Novel droplet size and complex refractive index measurement based on rainbow detection

Qi Pan (潘 琦)

College of Information & Control, Nanjing University of Information Science & Technology, Nanjing 210044, China

**Corresponding author: qpan@nuist.edu.cn*

Received July 13, 2012; accepted October 10, 2012; posted online December 28, 2012

A novel method to measure droplet size and complex refractive index simultaneously by rainbow detecting is presented. A new mathematic model for rainbow pattern of absorbing droplet is built. Based on this model, a series of new formulas to measure droplet imaginary part of refractive index is derived. Then a new inverse algorithm for simultaneously measure droplet size and the complex refractive index is presented, which is verified by simulation experiments under different conditions.

OCIS codes: 290.3200, 290.5820, 290.5850.

doi: 10.3788/COL201210.S22901.

Light scattering at the rainbow angle by droplet has been developed as a non-intrusive technique to measure droplet size and refractive index[1−3], with the assumption that the droplets are no absorbing. However, it is already found that particle absorbing coefficient, which is characterized by the imaginary part of refractive index[4], has a unique influence on absorbing droplet rainbow pattern.

In this letter, a potential use of rainbow detection to measure droplet imaginary part of refractive index is analyzed. Based on a new mathematic model, a series of new formulas to measure droplet imaginary part of refractive index is derived, and a new inverse algorithm for simultaneously measuring droplet size and the complex refractive index is also presented, which is verified by simulation experiments under different conditions.

The rainbow light is mainly formed by the rays experienced one internal reflection, with the ray path is shown in Fig. 1. According to Bouguer principle[4], the rainbow light of the absorbing droplet is attenuated exponentially with the ray path inside the droplet, so the formula is derived as

$$I = I_{\mathrm{r}}(x, m_1) \exp\left(\frac{-4\pi m_2 S}{\lambda}\right), \tag{1}$$

where $I_{\mathrm{r}}(x, m_1)$ is the rainbow light intensity when droplet has no absorption, and can be determined by size parameter x and real part of refractive index m_1; m_2 is imaginary part of refractive index; S is the ray path length inside the droplet of the rainbow light. According to Fig. 1, the relation of S and the light incident angle can be written as

$$S = 2L = 4a \sin\tau'. \tag{2}$$

As $I_{\mathrm{r}}(x, m_1)$ can be calculated by Airy theory, Eq. (1) can be written as

$$\begin{aligned} I =& 4I_0 \left[\frac{3a^2\lambda}{4h\cos(\theta-\theta_{\mathrm{rg}})}\right]^{\frac{2}{3}} f^2(z) \\ &\cdot \exp\left(\frac{-16\pi m_2 a}{\lambda}\sqrt{\frac{m_1^2-\cos\tau^2}{m_1^2}}\right), \end{aligned} \tag{3}$$

where $f(z)$ is the Airy integral; θ is the scattering angle; θ_{rg} is the geometry rainbow angle; h is the parameter related to real part of refractive index. It is known that the values of $f^2(z)$ are 1.0075 and 0.6165 at the first and second rainbow peak positions[5]. Put the two values into Eq. (3) respectively, and calculate the ratio as

$$\frac{I_1}{I_2} = 1.6342\left[\frac{\cos(\theta_2-\theta_{rg})}{\cos(\theta_1-\theta_{rg})}\right]^{\frac{2}{3}} \exp\left(\frac{-16\pi m_2 a}{\lambda m_1}\delta\right), \tag{4}$$

where I_1 and I_2 are the light intensities of the first and second rainbow peaks, respectively; θ_1 and θ_2 are the scattering angles of first and second rainbow peaks, respectively; δ is written as

$$\begin{aligned} \delta = \Big(&\sqrt{m_1^2-\cos\tau_{11}^2}+\sqrt{m_1^2-\cos\tau_{12}^2}-\sqrt{m_1^2-\cos\tau_{21}^2} \\ &-\sqrt{m_1^2-\cos\tau_{22}^2}\Big)/2, \end{aligned} \tag{5}$$

where τ_{11} and τ_{12} are incident angles corresponding to θ_1; τ_{21} and τ_{22} are incident angles corresponding to θ_2. Based on Airy theory, $\cos(\theta_2-\theta_{rg})/\cos(\frac{h\pi^2}{12})^{\frac{1}{3}}\cdot x^{-\frac{2}{3}}(\theta_1-\theta_{rg})$ can be generalized as

$$\frac{\cos(\theta_2-\theta_{rg})}{\cos(\theta_1-\theta_{rg})} = \frac{\cos\left[3.46687\left(\frac{h\pi^2}{12}\right)^{\frac{1}{3}} x^{-\frac{2}{3}}\right]}{\cos\left[1.08728\left(\frac{h\pi^2}{12}\right)^{\frac{1}{3}} x^{-\frac{2}{3}}\right]}. \tag{6}$$

In order to simplify Eq. (6), A is represented for $(\frac{h\pi^2}{12})^{\frac{1}{3}}\cdot x^{-\frac{2}{3}}$. And it can be calculated that when the droplet size parameter is big enough, the absolute value

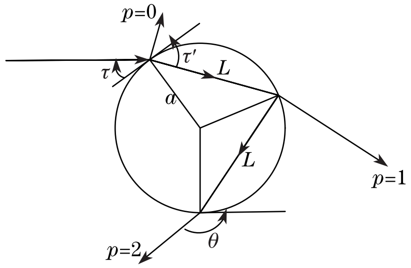

Fig. 1. Sketch of rainbow ray path.

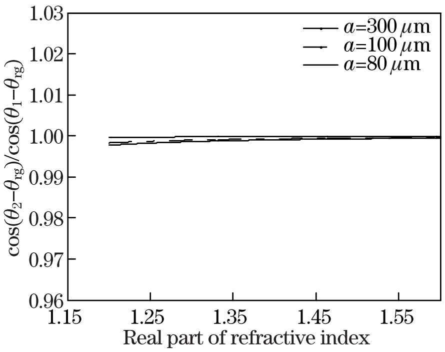

Fig. 2. Results of $\cos(\theta_2-\theta_{\rm rg})/\cos(\theta_1-\theta_{\rm rg})$ under different conditions (λ=0.632 μm).

of A is much smaller than 1. So Eq. (6) can be simplified by series expansion:

$$\begin{aligned}\frac{\cos(\theta_2-\theta_{rg})}{\cos(\theta_1-\theta_{rg})}&=\left[1-\frac{(3.46687A)^2}{2}+\frac{(3.46687A)^4}{24}\right]\times\\&\left[1+\frac{(1.08728A)^2}{2}+5\frac{(1.08728A)^4}{24}\right]+O\left(A^6\right)\\&=1-5.4185A^2+3.9422A^4+O\left(A^6\right)\\&\approx 1.\end{aligned}\qquad(7)$$

To verify Eq. (7), simulation is taken, which is shown in Fig. 2. It is shown that the bigger droplet size and real part of refractive index are, more close to 1 the value of $\cos(\theta_2-\theta_{\rm rg})/\cos(\theta_1-\theta_{\rm rg})$ is. The greatest difference between the value and 1 in Fig. 2 is smaller than 0.003, which indicates the approximation of Eq. (7) can be established in most cases.

Using Eq. (7), Eq. (4) can be simplified as

$$m_2=C_1\frac{m_1}{a\delta}\log\left(\frac{I_1}{C_2I_2}\right),\qquad(8)$$

where C_1 and C_2 are coefficients, which can be determined by calibration.

The rainbow pattern of absorbing droplet is influenced by droplet size and complex refractive index. It is already known that the rainbow peak position is mainly related to real part of refractive index, and the rainbow fringe space is mainly determined by droplet size. For droplet imaginary part of refractive index, which represents for droplet absorption, a simulated comparison is taken and shown in Fig. 3. It is shown that the rainbow peak position and the fringe space are not changed for droplets with different imaginary parts of refractive index, though the intensity is reduced. In other words, the inversing process of droplet size and real part of refractive index is insensitive to the imaginary part of the refractive index[6].

So according to Airy theory and geometric optics, by detecting the first and second rainbow peak positions, two formulas can be derived as

$$a=\frac{\lambda}{8}\left[\frac{3}{(m_1^2-1)}\left(\frac{4-m_1^2}{m_1^2-1}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}\left[\frac{2.37959}{\theta_2-\theta_1}\frac{180°}{\pi}\right]^{\frac{3}{2}},\qquad(9)$$

$$\begin{aligned}\theta_1&=4\cos^{-1}\left(\sqrt{\frac{4-m_1^2}{3m_1^2}}\right)-2\sin^{-1}\left(\sqrt{\frac{m_1^2-1}{3}}\right)\\&+\frac{1.0845}{4\left(m_1^2-1\right)^{\frac{1}{2}}}\left[\frac{3\lambda^2\left(4-m_1^2\right)^{\frac{1}{2}}}{a^2}\right]^{\frac{1}{3}}\frac{180°}{\pi}.\end{aligned}\qquad(10)$$

To solve Eqs. (9) and (10), an assumptive refractive index is taken into Eq. (9) to calculate the particle radius, then the refractive index is estimated according to Eq. (10). The above process is repeated until the best value of refractive index and radius are obtained. As the Airy theory is an approximate theory compared with exact light scattering theory, the difference between the two theories increases with the particle size decreasing. So when the particle size is small, an inversion algorithm based on Debye theory is used[7].

Using inversed droplet size and real part of refractive index, according to Eq. (8), the imaginary part of refractive index can be determined. So this novel rainbow detecting method can measure droplet size and the complex refractive index simultaneously.

Comparing with normal droplet absorption measurements, the method need not to know original incident light intensity, which greatly simplify the detecting setup. As the method can measure droplet size and the complex refractive index simultaneously, prior knowledge of the ray path of the measured object is not needed to know, which also simplifies the measuring process. This method is based on rainbow light detecting, the unique interference fringe structure of rainbow scattering is easy to observe, which avoids the receiving positioning problems in traditional methods. In addition, this method is a non-invasive and on-line measurement. However, this method is limited by the rainbow visibility, and it is applicable to small imaginary part of refractive index under certain wavelength.

Applying the method presented in this letter, we numerically simulated the rainbow parameter inversion under different droplet size and complex refractive index conditions, with the results shown in Figs. 4 and 5. The real rainbow scattering light distribution is simulated by Mie theory. It is shown that the inversed droplet size and real part of refractive index have a good precision compared with real value and keep stable under different imaginary part of refractive index conditions, which

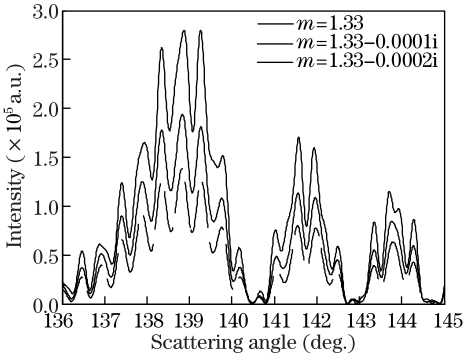

Fig. 3. Comparison of rainbow scattering pattern of droplets with different absorbing coefficients (a=50 μm, λ=0.506 μm).

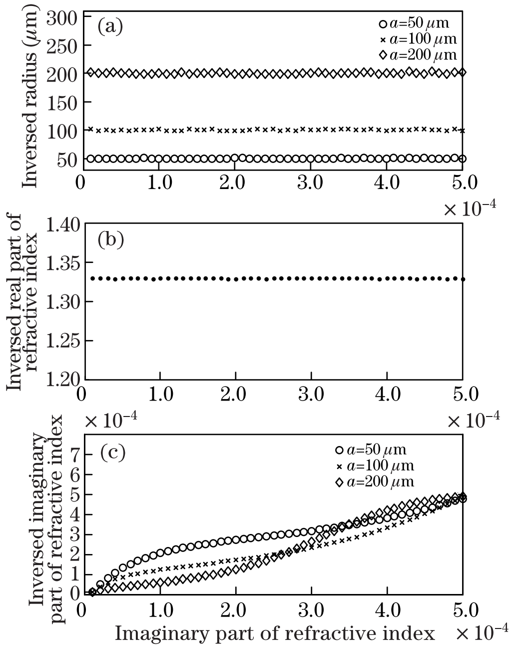

Fig. 4. Inversed results under different conditions (λ=0.506 μm, m_1=1.33).

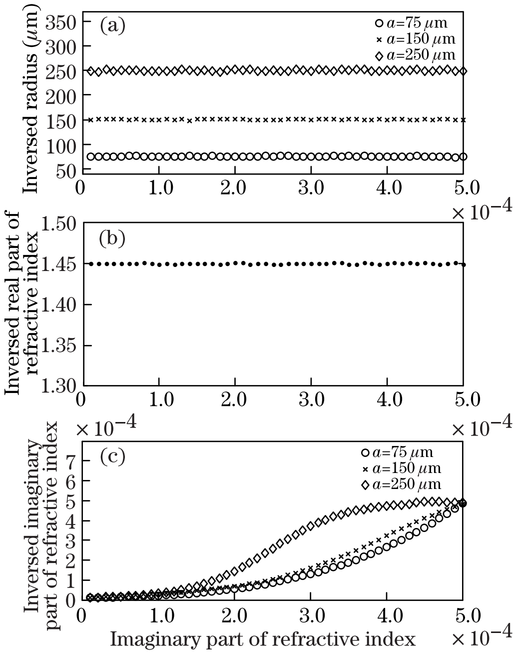

Fig. 5. Inversed results under different conditions (λ=0.506 μm, m_1=1.45).

indicates the imaginary part of refractive index influence the size and real part of refractive index inversing process little.

During the imaginary part of refractive index inversion, a calibration step is induced, which calibrated the inversed results at imaginary part of refractive index values of 0.0005 and 0, then the values of C_1 and C_2 can be determined. It is shown that the inversion results of the imaginary part of refractive index under different droplet sizes do not keep stable, but the inversion results are monotonically rising with the increase of the real value. This indicates that when droplet size changes greatly, the inversed results of imaginary part of refractive index is influenced by droplet size, and this systematic error is induced by Airy theory and approximate formula. It is also shown that because of the applied calibration method, the differences between the inversed results and corresponding real values are smaller in the region close to the calibration points and bigger in the region away from the calibration points, but in general the inversed error is rising with increase of droplet size differences. In other words, the method presented in this letter is still applicable when droplet sizes do not change dramatically. As droplet size increase will reduce the error induced by the Airy theory and formula approximation, the larger the droplet size is, the smaller the inversion error of imaginary part of refractive index is.

In conclusion, a novel method to measure droplet size and complex refractive index simultaneously by rainbow detecting is presented. It is based on a new mathematic model and new derived formulas to measure droplet imaginary part of refractive index, and a new inverse algorithm which can simultaneously measure droplet size and the complex refractive index, based on the findings that the inversing process of droplet size and real part of refractive index is insensitive to the imaginary part of the refractive index. The novel method is verified by simulation experiments, and the results show that the method is applicable when droplet sizes do not change dramatically.

This work was supported by the National Natural Science Foundation of China under Grant No. 51206082.

References

1. J. P. van Beeck and M. L. Riethmuller, Appl. Opt. **34,** 1633 (1995).
2. J. P. van Beeck, M. L. Riethmuller, G. Lavergne, Y. Biscos, and A. Atthasit, Proc. SPIE **4448,** 251 (2001).
3. P. Massoli, Appl. Opt. **37,** 3227 (1998).
4. H. C. van de Hulst, *Light Scattering by Small Particles* (John Wiley & Sons, New York, 1957) p 136.
5. J. P. van Beeck, "*Rainbow phenomena: development o a laser-based, non-intrusive technique for measuring droplet size, temperature and velocity*" PhD. Thesis (Technische Universiteit Eindboven, 1997).
6. Q. Pan and S. Wang, J. Southeast University (Nat. Sci. Ed.) (in Chinese) **38,** 468 (2008).
7. Q. Pan and S. Wang, Proc. SPIE **7375,** 73752T (2008).