Multilevel photon communication on BPPM with convolutional coding

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Multilevel photon combined with binary pulse position modulation (BPPM) (Manchester pulsed signals), is introduced in this letter. Initially, error probability derivation and explanation for four-level photon communications with that BPPM is presented. Next, the 2-level photon communications matching with BPPM is proposed. For performance comparison, it is done with that the conventional scheme by fixing the background noise and also increasing number of photon per slot. Successfully with applying convolutional coding for system improving, the proposed multilevel photon on BPPM with this coding scheme achieves higher gain. Finally, this work also benefits to improving for further performance when considering with multilevel error control coding as well.

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Basically optical modulation as one of those major parts of lightwave system has been developing continuously focusing more on advance technologies in order to provide better and higher information rates. That is to support the growth in user demand on broadband communication and for next generation networks. To emphasize on modulation techniques, multilevel photon communication as one of those proposed schemes in lightwave communication, has been focusing. It is proposed to achieve higher throughput or to increase the system efficiency to be better than that of traditional binary photon communications. Basically, regular pulse position modulation (PPM) has been developed consequently to be the multi-pulse version (MPPM), or the overlapping sense (OPPM), and the differential pulse style (DPPM)[1]. Subsequently based on that principle of PPM, the "multilevel concept" has been shown as a higher performance achieved method to add on, for instance as an invention in Ref. [2]. However apart from working on that PPM pulse format of modulation, another branch of "multilevel" PPM by consideration on other bound breaking idea such as multilevel dual-polarization signal constellations or polarized and amplitude hybrid, have been introduced in Refs. [3,4] respectively. In the similar direction of "multilevel" sense, the multilevel photon format from Ref. [5] is considered as another potential modulation focusing on light intensity within each pulse. Moreover, bandwidth-efficient techniques by using this multiple-level related technologies, has been also focusing in other various fields such as in free space channel, indoor broadband communications[6], quantum key distribution[7], and ultra wideband communication[8]. Thus, it becomes highly promising as a new modulation system when these two schemes are proposed jointly.

Eventually, this letter discovers above combination technique in order to improving overall system performance. In addition, matched convolutional codes as another promising solution, is also proposed in conjunction with those improved modulation scheme for obtaining further gain.

Considering firstly on a binary photon pulse or on-off keying system[1,9], transmitting symbol "l", the sender (T_x) delivers a rectangular pulse containing average number of photons α and pulse width of T_D seconds. Otherwise, transmitting symbol "0", sender will send no pulse. Then, the light signal denoted by the intensity is

$$\lambda_s(t) = \begin{cases} \dfrac{\alpha}{T_D}, & 0 \leqslant t \leqslant t_D \\ 0, & \text{elsewhere} \end{cases}. \qquad (1)$$

When the probability of those two symbols is equal, transmitted power in terms of transmitted photons is given in average by

$$P_2 = \alpha/(2T_D). \qquad (2)$$

Next from above background, a multilevel photon communication style is now realized. This modulation is similar to that a multilevel format of the traditional amplitude modulation (AM). It is only excepted on the signal constellations, those are specified by the light pulse intensity. Starting here with a signal set when considering on a 4-level photon style, the binary 2-tuples are placed onto four different rectangular pulses resulting with pulse width of T_D seconds and average number of photons (α_i) is then defined. Similarly, it is an analogous to that 4-AM modulation scheme (4 levels amplitude modulation)[10].

Refer to a binary pulse format, the Rx or receiver will count for the photons number in that pulse interval T_D. Then, it will estimate what is that the transmitted symbol. In the case of 4-level photon of Fig. 1, when all
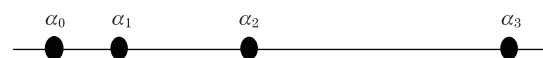

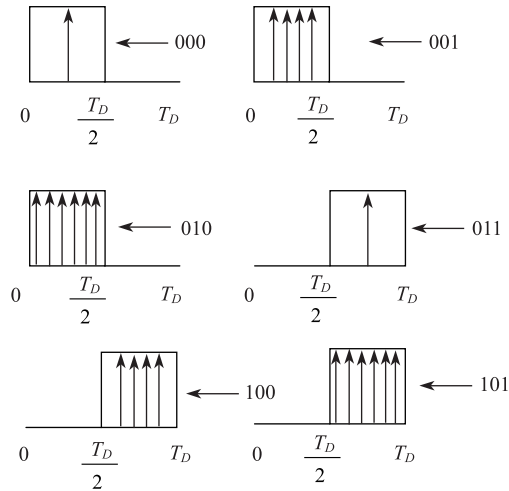Fig. 1. Format of a 4-level photon communication signals.

Fig. 2. Samples of PPM format with 4-level photon communications.

signal's occurrence probabilities are equal, the average T_x power is then

$$P_4 = \frac{1}{4T_D} \sum_{i=0}^{3} \alpha_i. \tag{3}$$

Considering for regular 4-AM with a photon channel, their signals of the occupied signal sets are equally spaced. For the considered 4-level photon communications from Fig. 1, signals are defined differently on their spaces. Meanwhile, signals α_i (i=0, 1, 2, 3) are given respectively by[11]

$$\begin{aligned} \alpha_0 &= K_b, \\ \alpha_1 &= \alpha + K_b, \\ \alpha_2 &= 4\alpha + K_b, \\ \alpha_3 &= 9\alpha + K_b, \end{aligned} \tag{4}$$

where α_3 corresponds to the peak power of light and K_b represents to the background noise. Obviously when α_0 equals to zero, there is no photon and means that there is no pulse in that time interval T_D.

In the following content, multilevel photon communications mentioned in previous background, is working jointly with the PPM scheme. Basically, this PPM bases on a concept of light pulse intensity that places on one of two beside time slots and it then represent to be a data block. From Fig. 2, a sample shows 4-level photon with two slots constructs a binary PPM (BPPM) or a word covering time with T_D seconds. In order to determine the data word, it is done by consideration on the pulse location in the frame and average number of photons. As from the previous mentioned background, there will be no pulse in that time interval T_D when α_0 is applied. Consequently, three groups of pulses as in Fig. 2 are solely used. Thus, each data word represents with three bits, as in six pulse formats, is the minimum requirement of this kind of modulation.

The performance of above proposed joint modulation scheme is presented as following. Firstly, channel model for photon communication is generally described by supposing a light pulse with that the average number of photons (α). That pulse is transmitted with existing noise

due to the background (and/or detector) dark current K_b during T_D second. Then, from Ref. [11] the probability of observing k photons at the receiver is

$$\Pr[k\,|\alpha] = \frac{(\alpha + K_b)^k e^{-(\alpha + K_b)}}{k!}. \tag{5}$$

For instance, the probability of observing k photons during the time interval at the receiver in this binary photon case, is given by

$$P[k\,|1] = \frac{(\alpha + K_b)e^{-(\alpha + K_b)}}{k!}, \tag{6}$$

$$P[k\,|0] = \frac{K_b e^{-K_b}}{k!}. \tag{7}$$

In this case, bit error probability is calculated by using threshold (m_T). Its decoding error may occur if $m < m_T$ when a "1" bit is sent, or in opposite if $m > m_T$ when a "0" bit is sent[9,10]. The bit probability of error (PE) for equally probable bits in decoding is thus,

$$\begin{aligned} \text{PE} =& \frac{1}{2} \sum_{k=0}^{m_T} \frac{(\alpha + K_b)^k e^{-(\alpha + K_b)}}{k!} \\ &+ \frac{1}{2} \sum_{k=m_T}^{\infty} \frac{K_b^k e^{-K_b}}{k!}, \end{aligned} \tag{8}$$

$$m_T = \frac{\alpha}{\log(1 + \frac{\alpha}{K_b})}. \tag{9}$$

Next, an error probability of a 4-level photon system from Eq. (8) is applied. Thus, the word probability of error (PWE) is that the PWE as

$$\begin{aligned} \text{PWE} =& 1 - \frac{1}{4} \sum_{k=0}^{m_{T1}} \frac{K_b^k e^{-K_b}}{k!} \\ &- \frac{1}{4} \sum_{k=m_{T1}}^{m_{T2}} \frac{(\alpha + K_b)^k e^{-(\alpha + K_b)}}{k!} \\ &- \frac{1}{4} \sum_{k=m_{T2}}^{m_{T3}} \frac{(4\alpha + K_b)^k e^{-(4\alpha + K_b)}}{k!} \\ &- \frac{1}{4} \sum_{k=m_{T3}}^{\infty} \frac{(9\alpha + K_b)^k e^{-(9\alpha + K_b)}}{k!}, \end{aligned} \tag{10}$$
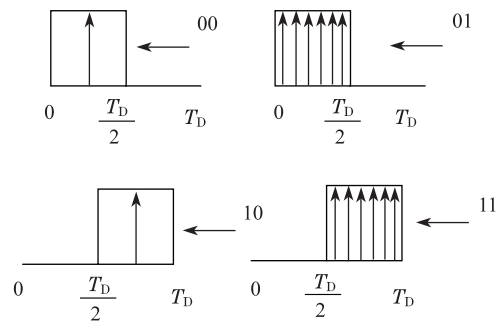

Fig. 3. Reformed 2-level photon patterns with PPM matching with punctuated convolutional coding.
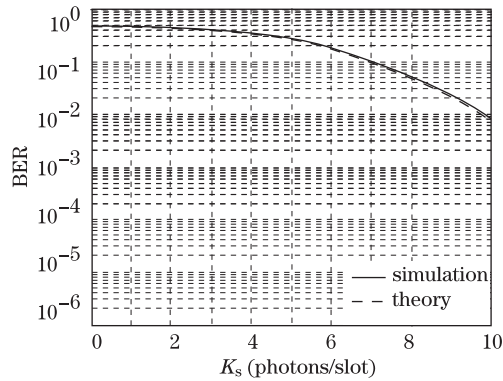
Fig. 4. (Color online) Analysis and simulation of 4-level photon communication system with BPPM in Possion channel, K_b is 4.
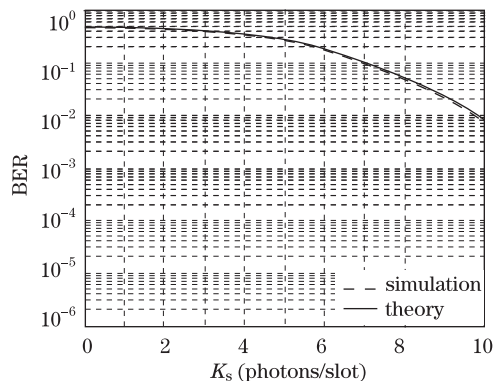

Fig. 5. (Color online) PWE of a 4-level photon communication versus its version with BPPM.

where m_{T1}, m_{T2}, and m_{T3} are m_T, $4m_T$, and $9m_T$, respectively. Bit PE from Ref. [9] is $\frac{\frac{M}{2}}{M-1}$PWE, where M is the total number of signal formats, thus

$$\text{PE} = \frac{2}{3}\text{PWE}. \tag{11}$$

Principally, error probability of BPPM is considered on background noises which occurs at each pulse slot. Next, it is comparable that probability of slots containing no pulse, equals or higher than that probability of signaled pulse. Then, a BPPM bit error probability is

$$\text{PE} = \sum_{k_1=0}^{\infty}\sum_{k_2=k_1+1}^{\infty}\text{Pos}(k_1, K_s + K_b)\text{Pos}(k_2, K_b)$$
$$+ \frac{1}{2}\sum_{k_1=0}^{\infty}\text{Pos}(k_1, K_s + K_b)\text{Pos}(k_1, K_b), \tag{12}$$

$$\text{Pos}(k, m) = \frac{m^k e^{-m}}{k!}, \tag{13}$$

where K_s, K_b are the signal and noise counts per pulse interval.

For a 4-level photon case combined with BPPM modulation, PWE can be derived from the pairwise error probability based on bit PE of BPPM and also with word error probability of 4-level photon case previously. Thus,

PWE of 4-level photon with BPPM is derived over Poisson distribution environment as

$$
\begin{aligned}
\text{PWE} =& 1 - \frac{1}{3}\sum_{k_1=1}^{m_{T1}}\sum_{k_2=0}^{k_1-1}\text{Pos}(k_1, K_s + K_b)\text{Pos}(k_2, K_b) \\
& - \frac{1}{3}\sum_{k_1=m_{T1}}^{m_{T2}}\sum_{k_2=0}^{k_1-1}\text{Pos}(k_1, 4K_s + K_b)\text{Pos}(k_2, K_b) \\
& - \frac{1}{3}\sum_{k_1=m_{T2}}^{\infty}\sum_{k_2=0}^{k_1-1}\text{Pos}(k_1, 9K_s + K_b)\text{Pos}(k_2, K_b) \\
& - \frac{1}{6}\sum_{k_1=0}^{m_{T1}}\text{Pos}(k_1, K_s + K_b)\text{Pos}(k_1, K_b) \\
& - \frac{1}{6}\sum_{k_1=m_{T1}}^{m_{T2}}\text{Pos}(k_1, 4K_s + K_b)\text{Pos}(k_1, K_b) \\
& - \frac{1}{6}\sum_{k_1=m_{T2}}^{\infty}\text{Pos}(k_1, 9K_s + K_b)\text{Pos}(k_1, K_b).
\end{aligned}
\tag{14}
$$

Next, Eq. (14) implies that the compared signals between of the first and the second slot of that BPPM, is done. Then, the threshold decision concept is followed. Finally, the bit probability of error is thus

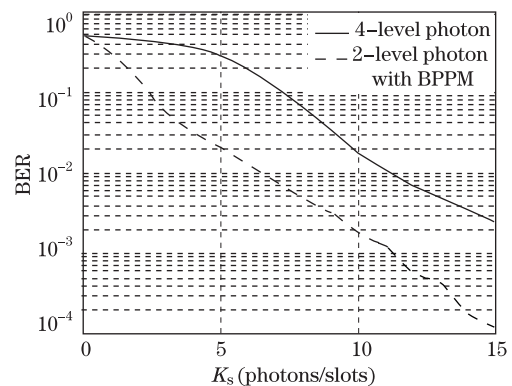$$\text{PE} = \frac{3}{5}\text{PWE}. \tag{15}$$

Fig. 6. BER of 4-level photon communication versus reformed 2-level photon format with BPPM at 2 bit/s/Hz.
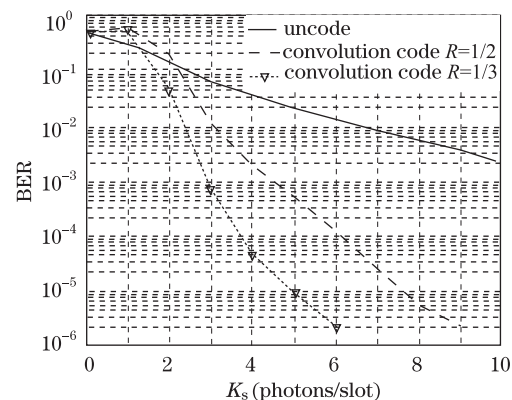

Fig. 7. Performance comparison of 2-level photon communication with BPPM and convolutional codes.

In order to investigate for additional system performance improving continuously from above main proposed method, error control coding is one of those possibilities. We then select a kind of root coding scheme based on convolutional method which could be expanded into higher advance coding scheme systematically. In the same hand, when the simple convolutional coding structure is proved, higher performance scheme such as turbo codes or low-density parity-check (LDPC) code could be certainly adopted at next. This code is generated by passing the information sequence to be transmitted through a linear finite-state shift register. Briefly, the shift register consists of K stages (constraint length) and n linear algebraic function generators. The input data are shifted into and along the shift register k bits at a time. The number of output bits for each k-bit input sequence is n bits. Consequently, codes rate defines $R_c = k/n$, consistent with the definition of the codes rate for a block codes. A convolutional codes is usually defined in terms of generator sequence, denoted by $g_1, g_2, \ldots, g_n$. The ith component of g_j, $1 \leqslant i \leqslant k$ and $1 \leqslant j \leqslant n$, is "1" if the ith element of the shift register is connected to the combiner corresponding to the jth bit in the output and "0" otherwise.

Generally for this scheme, coded sequence could be punctuated to match with fixed size of transmitted frame or packet yielding not much performance degradation. It is then a proper concept and selected to work with multilevel pulse modulation format in the following paragraphs, where 4-level scheme is reduced (or punctuated) to be halve. Finally for decoding of convolutional codes, the Viterbi algorithm is widely applied as also using in this work.

Obviously, from the principle of 4-level photon communication with BPPM, all pulse formats are not to support for all data patterns. To simplify for this initial step in order to using with previous mentioned code, the 4-level forms are reformed to be 2-level photon format as proposed in Fig. 3 matching with punctuated convolutional codes. The optimum average number of photon bases on square distance[11] between two α_i which clarified by $S = (X_1, X_2)$, where $X_1, X_2 \in (\alpha_1, \alpha_2, \alpha_3)$. The average numbers of photon are α_1 and α_3. The optimum is m_{T2} which studied and considered from three decision points of 4-level photon communication with BPPM[12].

Considering for uncoded 2-level photon communication with BPPM, number of slots per symbol is 2. Thus, bandwidth (B) is $2/(T_D/2)$ Hz. Number of bits per symbol is 2 bits/symbol, hence, bit rate (R_b) is $2/(T_D/2)$ bit/s. Bandwidth efficiency (ρ) is then

$$\rho = \frac{R_b}{B} = 2 \text{ bit/s/Hz.} \tag{16}$$

(This is simply verified to that of on/off keying scheme, where ρ is just halved or 1 bit/s/Hz). When an average background noise per slot is K_b, an average background for 2-level photon with BPPM is then $2K_b$. Next, convolution code is combined with this 2-level photon format with BPPM with code rate R_c. An average background noise per uncoded symbol applied from Ref. [13] is then derived as $K_b' = 2K_b/R_c$, and averaged background noise per uncoded bit is thus $K_b'' = K_b/R_c$.

Next, results of our combined multilevel modulation

method with coding for efficient bandwidth-utilization and performance improvement are shown in four points of view. Their analysis and simulation are done with specific conditions shown as the following.

a) In order to proving our proposed modulation scheme from above paragraphs, combination of 4-level photon communication with BPPM in Possion channel (K_b is 4) by varying average number of photon per slot (K_s), is firstly verified. Resulting in Fig. 4, both analysis and simulation results are coincided. Our proposed concept is then proved.

b) Focusing on modulation performance, a comparison of proposed multilevel photon scheme with PPM in photon channel is compared with that pure multilevel photon scheme. Figure 5 shows the word error rate between them showing as a function of K_s for fixed value of K_b at three. It denotes that word error of the proposed method presents with a lower probability. Moreover, it implies of obtaining higher precision on signal detection for those both adoptions, pulse and threshold decisions.

c) Comparison at fixed number of throughput at 2 bit/s/Hz, simulation of pure 4-level photon communication (2 bit/s and time interval as T) vs proposed 2-level photon communication with BPPM is resulted in Fig. 6. It is noted that simulation varies average photon per slot (K_s) and fixes average noise per slot (K_b) at 4. As a result, the latter scheme achieves higher performance. This can be concluded that our proposed method examines both optimum position and average number of photon as the reason.

d) For performance improvement with coding, next result for those 2-level photon patterns with BPPM in Fig. 3 matching with coding packet is illustrated in Fig. 7. It depicts the result comparison of proposed 2-level photon with BPPM between with and without convolutional codes. In this case, the average background noise per slot is 4. Generator matrix for code rate $R_c = 1/2$ is [1 0 1] and [1 1 1], and for $R_c = 1/3$ is [0 1 1], [1 0 1] and [1 1 1] with the same constraint length at 3. As a result, using matched-convolutional codes with 2-level photon communication with BPPM obtains gain accordingly to its used rate.

In conclusion, this letter proposes a successful concept of multilevel photon communication system with a BPPM scheme. It is a kind of bandwidth-efficient modulation method that improves system performance. Moreover, deployment of error coding scheme is also another technique that can be applied. Its total performance presents as a promising system that can be developed continuously for having the further optimal and more flexible structures. That is, for example, considering jointly with multilevel error control coding. Furthermore, this multilevel modulation is also surveying and found several potential applications. There are such as free space or turbulence optical channel, indoor broadband communications[6], quantum key distribution[7], and also in ultra wideband communication[8], where its propagation is similar with that lightwave environment.

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