Effective medium parameters for 1D photonic crystal containing single-negative material layers using the envelope function approach

Munazza Zulfiqar Ali

Physics Department, Quaid-i-Azam Campus, Punjab University, Lahore-54590, Pakistan

Corresponding author: munazzazulfiqar@yahoo.com

Received September 4, 2012; accepted January 9, 2013; posted online March 20, 2013

Nonlinear wave propagation in a 1D photonic crystal containing single-negative layers is investigated using the multiple-scale method. In this approach, the electric field is decomposed into a slowly varying envelope function and a fast Bloch-like function to obtain the analytic expressions of the effective parameters of an equivalent medium. The periodic structure has an equivalent left-handed medium for the envelope function. Gap soliton formation is discussed and compared with that associated with the Bragg gap.

OCIS codes: 050.5298, 060.4370, 070.7345.

doi: 10.3788/COL201311.040501.

Wave propagation in photonic crystals has been an active research area for more than a decade[1−5]. Conventional photonic crystals are synthesized based on the periodic arrangement of different materials with positive electric permittivity and magnetic permeability (right-handed medium). Gaps in such photonic crystals result from Bragg reflections. Experiments on left-handed metamaterials[6] have recently revealed the possibility of fabricating unconventional photonic crystals. Left-handed materials are characterized by simultaneously negative electric permittivity and magnetic permeability[7]. These materials are also known as double negative (DNG) materials. DNG metamaterials are composites of materials in which either permittivity or permeability is negative. Such materials are referred to as single-negative (SNG) materials. Inclusion of DNG and SNG materials in photonic crystals can result in many new and interesting phenomena for linear and nonlinear wave propagation[8−11]. A SNG characterized by negative permittivity is referred to as epsilon-negative (ENG), whereas that exhibiting negative permeability is called mu-negative (MNG). A pair of ENG and MNG can be referred to as a conjugate pair[12]. Wave propagation has recently been studied in photonic crystals containing alternate ENG and MNG layers[13−22]. The refractive index in a SNG material is imaginary, resulting in evanescent modes in a bulk medium. In a one-dimensional (1D) periodic structure containing alternate ENG and MNG layers, propagating modes can assume a structure described by the tight binding model in solid-state physics. An interesting phenomenon occurring in this structure is the appearance of a zero-ϕ gap. The properties of the zero-ϕ gap rather differ from those of a Bragg gap[16−19] because it results from a different mechanism. Some applications have also been shown to use these properties[17,19].

Most investigations on wave propagation in 1D periodic structures containing SNG materials have been conducted using layer-by-layer calculations. In this case, the wave propagation is locally treated within each layer; as we proceed to the other layer, the boundary conditions have to be applied at the interface between the two layers. An analytic solution for the wave propagation in a periodic medium can be obtained if specific conditions are satisfied. Analytic approaches provide a general view of wave propagation and more insights into the nature of the processes that occur. Here, we applied the envelope function approach[23−25], which is based on the multiple-scale method, to investigate the nature of the wave propagation in such structures. This approach allows the characterization of the periodic structure by few parameters, which is different from the conventional piecewise description of the structure. In this approach, the electric field can be decomposed into a fast Bloch-like component and a slow envelope function. In the slow envelope function, the periodic structure behaves as a homogeneous slab of material for linear and nonlinear wave propagation. The fast Bloch-like components determine the local behavior of the electric field within the repeating units. By using the transfer matrix approach for linear wave propagation, a periodic structure with alternating ENG and MNG layers can be effectively treated as a left-handed medium[21]. Using this approach, we can directly establish this similarity because the structure behaves as a homogenous medium for the envelope function. Most studies on such structures have examined linear wave propagation. The major advantage of the envelope function approach is that nonlinear wave propagation is easily and effectively treated.

Here, we consider a 1D structure of alternating layers a and b, which are assumed to be ENG and MNG, respectively. Given that SNG materials are inherently dispersive, the electric permittivity in layer a and magnetic permeability in layer b are represented by a Drude model, where absorption is negligible.

For layer a, which is assumed to be ENG, the linear values are defined as

$$\varepsilon_{\mathrm{a}} = 1 - \frac{\omega_{\mathrm{pe}}^2}{\omega^2},\ \mu_{\mathrm{a}} = 1. \tag{1}$$

For layer b, which is assumed to be MNG, these values are

$$\varepsilon_{\rm b}=1,\ \mu_{\rm b}=1-\frac{\omega_{\rm pm}^2}{\omega^2}. \tag{2}$$

Therefore, the refractive indices in the two layers are given by

$$n_{\rm a}=\sqrt{\varepsilon_{\rm a}\mu_{\rm b}},\ n_{\rm b}=\sqrt{\varepsilon_{\rm b}\mu_{\rm b}}. \tag{3}$$

Layer a behaves as an ENG layer in the frequency range $\omega<\omega_{\rm pe}$, and layer b behaves as a MNG layer in the frequency range $\omega<\omega_{\rm pm}$. Here, we have assumed that $\omega_{\rm pe}=\omega_{\rm pm}$. Thus, the two layers behave as SNG layers in the same frequency range. Although imaginary, the refractive index has the same value in both layers. To apply the envelope function approach, the contrast in therefractive index of the two media should be small (zero in the limiting case, Kogelnik approximation). In the current case, a zero-ϕ gap appears when no contrast is observed in the refractive index of the two media. The mismatch in the local phase shifts is produced by the different widths of the two layers.

The dispersion relation for the normally incident wave in a periodic structure containing alternate layers of ENG and MNG materials can be written as

$$\cos qd=\cosh(k_{\rm a}d_{\rm a})\cosh(k_{\rm b}d_{\rm a}) -\frac{1}{2}\Big(\frac{\eta_{\rm a}}{\eta_{\rm b}}+\frac{\eta_{\rm b}}{\eta_{\rm a}}\Big)\sinh(k_{\rm a}d_{\rm a})\sinh(k_{\rm b}d_{\rm b}), \tag{4}$$

where q is the Bloch wave vector. The local wave vectors in the two layers are imaginary, with absolute values given by

$$k_{\rm a}=\sqrt{|\varepsilon_{\rm a}\mu_{\rm a}|}k_0,\ \ k_{\rm b}=\sqrt{|\varepsilon_{\rm b}\mu_{\rm b}|}k_0, \tag{5}$$

where k_0 is the free space wave vector. The absolute values of the wave impedance in the two layers are

$$\eta_{\rm a}=\sqrt{\left|\frac{\mu_{\rm a}}{\varepsilon_{\rm a}}\right|},\ \eta_{\rm b}=\sqrt{\left|\frac{\mu_{\rm b}}{\varepsilon_{\rm b}}\right|}. \tag{6}$$

The horizontal axis shows the "qd."

Figure 1 shows the dispersion relation for the structural parameters within the frequency range in which the two layers behave as SNG media. At low frequencies, where $\eta_{\rm a}\neq\eta_{\rm b}$, $|\cos qd|>1$ corresponds to a gap and $|\cos qd|<1$ corresponds to a propagating region. At a certain frequency, the wave impedances in the two layers are identical, i.e., $\eta_1=\eta_2$. Considering that the widths of the two layers differ in the present case, a mismatch in the local phase shifts, i.e., $k_{\rm a}d_a\neq k_{\rm b}d_{\rm b}$, occurs at this wave impedance matching frequency, which corresponds to $|\cos qd\ |>1$. Furthermore, a zero-ϕ gap is observed.

The nonlinear wave equation for the normally incident wave in this periodic structure can be written in cgs units as

$$-c^2\frac{\partial^2E(x,t)}{\partial x^2}+\mu(x)\varepsilon(x)\frac{\partial^2E}{\partial t^2} =4\pi\mu(x)\chi^{(3)}(x)\frac{\partial^2}{\partial t^2}[E(x,t)]^3. \tag{7}$$

We assume that the nonlinearity results from the nonlinear electric polarization in either the ENG or MNG layer. The nonlinear polarization starts because of third-order susceptibility, which is the most commonly occurring nonlinearity exhibited by centrosymmetric and non-centrosymmetric media. Magnetic permeability is assumed linear in both layers. Equation (7) is solved using the multiple-scale method by introducing

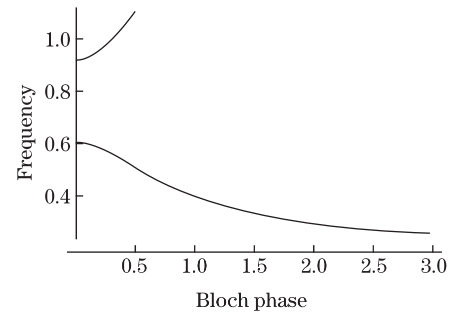

Fig. 1. Dispersion curve for $\omega_{\rm pe}=\omega_{\rm pm}=3.3\times10^{11}$ rad/s, $d_{\rm a}$=0.7×10^{-3} m, $d_{\rm b}$=0.3×10^{-3} m, and d =1×10^{-3} m in the frequency range within which the layers behave as SNG media. On the vertical axis, frequency is in dimensionless units, i.e., $W=\frac{\omega d}{c}$.

$$\begin{aligned}&x_i=s^ix\\ &\qquad\qquad i=0,1,2,3\cdots,\quad s<<1\\ &t_i=s^it\\ &E(x_i,t_i)=se_1(x_i,t_t)+s^2e_2(x_i,t_t)+s^3e_3(x_i,t_i)+\cdots\end{aligned} \tag{8}$$

The approach[24] was previously used for a periodic structure of right-handed layers. The procedure involves substitutions in Eq. (7) from Eq. (8). The coefficients of s with equal powers on both sides of the equation are then equated. Equating the coefficient of s splits the electric field into a slow envelope function $a(x_1,x_2\cdots;t_1,t_2..)$ and a fast Bloch function $\phi_{\rm m}(x_0)$. A number of features have emerged because of the inclusion of SNG layers. The product $\varepsilon(x)\mu(x)$ is negative in both layers. The propagating modes in this structure can appear if the orthogonality condition for the normalized Bloch functions $\phi_{\rm m}$ is

$$\int\phi_{\rm m'}^*\varepsilon(x)\mu(x)\phi_{\rm m}=-\delta_{\rm mm'}. \tag{9}$$

This condition and the coefficient of s^2 give

$$\frac{\partial a}{\partial t}-\omega_{\rm m}'\frac{\partial a}{\partial x}=0, \tag{10}$$

where $\omega_{\rm m}'$ is the group velocity of the envelope function. In a 1D periodic structure containing a right-handed material[24],

$$\frac{\partial a}{\partial t}+\omega_{\rm m}'\frac{\partial a}{\partial x}=0. \tag{11}$$

Equations (10) and (11) suggest that in a structure of alternating ENG and MNG layers, the envelope function moves at a group velocity opposite to the direction of the x-axis (negative group velocity). By contrast, the envelope function travels in the direction of the x-axis (positive group velocity)in a structure of right-handed layers. Given that the wave vector of the propagating mode lies along the positive direction of the x-axis and the phase velocity is along the direction of the wave vector, a positive group velocity indicates that the phase velocity and group velocity have the same direction. In addition, a negative group velocity indicates that the phase velocity and group velocity have opposite directions. One of the basic characteristics of a left-handed medium is that the phase velocity and the group velocity are in opposite directions[7]. Thus, the structure considered in this study is an equivalent left-handed medium.

Collecting the coefficient of s^3 and following the same procedure, a modified nonlinear Schrödinger equation is obtained.

$$\mathrm{i}\frac{\partial a}{\partial t} - \frac{1}{2}\omega_{\mathrm{m}}^{''}\frac{\partial^2 a}{\partial z^2} - \alpha_{\mathrm{m}}|a|^2 a = 0, \qquad (12)$$

where $z = x + \omega_{\mathrm{m}}^{'}t$, $\omega_{\mathrm{m}}^{''}$ is the group velocity dispersion, and α_{m} is the effective nonlinear coefficient.

In a right-handed periodic medium[24], the signs of the second and third terms in Eq. (12) are positive. This equation accepts a soliton solution when α_{m} and $\omega_{\mathrm{m}}^{''}$ have the same signs, i.e., both are negative or positive. In the present case, the positive values of α_{m} and $\omega_{\mathrm{m}}^{''}$ correspond to the negative values of α_{m} and $\omega_{\mathrm{m}}^{''}$ because the signs of two terms were reversed. Therefore, the effective medium behaves as a left-handed medium.

For example, if the Bloch functions lie on the edges of the zero-ϕ gap where the group velocity vanishes, $z = x$, and the envelope function is at rest in space. Assuming a harmonic time dependence for the envelope function, the following equation is obtained

$$a(x, t) = \psi(x)\mathrm{e}^{-\mathrm{i}\delta t}, \qquad (13)$$

where δ can be considered as the detuning, i.e., $\delta = \omega - \omega_{\mathrm{m}}$.

Thus, we obtain an equation of the form

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}X^2} - B^2\psi + 2\frac{B^2}{A^2}|\psi|^2\psi = 0, \qquad (14)$$

$$A = \sqrt{\frac{2\delta}{\alpha_{\mathrm{m}}}},\ B = \sqrt{\frac{2\delta}{\omega_{\mathrm{m}}^{''}}}. \qquad (15)$$

To obtain real values of A and B, δ should have the same sign as those of α_{m} and $\omega_{\mathrm{m}}^{''}$, s shown in the case considered in Ref. [24]. At the edge of a gap, a positive detuning corresponds to a frequency that lies above the lower edge. By contrast, negative detuning corresponds to a frequency that lies below the upper gap edge inside the gap. For Bragg gaps, gap soliton formation has been observed at its edges[26]. Gap solitons form at the lower edge of the Bragg gap for a negative effective nonlinear coefficient and at the upper edge for a positive effective nonlinear coefficient. In the present case, the gap soliton at the lower edge of the gap forms when the effective nonlinear coefficient is positive. At the upper edge, this phenomenon occurs when the nonlinearity is negative. This effect has been studied on a 1D structure consisting of alternate left-handed and right-handed layers by using the transfer matrix approach[9,11].

In conclusion, nonlinear wave propagation in a periodic structure containing alternate MNG and ENG layers is examined using the envelope function approach in which the periodic structure is a homogenous medium. This homogeneous medium possesses characteristics similar to those of a left-handed medium. The group and phase velocities are opposite in directions, and the effective nonlinearity of the medium is opposite to that of the medium of right-handed layers. A similarity between the periodic structures of SNG layers and a homogenous left-handed medium is indicated in Ref. [21] for linear wave propagation. The approach used in the current study further shows this similarity for nonlinear wave propagation.

References

1. E. Yablonovitch, Phys. Rev. Lett. **58,** 2059 (1987).
2. S. John, Phys. Rev. Lett. **58,** 2486 (1987).
3. C. M. Soukoulis, *Photonic Band Gap Materials* (Springer, Berlin, 1996).
4. J. D. Joannopoulos, Nature **386,** 143 (1997).
5. S. F. Mingaleev and Y. S. Kivshar, Phys. Rev. Lett. **86,** 5474 (2001).
6. R. A. Shelby, D. R. Smith, and S. Schultz, Science **292,** 77 (2001).
7. V. G. Veselago, Sov. Phys. Usp. **10,** 509 (1968).
8. A. A. Zharov, I. V. Shadrivov, Y. S. Kivshar, Phys. Rev. Lett. **91,** 037401 (2003).
9. T. Pan, C. Tang, L. Gao, and Z. Li, Phys. Lett. A **337,** 473 (2005).
10. J. B. Pendry and S. A. Ramakrishna, J. Phys. Condens. Matter **15,** 6345 (2003).
11. M. Z. Ali and T. Abdullah, Phys. Lett. A **351,** 184 (2006).
12. A. Alu and N. Engheta, IEEE Trans. Microw. Theory Technol. **52,** 199 (2004).
13. L. Wang, H. Chen, and S. Zhu, Phys. Lett. A **350,** 410 (2006).
14. K. Y. Kim, Opt. Lett. **30,** 430 (2005).
15. M. Hotta, M. Hano, and I. Aawi, IEICE. Trans. Electron. **E88-C,** 275 (2005).
16. L. G. Wang, H. Chen, and S. Y. Zhu, Phys. Rev. B **70,** 245102 (2004).
17. H. Jiang, H. Chen, H. Li, Y. Zhang, J. Zi, and S. Zhu, Phys. Rev. E **69,** 066607 (2004).
18. H. Jaing, H. Chen, H. Li, and Y. Zhang , Chin. Phys. Lett. **22,** 884 (2005).
19. H. Jiang, H. Chen, H. Li, and Y. Zhang, J. Appl. Phys. **98,** 013101 (2005).
20. G. Guan, H. Jiang, and H. Li, Appl. Phys. Lett. **88,** 211112 (2006).
21. D. R. Fredkin and A. Ron, Appl. Phys. Lett. **81,** 1753 (2002).
22. S. M. Wang, C. J. Tang, T. Pan, and L. Gao, Phys. Lett. A **348,** 424 (2006).
23. J. E. Sipe and H. G. Winful, Opt. Lett. **13,** 132 (1988).
24. C. M. de Sterke and J. E. Sipe, Phys. Rev. A **38,** 5149 (1988).
25. C. M. de Sterke and J. E. Sipe, Phys. Rev. A **39,** 5163 (1989).
26. W. Chen and D. L. Mills, Phys. Rev. Lett. **58,** 160 (1987).