

# A method for determining cirrus height with multiple scattering

Xinglong Xiong (熊兴隆), Meng Li (李 猛)\*, Lihui Jiang (蒋立辉), and Shuai Feng (冯 帅)

Tianjin Key Laboratory for Advanced Signal Processing, Civil Aviation University of China, Tianjin 300300, China

\*Corresponding author: 867750570@163.com

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An approach for determining cirrus height with multiple scattering effect using data from a Mie scattering lidar is proposed. We compute the exact extinction coefficients of cirrus via altitude. The regulated height of cirrus is obtained through multiple scattering factors. Experimental result demonstrates that the proposed approach can be used to determine effectively cirrus height with multiple scattering.

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Lidar is used for remote sensing and has been extensively applied for target detection. Raman and Mie scattering lidars are currently the most commonly used lidars. Compared with the Mie scattering signal, the return signal of the Raman lidar is 1000 to 10000 times weaker, and its structure is significantly complex<sup>[1,2]</sup>. Therefore, Mie scattering lidar is used in civil aviation as the main tool in measuring cirrus height. Well-known methods for measuring cirrus height are the zero-crossing method<sup>[3]</sup> and its variants<sup>[4,5]</sup>. These methods can be used for single scattering, but produces a relatively large error<sup>[6–8]</sup> in the retrieved cirrus height for serious multiple scattering in cirrus clouds. This problem is solved in this letter.

Platt *et al.*<sup>[9]</sup> firstly stressed the consideration of multiple scattering effect in cirrus clouds. Kunkel *et al.*<sup>[10]</sup> proposed experimental models on multiple scattering. In 1995, Platt *et al.*<sup>[9]</sup> proposed multiple scattering factors, with simpler and more practical description of the degree of multiple scattering. The calculation methods for multiple scattering factors include Platt's<sup>[9]</sup> and Chen's methods<sup>[11]</sup>. Platt's method used distance as variable to calculate multiple scattering factors, yielding more accurate results. The calculation methods used in the models are the Monte Carlo (MC) method and its variants<sup>[12–14]</sup>. The MC method is a powerful numerical technique for multiple scattering computing and exhibits many advantages that can be used to calculate multiple scattering factors, which shows good agreement with experimental results<sup>[15]</sup>.

In this letter, we propose an approach for determining cirrus height based on multiple scattering factors.

Compared with the result from a conventional method by checking the zero-cross variation in elastic-scattering signal, the obtained cirrus height using the proposed approach is more accurate. The cirrus-bottom height is firstly determined using the zero-crossing method, and the extinction coefficient of the cirrus bottom is then obtained using the method given in Ref. [16]. The nonlinear equation with cirrus lidar ratio and cirrus-top extinction coefficient as variables is solved, and the lidar ratio can be obtained. The extinction coefficients of cirrus are calculated using Fernald's method<sup>[17,18]</sup>. We acquire multiple scattering factors using the MC and Platt's methods. With multiple scattering factors and the extinction coefficient curve, the error of the cirrus-bottom height is computed using the geometric method. The precise height of the cirrus bottom can then be derived. Simulation results of real Mie scattering lidar data show that the proposed approach effectively determines cirrus height with multiple scattering.

The extinction coefficients are derived as follows. The lidar ratio is solved firstly. Then, the extinction coefficient is retrieved using Fernald's method.

We derive the lidar ratio through the following steps. 1) The initial heights of the cirrus top and bottom, named as  $z_t$  and  $z_b$ , respectively, can be acquired using the zero-crossing method. The extinction coefficient of the cirrus bottom  $X_1$  is determined according to Ref. [16]; 2) Using the extinction coefficient of the cirrus top  $X_2$  and the cirrus lidar ratio  $S_a$  as variables, a nonlinear equation is established as

$$\left\{ \begin{array}{l} \int_{z_b}^{z_t} \sigma_a(z') dz' = \int_{z_b}^{z_t} \left\{ -S_a \sigma_m(z) / S_m + \frac{S(z) \cdot \exp \left[ 2(S_a/S_m - 1) \cdot \int_z^{z_t} \sigma_m(z') dz' \right]}{S(z_t) / [X_2 + S_a \sigma_m(z_t) / S_m] + 2 \int_z^{z_t} S(z) \cdot \exp \left[ 2(S_a/S_m - 1) \cdot \int_z^{z_t} \sigma_m(z') dz' \right] dz} \right\} dz \\ X_1 = -S_a \sigma_m(z_b) / S_m + \frac{S(z_b) \cdot \exp \left[ 2(S_a/S_m - 1) \cdot \int_{z_b}^{z_t} \sigma_m(z') dz' \right]}{S(z_t) / [X_2 + S_a \sigma_m(z_t) / S_m] + 2 \int_{z_b}^{z_t} S(z) \cdot \exp \left[ 2(S_a/S_m - 1) \cdot \int_{z_b}^{z_t} \sigma_m(z') dz' \right] dz} \\ \tau = \int_{z_b}^{z_t} \sigma_a(z') dz' = -\ln [T(z_b, z_t)] \\ T(z_b, z_t) = [P_t z_t^2 / P_b z_b^2]^{1/2}. \end{array} \right. \quad (1)$$

Solving Eq. (1), the lidar ratio of cirrus can be obtained.

In Eq. (1),  $S(z)$  can be calculated as  $S(z) = P(z)z^2$ , where  $P(z)$  is the power of the lidar return signals.  $T$  is the transmission of cirrus clouds, and  $\tau$  is the optical depth.  $\sigma_a$  and  $\sigma_m$  represent the aerosol (cirrus) extinction coefficient and the extinction coefficient of the atmosphere molecule, respectively.  $S_m = 8\pi/3$ , which is

$$\sigma_a(z) = \begin{cases} -S'_a\sigma_m(z)/S_m + \frac{S(z) \cdot \exp[2(S'_a/S_m - 1) \cdot \int_z^{z_b} \sigma_m(z') dz']}{S(z_b)/[X_1 + S'_a\sigma_m(z_t)/S_m] + 2 \int_z^{z_b} S(z) \cdot \exp[2(S'_a/S_m - 1) \cdot \int_z^{z_b} \sigma_m(z') dz'] dz} & (0 < z < z_b) \\ -S_a\sigma_m(z)/S_m + \frac{S(z) \cdot \exp[2(S_a/S_m - 1) \cdot \int_z^{z_t} \sigma_m(z') dz']}{S(z_t)/[X_2 + S_a\sigma_m(z_t)/S_m] + 2 \int_z^{z_t} S(z) \cdot \exp[2(S_a/S_m - 1) \cdot \int_z^{z_t} \sigma_m(z') dz'] dz} & (z_b < z < z_t) \end{cases} \quad (2)$$

Outside cirrus, the lidar ratio  $S'_a$  is assumed to have a constant value of 50 because it only slightly influences the extinction coefficient of cirrus. In cirrus, the lidar ratio  $S_a$  can be calculated using Eq. (1).

The multiple scattering factor  $\eta$  is an important parameter representing the degree of multiple scattering. Outside cirrus,  $\eta$  is assumed to be 1. Inside cirrus, Ref. [15] indicates that the multiple scattering factor  $\eta$  can be calculated as

$$\begin{cases} \eta(z) = 1 - \frac{\ln(m)}{2\delta(z)} \\ \delta(z) = -\int_0^z \sigma_a(z) dz \end{cases}, \quad (3)$$

where  $m$  represents the ratio of multiple scattering and signal scattering, which can be calculated using the MC method according to Ref. [15]. The parameters of the MC simulation are presented in Table 1. Using Eq. (3), the multiple scattering factor  $\eta(z)$  can be obtained.

Considering multiple scattering, the formulation of lidar is expressed as

$$P(z) = P_0 C z^{-2} [\beta_m(z) + \beta_a(z)] \cdot \exp\left\{-2 \int_{z_0}^z \eta[\sigma_a(z) + \sigma_m(z)] dz\right\}, \quad (4)$$

where  $\beta_a$  and  $\beta_m$  represent the aerosol (cirrus) backscatter coefficient and the backscatter coefficient of the atmosphere molecule, respectively.

Through the geometric method demonstrated in Fig. 1, the error of cirrus height caused by multiple scattering

**Table 1. Parameters of MC Simulation**

Parameter	Value
Wavelength (nm)	532
Beam Divergence Half Angle (mrad)	0.05
Telescope Receiver Area (m <sup>2</sup> )	1
Field of View (FOV) (mrad)	1
Single Scatter Albedo $\omega$	1
Phase Function	Heymsfield and Platt Model
Refractive Index	$1.31 + 2.5 \times 10^{-9}i$
Photo Number (m)	$10^6$
Max Scattering Order $N$	5

assumed according to Rayleigh scattering theory. The first equation is obtained by integrating both sides of Fernald's retrieval equation. The second equation is a form of Fernald's formulation, with  $z=z_b$ . The third and fourth equations were given in Ref. [11].

After deriving the lidar ratio, we express the extinction coefficient given by Fernald as

can be presented as

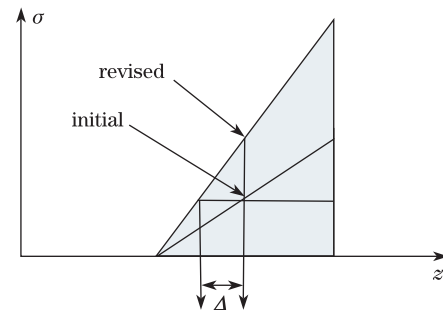
$$\begin{cases} \Delta_b = \frac{\eta(z) \left(\frac{X_1}{\eta} - X_1\right)}{\frac{d\sigma(z)}{dz}}, & z = z_b \\ \Delta_t = \frac{\eta(z) \left(\frac{X_2}{\eta} - X_2\right)}{-\frac{d\sigma(z)}{dz}}, & z = z_t \end{cases} \quad (5)$$

The regulated height of the cirrus top and bottom can be expressed as

$$\begin{cases} z'_b = z_b - \Delta_b \\ z'_t = z_t + \Delta_t \end{cases} \quad (6)$$

After having been regulated by multiple scattering factors, the extinction coefficient curve increases. Using the geometric method, we can obtain the deviation in distance. In Eq. (5),  $\Delta_b$  represents the error of the cirrus-bottom height, and  $\Delta_t$  represents the error of the cirrus-top height. When the multiple scattering factor  $\eta$  is close to 1,  $\Delta$  approaches 0. Therefore, if the multiple scattering is not serious, the initial height is almost equal to the regulated height, and the demonstrated approach is still effective. If  $\eta$  is smaller, the error will be relatively large. The extinction coefficient curve significantly influences  $\Delta$ . If the curve of  $\sigma(z)$  has precipices on the point of the cirrus bottom or top, the error  $\Delta$  will be small. Otherwise,  $\Delta$  will be large.

Two experiments were conducted on real lidar return signals offered by the Anhui Institute of Optics and Fine Mechanics, Chinese Academy of Sciences. The power of the lidar return via altitude is presented in Figs. 2(a) and (c). The extinction coefficients via altitude without



**Fig. 1. Geometric diagram of the error  $\Delta$  of cirrus height caused by multiple scattering.**

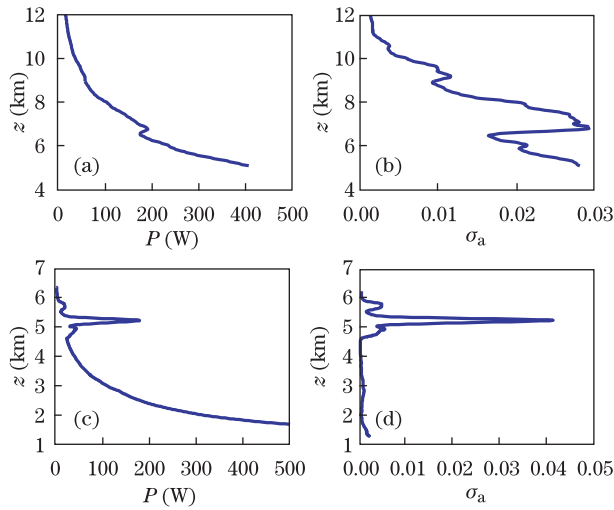


Fig. 2. (a) Experimental lidar signals and (b) extinction coefficient with serious multiple scattering. (c) Experimental lidar signals and (d) extinction coefficient with little multiple scattering.

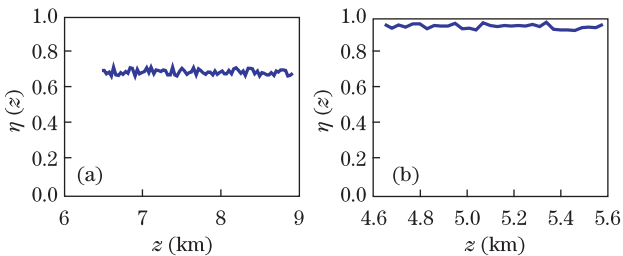


Fig. 3. Multiple scattering factor  $\eta(z)$  of (a) the first and (b) the second experiments calculated using the MC method.

multiple scattering effect are shown in Figs. 2(b) and (d).

The power of the lidar return and the extinction coefficient via altitude in the first experiment are presented in Figs. 2(a) and (b), in which multiple scattering is serious. According to the aforementioned steps, the initial heights of the cirrus bottom and top are 6.48 and 8.91 km, respectively. From Fig. 3(a), the multiple scattering factors  $\eta(6.48)$  and  $\eta(8.91)$  are 0.707 and 0.685, respectively. The error of the cirrus-bottom height  $\Delta_1$  is 0.18 km and that of the cirrus-top height  $\Delta_2$  is 0.53 km. After regulation, the bottom height  $z'_t$  is 6.30 km, and the top height  $z'_b$  is 9.44 km. Given that  $\eta$  is much smaller in this experiment, the errors become 0.18 and 0.53 km. These large values reflect that multiple scattering may seriously affect cirrus height.

The power of the lidar return and the extinction coefficient via altitude in the second experiment are shown in Figs. 2(c) and (d), in which multiple scattering is not serious. Similar to the aforementioned steps, the regulated heights of the cirrus bottom and top are 4.65 and 5.58 km, respectively. According to Fig. 3(b), the calculated  $\eta(4.65)$  is 0.946 and  $\eta(5.58)$  is 0.96. Given that the results are close to 1, the errors of the cirrus

bottom and top are 0.002 and 0.001 km, respectively. These errors can be neglected completely, because the resolution ratio of cirrus height is 0.03 km. Therefore, in the case of little multiple scattering, the regulated heights are nearly equal to the initial values obtained using the zero-crossing method.

In conclusion, we present a technique for determining cirrus height, considering multiple scattering effects. Through the multiple scattering factors and exact extinction coefficient curve of cirrus, the regulated height of cirrus can be obtained. The proposed technique fully considers multiple scattering, resulting in accurate cirrus height. The two experiments on real lidar return signals show that the demonstrated approach is effective.

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