Optical frequency ruler with moving fluid

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A configuration consisting of a double slit and pipe in conjunction with moving fluid is proposed to manipulate a broadband light source through the interference spectrum. Theoretical analysis shows that, by varying the speed of the fluid, the scheme can act as a special dynamic frequency filter and produce regular dark lines that may serve as an optical frequency ruler.

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Spectral changes caused or induced by different mechanisms, such as the aperture geometry[1,2], spectrum correlation[3], material interaction[4,5], or scattering[6] have gained significant research interest, and a variety of applications[7−9] using these effects have been suggested. When a broadband light source passes a normal double slit, the spectrum detected at the far-field changes drastically across the observation plane, depending on the spectral degree of coherence (correlation induced effect) and/or the observation position (diffraction induced effect)[10]. This phenomenon, which has been verified experimentally, is called the spectral interference law for superposition of beams[11]. In this letter, we suggest an improved version of the double slit scheme that can offer distinct advantages such as flexible control of the spectrum and a detected location that is fixed at the center of the detection plane (not across the space). Consider a double slit and a transparent U-shaped pipe in which a fluid can flow, as in Fig. 1(a); here water is taken as an example. If we limit the incident spectrum to the visible range, which falls in the transparent window of water, absorption can be neglected[12] because the absorption coefficient is less than 0.01 cm^{-1}. However, material dispersion must be considered for a broadband source; the wavelength (or frequency) dependence of the refraction index $n(\lambda)$ (or $n(\nu)$) is taken from Hale *et al.*[12]. For water in the visible region, $n(\lambda)$is around 1.33. To produce an interference spectrum, the double slit is used to mask part of the pipe (the shaded region), and only the areas of the pipe facing the slit apertures are exposed to the front, as shown in Fig. 1(b). When a spatially completely coherent light is incident from the left through the pipe, as in Fig. 1(a), the light moves with the water in part A of the pipe but against the water in part B. We labeled the water speed and the light speed in still water u and c_{w} respectively; here $c_{\mathrm{w}} = c/n(\nu)$. Because u may vary widely in the proposed scheme, Einstein's addition theorem of relative velocity[13] is utilized to give the light velocities in parts A and B of the pipe as

$$v_{\mathrm{A}} = \frac{c_{\mathrm{w}} + u}{1 + (uc_{\mathrm{w}}/c^2)}, \quad v_{\mathrm{B}} = \frac{c_{\mathrm{w}} - u}{1 - (uc_{\mathrm{w}}/c^2)}. \tag{1}$$

A comparable setup without a double slit was used as an interferometer for monochromatic light by Fizeau in 1851 and results successfully showed a the relativistic effect, called Fresnel drag, which was predicted by Fresnel in 1818[13]. In the present study, the phase difference introduced by the moving water between the two slits is

$$\Delta\phi = \phi_{\mathrm{B}} - \phi_{\mathrm{A}} = 2\pi l\nu(1/v_{\mathrm{B}}v_{\mathrm{A}}), \tag{2}$$

where l is the length of the pipe and ν is the frequency. $\Delta\varphi$ in Eq. (2) can be changed by varying u in Eq. (1). Without losing generality, we can consider the situation along the x direction only in Fig. 1. The aperture function in Fig. 1(a), with the phase difference $\Delta\phi$, can be written as

$$g(x') = \exp(\mathrm{j}\Delta\phi)\cdot\Pi\left(\frac{x'+\frac{a}{2}}{b}\right) + \Pi\left(\frac{x'-\frac{a}{2}}{b}\right), \tag{3}$$

where $\Pi(x')$ is the rectangular function defined as $\Pi(x'/b) = 1$ for $|x'| \leqslant b/2$ and $\Pi(x'/b) = 0$ for $|x'| > b/2$. The Fourier transform is

$$F\left[g\left(x'\right)\right] \\ = \exp(\mathrm{j}\pi af_x)b\sin c\left(\pi bf_x\right)\left\{1+\exp[\mathrm{j}(-2\pi af_x + \Delta\phi_{eo})]\right\}, \tag{4}$$

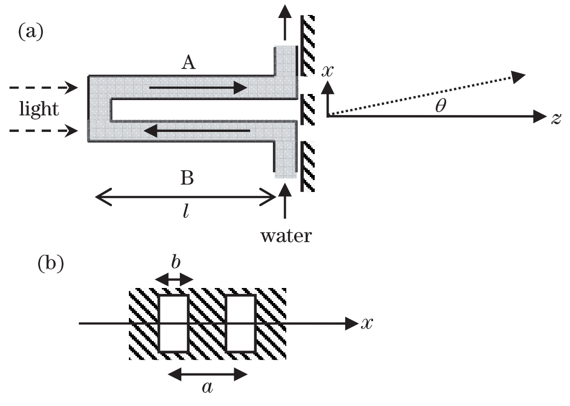

Fig. 1. (a) Top view of the proposed setup. A double slit with a pipe is used to produce the interference spectrum. The light incident from the left runs with the water in part A, but against it in part B. (b) Front view of the setup, viewed from the detection direction (+z).

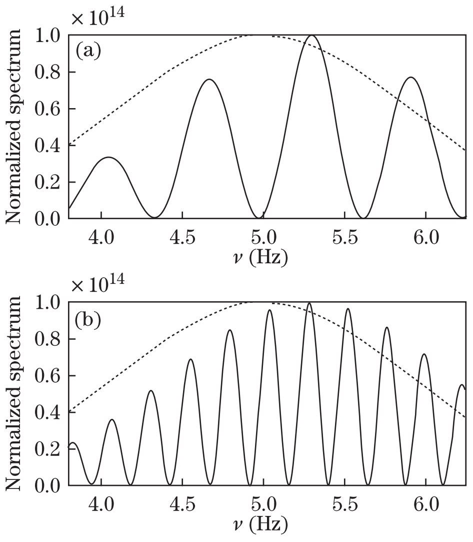

Fig. 2. Plots of incident spectrum $I^{(i)}(v)$(dotted line) and modulated spectrum $I_0(v)$ (solid line) at the center direction as a function of water speed. (a) u= 8.75 $\times 10^3$ and (b) 2.3 $\times 10^4$ m/s. The circles on the abscissa are zeros of the modifier in Eq. (6).

where sinc function is defined as $\sin c(x) = \sin(x)/x$ and f_x is the spatial frequency variable. Assuming that $I^{(i)}(v)$ is the incident spectral distribution, the interference spectrum detected at θ is obtained as[4,10,14] $I(\theta, v) \propto I^{(i)}(v) \times v^2 \times |F[g(x')]|^2$ with $f_x = \nu x/cz = \nu \tan(\theta)/c$. where θ is the angle between the z-axis and the detection direction, as shown in Fig. 1(a), and c is the light speed in a vacuum. After some algebra, the above formula can be expressed as

$$I(\theta, v) \propto I^{(i)}(v) \times v^2 \mathrm{sinc}^2\left(\frac{\pi b \tan(\theta) v}{c}\right)$$
$$\cos^2\left(\frac{\Delta\varphi}{2} + \frac{\pi a \tan(\theta) v}{c}\right) \equiv I^{(i)}(v) \times M(\theta, v), \quad (5)$$

where a and b are the distance between two slits and the slit width respectively, as in Fig. 1(b). Because Eq. (5) is obtained under the assumption of far-field condition, parameters a, b, and $\tan(\theta) = x/z$ must satisfy the Fraunhofer approximation[15] as follows: $\tan(\theta) \ll 1$ and $N_\mathrm{c} = a^2/\lambda_\mathrm{c} z = a^2 \nu_\mathrm{c}/cz \leqslant 1.0$, where N_c is the Fresnel number at the center frequency ν_c. If we take $\tan(\theta) = 0.01$, a= 1.0 mm, and v_c= 5 $\times 10^{14}$ Hz, for example, the measurement distance required to assure the applicability of Eq. (5) is $z \geqslant 1.7$m. All of the terms in the right-hand side of $I^{(i)}(v)$ in Eq. (5) are referred to as the modifier $M(\theta, v)$, because it modifies the incident spectrum to give the detected spectrum at θ. As mentioned above, the superposed spectrum $I(\theta, v)$ in Eq. (3) is changed if the observation position, or direction angle θ, varies. By introducing another adjustable phase change $\Delta\phi$, spectrum manipulation at a fixed θ is possible, thereby facilitating future usage. If we want to control the spectrum at the center direction, by setting θ= 0 in Eq. (3) and using $\mathrm{sinc}(0) = 1$, the detected center spectrum must be

$$I_0(v) \equiv I(\theta = 0, v) \propto I^{(i)}(v) \times v^2 \times \cos^2\left(\frac{\Delta\varphi}{2}\right)$$
$$= I^{(i)}(v) \times M_0(v), \quad (6)$$

where $M_0(v) \equiv M(\theta = 0, v) = v^2 \times \cos^2(\Delta\phi/2) = v^2 \times \cos^2[\pi l \nu(1/v_\mathrm{B} - 1/v_\mathrm{A})]$ is the modifier at the central direction, which is mainly a sinusoidal cosine square term multiplied by a monotonically increasing v^2 term. Let us consider a broadband light source, assuming that it can be described with a Gaussian profile $I^{(i)}(v) = \exp\left\{-[(v - v_\mathrm{c})/\delta\nu]^2\right\}$ with a center frequency of v_c= 5 $\times 10^{14}$ Hz and bandwidth of $\delta\nu$= 1.25 $\times 10^{14}$ Hz, corresponding to a center wavelength of λ_c=600 nm and a spectral width of $\Delta\lambda$=320 nm, from 480 to 800 nm. Figures 2(a) and (b) show the interference spectrum $I_0(v)$ (solid line) and the incident spectrum $I^{(i)}(v)$ (dotted line) for two water speeds, u= 8.75 $\times 10^3$ and 2.3 $\times 10^4$ m/s with l = 10 cm. While many spikes may be observed because of the oscillatory behavior of the cosine function, they are not symmetrical to $v = v_\mathrm{c}$ because of the v^2 term. Higher water speeds cause more spikes in the interference spectrum, which can be understood by observation that a high speed u in Eq. (1) leads to large phase differences $\Delta\varphi$ in Eqs. (2) and (6), and, accordingly, a narrow period in the modifier. Determination of a possible application for these interference spectra is interesting. The spikes are separated by vanishing frequencies marked as circles on the abscissa of Fig. 2, which are zeros of the cosine function in $M_0(v)$ of Eq. (4). These regularly disappearing frequencies, or dark lines, from a broadband light source may be used as frequency references similar to marks of a ruler; thus the notion of an "optical spectral ruler", is suggested in of this work. In the present case, the spaces between lines may be controlled by varying water speed. Similar reference lines and spectral behaviors nay be obtained through the use of the Fabry-Perot etalon or the optical comb technique[16], but the line width is usually fixed in these cases by the finesse or Q factor of the cavity[17]. The advantages of the present scheme are that no cavity is necessary and the spacing between dark lines is tailorable as shown in the Fig. 2. However narrower line spacings require high water speeds, which are technically challenging to produce. While very high water speeds are difficult to achieve, the proposed scheme may still be applicable to superfluid that flow very quickly without viscosity.

In conclusion, we propose a configuration utilizing a double slit with moving fluid to produce a variable-phase interference spectrum. For a broadband light source, the spectrum can be manipulated by changing the velocity of the fluid. The spectrum can be controlled at a fixed central direction, which considerably broadens its potential practical applications and the vanishing frequencies (or wavelengths) observed may serve as an optical spectral ruler with variable line width. The scheme presented here provides dynamic reference frequencies without the use of a resonant cavity. Considering that the number of spikes in the spectrum changes with the fluid velocity, the setup may also be used as an optical tachometer for measuring water speeds by counting of the number of spikes produced.

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