

$$y=x-\mathrm{i}C\gamma\rho_\pi, \tag{7}$$

where $y=\mu E_\pi^I/\hbar\sqrt{T}$ and $x=\mu E_\pi^T/\hbar\sqrt{T}$ are the normalized incident and output fields, respectively; $C=LN\omega\mu^2/2\hbar c\varepsilon_0 T\gamma$ is the usual cooperation parameter. The nonlinear term on the right-hand side of Eq. (7) is indispensable to the occurrence of OB. The steady-state values of ρ_π is obtained and shown as Eq. (4a).

In the numerical calculations, all parameters are dimensionless and normalized by γ. First We assume that both applied fields are resonant with the corresponding atomic transitions, i.e., $\Delta=0$. We set the relative phase $\phi=0$, to investigate the effect of the amplitude of the coupling field (Ω_σ) on OB. From Fig. 2(a), one may find that in the resonant situation, when Ω_σ increases, the width of the hysteresis cycle decreases, i.e., it is more difficult to observe OB. In a closed-loop system, the relative phase ϕ affects the atomic dynamics (see Eqs. (1) and (4)), so it allows us to control optical properties of the medium by the relative phase. We choose the Rabi frequency of the coupling field is $\Omega_\sigma=5$ to investigate the influence of the relative phase on OB. It is shown in Fig. 2(b) that, when $\phi=0$ (black/solid curve), the transmitted light is a single value function of the input light, and there is no OB. When ϕ is increased, optical bistability appears. Our numerical calculations show that the critical value of the relative phase where OB appears is $\phi=0.27\pi$. The hysteresis cycle may be broadened considerably while ϕ is increased. When ϕ is further increased, multistability appears. The numerical calculations show that multistability exists when the relative phase $\phi\in(0.4785\pi,0.5215\pi)$. In Fig. 2(b). we chose $\phi=\pi/2$ and show optical multistability behavior (pulper/dash-dot-dot curve).

In order to get the physical origin of such effects, we now investigate the polarization of the $\pi-$polarized field. In Fig. 3, we plot the $\pi-$polarized absorption and dispersion. In the resonant situation, when $\phi=0$, from Fig. 3(a) it is found that $\mathrm{Im}(\rho_\pi)>0$, i.e., the medium is

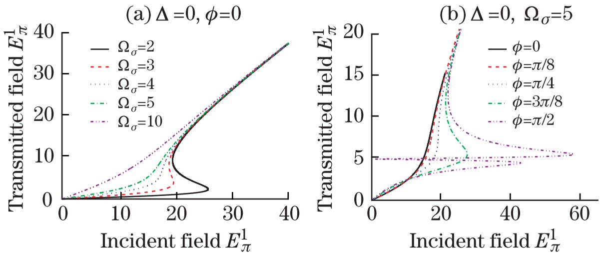

Fig. 2. (Color online) Transmitted light versus incident light in the resonant situation. (a) Relative phase $\phi = 0$; (b) $\Omega_\sigma = 5$. The other parameters are $\Delta = 0$, and $C = 200$.

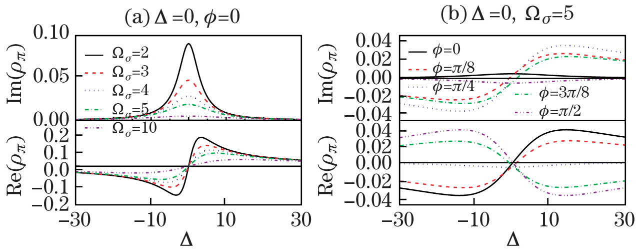

Fig. 3. (Color online) Absorption and dispersion of $\pi-$polarized field (a) when $\phi = 0$ and (b) when $\Omega_\sigma = 5$. $\Omega_\pi = 1$ and the other parameters are the same as Fig. 2.

an absorber. The increasement of Ω_σ leads to a weaker absorption and dispersion. This may cause the disappearance of OB in Fig. 2(a). The effect of the relative phase on the polarization is shown in Fig. 3(b). When $\phi = \pi/4$, an anomalous "dispersion like" absorption profile appears. When $\phi = \pi/2$, it is shown that $\text{Im}(\rho_\pi) < 0$, i.e., the medium changes into an amplifier. Thus, the OB behavior will change when the relative phase increases. When ϕ is increased from $\pi/2$ to π, the polarization will make inverse changes to that ϕ rises from 0 to $\pi/2$, as a result, the hysteresis cycles behave adversely. Therefore, one can control the OB behavior via the relative phase.

Next, we considerer the off-resonant situation (we choose $\Delta = 10$ for an example). We find that when the Rabi frequency of coupling $\sigma-$polarized field is not too strong, e.g., from Figs. 4(a) and (b), increasing ϕ from 0 to $\pi/2$ leads to wider hysteresis cycles, and increasing ϕ from $\pi/2$ to π leads to narrower hysteresis cycles; when $\phi = \pi/2$, multistability appears. When Ω_σ is strongly increased, e.g., when $\Omega_\sigma = 30$ in Fig. 4(c), OB will not happen, however multistability still appears when $\phi = \pi/2$.

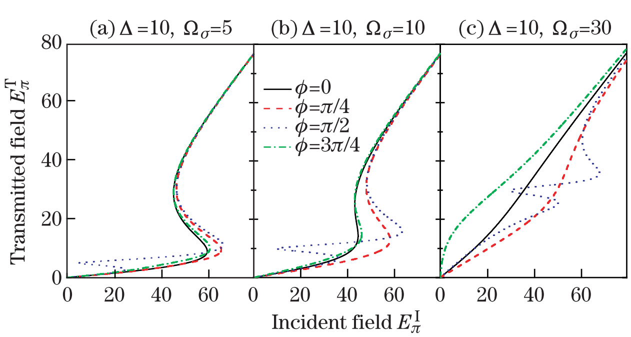

Fig. 4. (Color online) Transmitted light versus incident light in the off-resonant situation ($\Delta = 10$). (a) $\Omega_\sigma = 5$, (b) $\Omega_\sigma = 10$, and (c) $\Omega_\sigma = 30$. The other parameters are the same as Fig. 2.

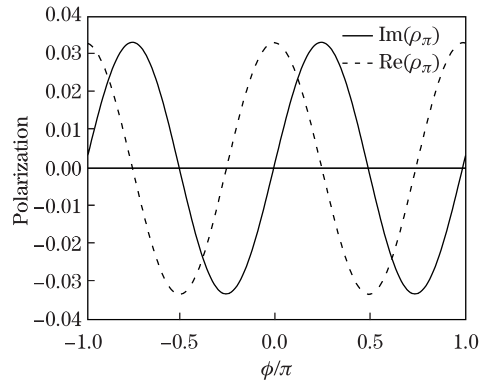

Fig. 5. $\text{Im}(\rho_\pi)$ (solid curve) and $\text{Re}(\rho_\pi)$ (dashed curve) versus the relative phase ϕ. The parameters are $\Omega_\pi = 1$, $\Omega_\sigma = 10$, and $\Delta = 10$.

If we compare the hysteresis cycles with the same relative phase from Figs. 4(a) to (c), we find that increasing the coupling field intensity leads to a lower OB threshold, however the hysteresis cycles become narrower and finally disappear. From the π-polarization, we find that OB changes from dispersive ($\phi = 0$) to absorptive ($\phi = \pi/4$) and again to dispersive ($\phi = \pi/2$), as shown in Fig. 5.

In conclusion, in a duplicated two-level system contained in a unidirectional ring cavity, we studied the optical bistability and multistability via amplitude and phase control. Increasing the relative phase ϕ from 0 to $\pi/2$ could increase the width of bistable hysteresis loop, and multistability occurs if $\phi = \pi/2$. By adjusting the relative phase, one can switch between bistability and multistability. Increasing the amplitude of the orthogonally polarized coupling field may reduce the OB threshold, however the width of the hysteresis cycle also decreases. Similar level structure can be found in quantum dots[37]. These results may be helpful for experimental studied of amplitude and phase control of optical bistability.

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