

Controlling the propagation of optical rogue waves in nonlinear graded-index waveguide amplifiers

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A unified theory to construct exact optical rogue wave solutions of (1+1)-dimensional nonlinear Schrödinger equation with varying coefficients is proposed. The dynamics of the first-order optical rogue waves in nonlinear graded-index waveguide amplifiers exhibiting self-focusing or self-defocusing Kerr nonlinearity are also investigated. Moreover, under the suitable parameter condition, the propagation characteristics of the rogue waves in the nonlinear optical media are discussed. The properties of the optical rogue waves, such as width, amplitude, and position, can be controlled in the nonlinear optical media.

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Rogue waves, whose amplitude are two, three, or more times higher than the average wave crests^[1], have received a great deal of attention for understanding their complex physics and wealth of potential applications in recent years. The rogue waves have been considered as myths for a long time (they appear from nowhere and disappear without a trace), and are potentially destructive (their amplitudes are much higher than the average wave crests)^[2,3]. Intensive efforts have been devoted to explain how rogue waves are excited. Generally speaking, modulational instability is considered as the main mechanism for the appearance of rogue waves. In addition, soliton collision with energy exchange can lead to the creation of rogue waves. The nonlinear Schrödinger equation (NLSE) can describe many dynamical features of rogue waves. With the increase in theoretical and experimental studies on this topic, the rogue waves are now observed in many fields of nonlinear science, such as the nonlinear oceanography^[4–6], the nonlinear optics^[7–11], and the mean-field theory of Bose-Einstein condensates^[12,13]. The rogue waves in the open ocean were firstly measured on the oil platform in Norway in 2005^[5]. The initial experiments studied optical rogue waves in the supercontinuum generation in 2007^[8]. Recently, Kibler *et al.*^[10] experimentally observed multiple periodic solutions in optical fibre. Matter rogue waves have also been observed in Bose-Einstein condensates embedded in an optical lattice^[12].

To date, most of the experimental and theoretical studies are focused on optical pulses propagating in the nonlinear optical material governed by the (1+1)-dimensional NLSE^[2,3,7]. However, only few studies have focused on the higher-dimensional cases^[14–16]. To the best of our knowledge, this letter is the first to report on the position and direction control of optical rogue waves in nonlinear graded-index waveguide amplifiers.

Our goal is to construct the exact optical rogue wave solutions of the (1+1)-dimensional NLSE with varying coefficients and investigate the propagation dynamics of the optical rogue waves in a planar graded-index waveguide amplifiers. Then, we discuss the control of the rogue

waves.

We start by considering the propagation of a continuous-wave (CW) optical beam inside a planar, graded-index nonlinear waveguide amplifier with the refractive index given as

$$n(z, x) = n_0 + n_1 f(z)x^2 + n_2 \gamma(z)I(z, x), \quad (1)$$

where $I(z, x)$ is the optical intense. Here, the first two terms describe the linear part of the refractive index, and the last term represents Kerr-type nonlinearity. We consider the generalized NLSE with distributed coefficients as

$$i \frac{\partial u}{\partial z} + \frac{1}{2} \frac{\partial^2 u}{\partial x^2} + \frac{f(z)}{2} x^2 u + \sigma \gamma(z) |u|^2 u = i \frac{g(z)}{2} u, \quad (2)$$

which describes the propagation of the optical beam inside a planar graded-index waveguide amplifier with the refractive index $n = n_0 + n_1 f(z)x^2 + n_2 \gamma(z)I$ ^[17,18]. Here $\sigma = n_2/|n_2| = \pm 1$ corresponds to self-focusing (+) and self-defocusing (–) nonlinearities of the waveguide, respectively, and $f(z)$, $\gamma(z)$, and $g(z)$ are the functions of the normalized distance z . The dimensionless tapering function $f(z)$ can be negative or positive, depending on whether the graded-index medium acts as self-focusing ($\gamma > 0$) or self-defocusing ($\gamma < 0$) Kerr nonlinearities. Here, u is the beam envelope, z is the propagation distance, and x is the spatial coordinate. Further, $g(z)$ stands for the gain ($g > 0$) or the loss ($g < 0$). All the above variables u , z , x , and $g(z)$ have been respectively normalized by $(k_0 |n_2| L_D)^{-1/2}$, L_D , ω_0 , and L_D^{-1} , with the wavenumber $k_0 = 2\pi n_0/\lambda$ at the input wavelength λ , the diffraction length $L_D = k_0(2k_0^2 n_1)^{-1/2}$, and the characteristic transverse scale $\omega_0 = (2k_0^2 n_1)^{-1/4}$.

We start by seeking a solution to Eq. (2) of the form

$$u(z, x) = A(z)U[X(z, x), Z(z)] \exp[i\varphi(z, x)], \quad (3)$$

$$\varphi(z, x) = \frac{1}{2}a(z)x^2 + b(z)x + c(z), \quad (4)$$

where $A(z)$ and $\varphi(z, x)$ are the dimensionless amplitude and global phase, respectively. Further, the transformed

field $U(X, Z)$ satisfies the following standard, homogeneous NLSE:

$$iU_z + \frac{1}{2}U_{xx} + |U|^2U = 0, \quad (5)$$

with a set of differential equations being satisfied. After solving these differential equations, one obtains the similarity variable, position, amplitude, effective propagation distance, and phase of the beam given as

$$X(z, x) = \frac{x - x_c(z)}{\omega(z)}, \quad x_c(z) = \left(\int_0^z \frac{b_0}{\omega(s)^2} ds + x_0 \right) \omega(z), \quad (6)$$

$$A(z) = \frac{1}{\omega(z)\sqrt{\sigma\gamma(z)}}, \quad Z = \int_0^z \frac{1}{\omega(s)^2} ds + Z_0. \quad (7)$$

The parameters related to the phase are given by

$$a(z) = \frac{\omega_z}{\omega(z)}, \quad b(z) = \frac{b_0}{\omega(z)}, \quad c(z) = -\frac{b_0^2}{2} \int_0^z \frac{1}{\omega(s)^2} ds, \quad (8)$$

where $a(z)$, $b(z)$, and $c(z)$ are the parameters related to the phase-front curvature, the frequency shift, and the phase offset, respectively. The real function $\omega(z)$ is the width of the beam, and the subscript 0 denotes the initial values of the given variables. Note that the transformation of the inhomogeneous NLSE into the homogeneous one is obtained with the need to satisfy the following constraint condition

$$g = -\frac{\omega_z}{\omega} - \frac{\gamma_z}{\gamma}, \quad (9)$$

$$f = \frac{\omega_{zz}}{\omega}. \quad (10)$$

Equation (5) has been extensively investigated, and many kinds of exact solutions have been obtained. One of these solutions, the rational-oscillatory solution, has some intriguing properties. In this letter, we are going to study this rogue wave solution in a nonlinear optical system. Here, the optical rogue wave solution of Eq. (2) can be readily obtained by using the similarity transformation (Eq. (3)), as the rational-oscillatory solution of Eq. (5) has already been obtained by Peregrine in 1983^[19]. Note that the rational-oscillatory solution (also called Peregrine soliton) has been demonstrated in optics recently by Kibler *et al.*^[20,21]. Next, we apply the Galilean transformation to the first-order rational-oscillatory solution as

$$U = \left[1 - \frac{4 + 8i(Z - Z_c)}{1 + 4[X - v(Z - Z_c)]^2 + 4(Z - Z_c)^2} \right] \times \exp \left[i \left(1 - \frac{v^2}{2} \right) (Z - Z_c) + ivX \right], \quad (11)$$

where Z_c and v are two arbitrary constants, $X_c = v(Z - Z_c)$ is the center of the rogue wave, and $Z = Z_c$ is the position where the rogue wave emerges and disappears after that. By the Galilean transformation, the parameter Z_c is added in solution of Eq. (11). However, we find that the parameter Z_c is important to control the

rogue waves, which can be found in the following analysis. This kind of analysis for Eq.(1) is new.

Therefore, the one-to-one correspondence of Eq. (3) and rational-oscillatory solution of Eq. (11) admit us to obtain variable parametric rational-oscillatory solution of Eq. (1), that is, the function U in solution (3) is replaced by the detailed form of Eq. (11). This kind of variable parametric rational-oscillatory solution for Eq. (1) is hardly reported.

In the following, we demonstrate that the optical rogue waves can be restricted to emerge in the desirable position by adjusting the corresponding parameters. We consider some specific cases in order to gain further insights into the dynamical behavior of the optical rogue wave. We begin by observing that Eq. (10) is formally identical to a wave equation governing the modes of an inhomogeneous planar waveguide with the refractive index profile given by the function $f(z)$. It then follows from the theory of sech^2 -profile waveguide^[22] that the lowest-order mode of such a waveguide has the form

$$f(z) = 1 - 2\text{sech}^2(z), \quad w(z) = \text{sech}(z). \quad (12)$$

According to the exact expression of pulse's width, we can obtain the explicit expressions of the similarity variable, effective propagation distance, and the phase. Here, we respectively address the functions of the effective propagation distance and pulse's center position as

$$\begin{aligned} Z &= \frac{1}{2} \cosh(z) \sinh(z) + \frac{1}{2} z + Z_0, \quad x_c \\ &= \frac{1}{2} \frac{b_0 \cosh(z) \sinh(z) + b_0 z + 2x_0}{\cosh(z)}. \end{aligned} \quad (13)$$

If we take $g(z) = g_0 \tanh(z)$, from the compatibility condition in Eq. (9), the required cubic nonlinearity parameter is found to be $\gamma(z) = \cosh(z)^{1-g_0}$, where g_0 is an arbitrary constant. The gain, width, and tapering profiles are displayed in Fig. 1. As can be seen, the tapering function $f(z)$ crosses zero near $z = 1$, implying that the linear inhomogeneity of the waveguide should change from the focusing to de-focusing type. The required normalized gain $g(z)$ is zero initially and tends toward 0.01 asymptotically for large z . Such a gain distribution can be realized. Then the other parameters are given by

$$A(z) = \frac{1}{\omega\sqrt{\sigma\gamma}}, \quad (14)$$

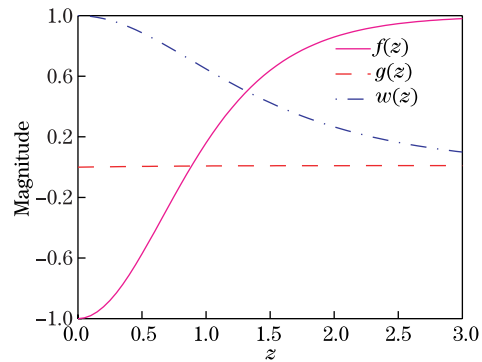


Fig. 1. (Color online) Gain, width, and tapering profiles, plot as functions of z , for the specific choices of $f(z)$.

$$\begin{aligned}
 a(z) &= -\tanh(z), & b(z) &= \frac{b_0}{\operatorname{sech}(z)}, \\
 c(z) &= -\frac{1}{4}b_0^2[\cosh(z)\sinh(z) + z].
 \end{aligned}
 \quad (15)$$

On one hand, from the first expression of Eq. (13), the effective propagation distance has a relation to the original propagation distance, one has $Z > z$ and $Z \rightarrow \infty$ with $z \rightarrow \infty$, which implies that optical rogue wave will be excited at somewhere and then vanish quickly. Figure 2 displays the evolution of the first-order rogue wave in nonlinear optical material. The excitation of the center-of-mass motion of the rogue wave follows a curved line when one takes the parameter $b_0 = 2$ (Fig. 2). Conversely, when one takes the parameter $b_0 = -2$ (Fig. 3), the excitation process of the center-of-mass of the rogue wave follows a curved line which points to a different direction in comparison with the red dashed line plotted in Fig. 2. In addition, we consider a special situation with $f(z) = f_0$ (which is a constant). In this case, from Eq. (10) we can see the beam with $\omega(z) = C_1 \exp[\sqrt{f_0}z] + C_2 \exp[-\sqrt{f_0}z]$, where we chose $\omega(0) = 1$. To Simplify, we take $C_1 = C_2 = 1/2$. If we take $\gamma(z) = \gamma_0$, (where γ_0 is an arbitrary constant), from the compatibility condition in Eq. (9), the required gain parameter is found to be $g = \sqrt{f_0}$. The gain, width, and tapering profiles are displayed in Fig. 3. Such distributions can be realized easily. According to the exact expression of pulse width, we can obtain the explicit expressions of the similarity variable, effective propagation distance, and the phase. Here, we respectively address the functions of the effective propagation distance and pulse's center position as

$$\begin{aligned}
 Z &= \frac{1}{2} \frac{\exp[2\sqrt{f_0}z]}{\sqrt{f_0}} + Z_0, \\
 x_c &= \frac{1}{2} \frac{b_0 \exp[\sqrt{f_0}z]}{\sqrt{f_0}} + \frac{x_0}{\exp[\sqrt{f_0}z]}.
 \end{aligned}
 \quad (16)$$

Then, the other parameters are given by

$$A(z) = \frac{\exp[\sqrt{f_0}z]}{\sqrt{\sigma\gamma_0}}, \quad (17)$$

$$a(z) = -\sqrt{f_0}, \quad b(z) = b_0 \exp[\sqrt{f_0}z],$$

$$c(z) = -\frac{1}{4} \frac{b_0^2 \exp[2\sqrt{f_0}z]}{\sqrt{f_0}}. \quad (18)$$

As before, from the first expression of Eq. (16), the effective propagation distance has a relation to the original propagation distance, one has $Z > z$ and $Z \rightarrow \infty$ with $z \rightarrow \infty$, which implies the optical rogue waves will be excited at somewhere and then vanish quickly. Figure 4 displays the evolution of the first-order rogue wave in nonlinear optical material. The excitation of the center-of-mass motion of the rogue wave follows a curved line when one takes the parameter $b_0 = 2$ (Fig. 5). Conversely, when one takes the parameter $b_0 = -2$ (Fig. 6), the excitation process of the center-of-mass motion of the rogue wave follows a curved line which points to a different direction in comparison with the red dashed line plotted in Fig. 5.

Above all, the optical rogue wave can be controlled to

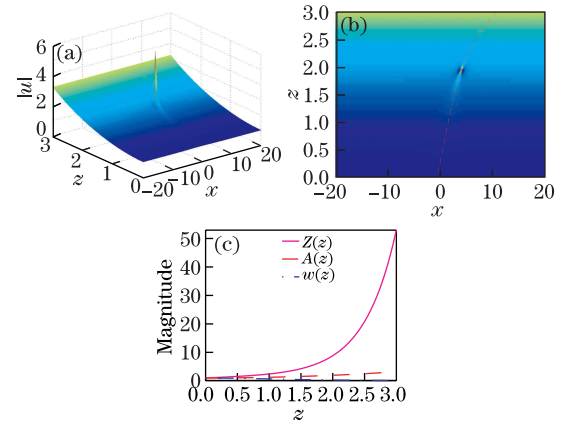


Fig. 2. (Color online) (a) Emergence of the optical rogue wave with the parameters $b_0 = 2$, $x_0 = 0.1$, $g_0 = 0.01$, $\sigma = 1$, $v = 0.1$, $Z_c = 8$, and other parameters are specified in the text; (b) corresponding contour plot (red dashed line denotes the center-of-mass motion of the optical pulse); (c) corresponding width, the effective propagation distance, and amplitude.

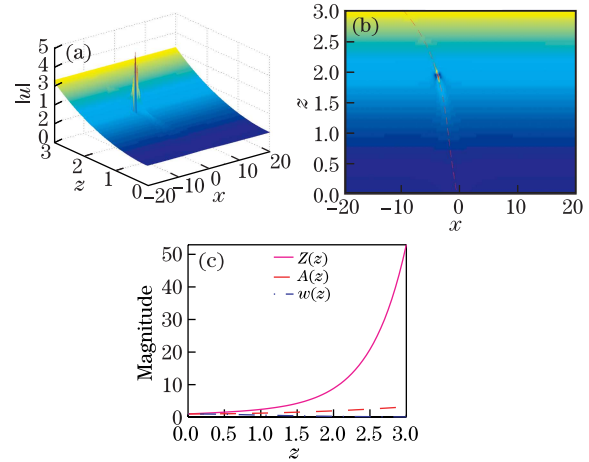


Fig. 3. (Color online) (a) Emergence of the optical rogue wave with the parameters $b_0 = -2$, $x_0 = 0.1$, $g_0 = 0.01$, $\sigma = 1$, $v = 0.1$, $Z_c = 8$, and other parameters are specified in the text; (b) corresponding contour plot (red dashed line denotes the center-of-mass motion of the optical pulse); (c) corresponding width, the effective propagation distance, and amplitude.

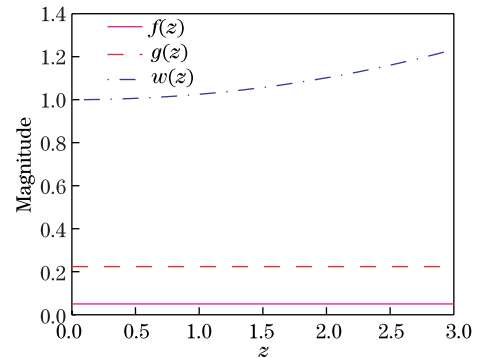


Fig. 4. (Color online) Gain, width, and tapering profiles, plot as functions of z , for the specific choices of $f(z)$.

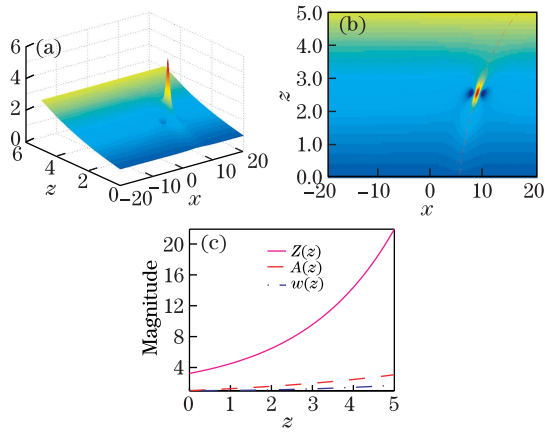


Fig. 5. (Color online) (a) Emergence of the optical rogue wave with the parameters $f_0 = 0.05$, $b_0 = 2$, $x_0 = 1$, $\gamma_0 = 1$, $\sigma = 1$, $v = 0.1$, $Z_0 = 1$, $Z_c = 8$, and other parameters are specified in the text; (b) corresponding contour plot (red dashed line denotes the center-of-mass motion of the optical pulse); (c) corresponding width, the effective propagation distance, and amplitude.

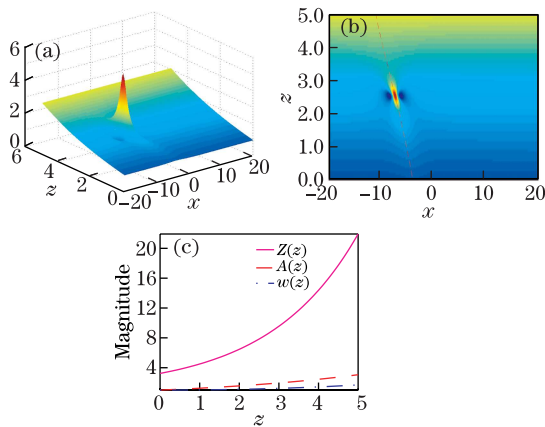


Fig. 6. (Color online) (a) Emergence of the optical rogue wave with the parameters $f_0 = 0.05$, $b_0 = -2$, $x_0 = 0.1$, $\gamma_0 = 1$, $\sigma = 1$, $v = 0.1$, $Z_0 = 1$, $Z_c = 8$, and other parameters are specified in the text; (b) corresponding contour plot (red dashed line denotes the center-of-mass motion of the optical pulse); (c) corresponding width, the effective propagation distance, and amplitude.

emerge in the desirable position and direction through adjusting the corresponding parameters: $f(z)$ and $b(z)$, which means that we can control the propagation of optical rogue wave by the materials and the initial optical pulse.

In conclusion, we construct the relation between the pulse propagation and exact optical rogue wave solutions of (1+1)-dimensional NLSE with varying coefficients. Moreover, we investigate the dynamic of the first-order optical rogue wave in waveguide amplifiers. We find that the properties of the optical rogue waves, such as width, amplitude, and position, are controllable in the nonlinear optical media. Under the parameter condition, we discuss the propagation behaviors of controllable rogue waves. The results well supplement our comprehension of the rogue waves, that is it “appears from nowhere and

disappears without a trace”; rogue waves can be controlled as discussed by a similar transform method in this letter. Moreover, these results may have potential values for the optical pulses, optical rogue waves. However, due to the lack of experimental and designed basis related to these theoretical results, we cannot provide further details about the real physical application. More practical implementations of these theoretical results can be identified in future studies.

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