Light delay and advancement by coupled double-ring resonators

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Through finding the derivatives of the group delay and transmittance of the coupled double-ring resonators (CDRRs) with respect to different structure parameters, the dispersion and filtering characteristics of CDRRs varying with different coupling and attenuation/gain coefficients are analyzed systematically. The parameter ranges in that obvious slow and fast light can be achieved are given, and large light delay and advancement without serious attenuation and great distortion are obtained.

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Obvious slow and fast light, and even negative group velocity control have been implemented these years. Group velocity control is not only of great application potential, but also profound physics significance. Recently, the group velocity control of light in matter has been one of hot research areas in photonic domain. Group velocity control occurs mainly in dispersion media and dispersion structures. With the development of integrated optics, microring resonators as a kind of group velocity control devices based on the dispersion structure have been widely studied. They are promising for applications such as optical delay lines[1,2], light storage[3,4], and laser gyroscopes[5,6], etc.

Using lossy microring resonators to realize fast or slow light often suffers serious attenuation and great distortion[7−9]. This usually poses a great difficulty to the application of group velocity control by microring resonators. Introducing gain to microring resonators can not only overcome severe attenuation and distortion, but also enrich filtering and dispersion characteristics of microring resonators[10−13].

Coupled double-ring resonators (CDRRs) are very useful in group velocity control due to their simple structure and rich transmission capacity. Characteristics of CDRRs are influenced by coupling coefficients and attenuation/gain coefficients, and the influences of these parameters are different under different conditions. So it is very complicated to tell how dispersion and transmittance vary with these parameters.

In this letter, we show that the disciplinarians of dispersion and filtering characteristics of CDRRs varying with different parameters can be summarized under different conditions. The group velocity controls are optimized and large delay and advancement are achieved.

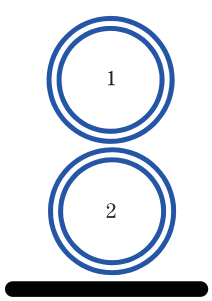

Fig. 1. Structure of CDRR.

A CDRR consists of two coupled mirorings and an excitation waveguide, and its structure is shown in Fig. 1.

The transfer functions of the first ring and the whole system, $\tau_1(\phi_1)$ and $\tau_2(\phi_1,\phi_2)$ are

$$\tau_1(\phi_1)=\frac{t_1-a_1\exp(\mathrm{i}\phi_1)}{1-t_1a_1\exp(\mathrm{i}\phi_1)}=|\tau_1|\exp\left[\mathrm{i}\phi_1^{(\mathrm{eff})}\right], \tag{1}$$

$$\tau_2(\phi_1,\phi_2)=\frac{t_2-a_2\tau_1\exp(\mathrm{i}\phi_2)}{1-t_2a_2\tau_1\exp(\mathrm{i}\phi_2)}=|\tau_2|\exp\left[\mathrm{i}\phi_2^{(\mathrm{eff})}\right], \tag{2}$$

where ϕ_i and a_i are the phase shifts and the amplitude transmission factors of the round-trip propagation respectively, t_i are the coupler transmission coefficients, $\phi_i^{(\mathrm{eff})}$ are the effective phase shifts, and $i=1,2$ specifies the first or second ring. In this letter, we only consider CDRRs whose two rings have identical perimeters and refractive indexes. So we can define $\phi_1=\phi_2=\phi$.

The first-order derivative of the effective phase shift of a waveguide structure corresponds to its group delay[14]. For a CDRR, at resonance wavelength when the first ring is under-coupled ($t_1>a_1$), it can be written as[14]

$$\frac{\mathrm{d}\phi_2^{(\mathrm{eff})}}{\mathrm{d}\phi_2}\Big|_{\phi_1=\phi_2=0}=\frac{a_2|\tau_{1,0}|\left(t_2^2-1\right)}{(t_2-a_2|\tau_{1,0}|)(1-t_2a_2|\tau_{1,0}|)}\frac{t_1\left(1+a_1^2\right)-2a_1}{(t_1-a_1)(1-a_1t_1)}, \tag{3}$$

and when the first ring is over-coupled ($t_1<a_1$), we have

$$\frac{\mathrm{d}\phi_2^{(\mathrm{eff})}}{\mathrm{d}\phi_2}\Big|_{\phi_1=\phi_2=0}=\frac{a_2|\tau_{1,0}|\left(1-t_2^2\right)}{(t_2+a_2|\tau_{1,0}|)(1+t_2a_2|\tau_{1,0}|)}\frac{t_1\left(1+a_1^2\right)-2a_1}{(t_1-a_1)(1-a_1t_1)}, \tag{4}$$

where $\tau_{1,0} \equiv \tau_1(0)$. For the case of $\mathrm{d}\phi_2^{(\mathrm{eff})}/\mathrm{d}\phi_2 > 0$, the CDRR has normal dispersion and gives out slow light, the larger $\mathrm{d}\phi_2^{(\mathrm{eff})}/\mathrm{d}\phi_2$ is, the greater the delay. For $\mathrm{d}\phi_2^{(\mathrm{eff})}/\mathrm{d}\phi_2 < 0$, anomalous dispersion results in fast light, the smaller $\mathrm{d}\phi_2^{(\mathrm{eff})}/\mathrm{d}\phi_2$, the greater the advancement. Large positive or negative group delay can be achieved by microring resonators due to interaction between pulses and the resonators[7−9].

When the first ring is under-coupled ($t_1 > a_1$), the first-order derivative of the group delay and transmittance with respect to a_2 can be written as

$$\frac{\mathrm{d}\left(\frac{\mathrm{d}\phi_2^{(\mathrm{eff})}}{\mathrm{d}\phi_2}\right)}{\mathrm{d}a_2} = \frac{t_2|\tau_{1,0}|\left(t_2^2-1\right)\left(1-a_2^2|\tau_{1,0}|^2\right)\left[t_1\left(1+a_1^2\right)-2a_1\right]}{(t_1-a_1)(1-t_1a_1)\left[(t_2-a_2|\tau_{1,0}|)(1-t_2a_2|\tau_{1,0}|)\right]^2}, \tag{5}$$

$$\frac{\mathrm{d}\tau_{2,0}^2}{\mathrm{d}a_2} = \frac{2|\tau_{1,0}|\left(t_2^2-1\right)(t_2-a_2|\tau_{1,0}|)}{(1-t_2a_2|\tau_{1,0}|)^3}, \tag{6}$$

where $\tau_{2,0}^2 \equiv |\tau_2(0)|^2$. The non-lasing condition of ring 1 and CDRR are $t_1 < 1/a_1$ and $t_2 < 1/(a_2\tau_{1,0})$[14], respectively. In addition, to realize significant mode splitting and large dispersion, we should choose $t_1 < 2a_1/(1+a_1^2)$[15]. When $a_2|\tau_{1,0}| < 1$, it can be seen from Eq. (5) that $\mathrm{d}\phi_2^{(\mathrm{eff})}/\mathrm{d}\phi_2$ increases with a_2. From Eq. (6) we know that when $t_2 > a_2|\tau_{1,0}|$, $\tau_{2,0}^2$ decreases with a_2, and when $t_2 < a_2|\tau_{1,0}|$, $\tau_{2,0}^2$ increases with a_2. Whereas for the case of $t_2 < 1 < a_2|\tau_{1,0}|$, $\mathrm{d}\phi_2^{(\mathrm{eff})}/\mathrm{d}\phi_2$ decreases with a_2, $\tau_{2,0}^2$ increases with a_2. Similarly, disciplinarians of $\mathrm{d}\phi_2^{(\mathrm{eff})}/\mathrm{d}\phi_2$ and $\tau_{2,0}^2$ varying with a_2 at other conditions, and varying with a_1, t_1 and t_2 at various conditions can be summarized also.

Detailed analysis indicates that when there is gain in the first ring, only slow light can be obtained. In addition, when $a_2 < 1$, $a_2|\tau_{1,0}| > 1 > a_1a_2$, and $t_1 < t_1^{(\mathrm{cr})}$, that large $\mathrm{d}\phi_2^{(\mathrm{eff})}/\mathrm{d}\phi_2$ and $\tau_{2,0}^2$ is about 1 can be achieved. Here $t_1^{(\mathrm{cr})}$ is a parameter which can be used to distinguish between coupled-resonator-induced transparency (CRIT) and coupled-resonator-induced absorption (CRIA)[15], and it can be expressed as

$$t_1^{(\mathrm{cr})} = \frac{2t_2\left(1-a_1^2a_2^2\right)}{2a_1t_2\left(1-a_2^2\right)+a_2\left(1-a_1^2\right)\left(1+t_2^2\right)}. \tag{7}$$

We choose a series of parameters satisfying above condition as t_1=0.9995, a_1=1.0001, a_2=0.99, and t_2=0.78, and calculate pulse responses of a CDRR with these parameters. The ring radius is 300 μm, the length of the straight waveguide is 1 cm, and the effective refractive indexes of the ring and straight waveguide are both 3. The input pulse is Gaussian and can be expressed as

$$A(t) = \exp\left(-\frac{t^2}{t_d^2}\right)\exp\left(\mathrm{i}2\pi ct/\lambda_0\right), \tag{8}$$

where λ_0 =1 550.12795 nm is the resonance wavelength of the CDRRs, and t_d=9 ns is the pulse width. The calculated output pulses are shown in Fig. 2. A delay of 7.2042 ns is obtained.

According to Eq. (3), when the first ring is under-coupled ($t_1 > a_1$), and the whole structure is over-coupled ($t_2 < a_2|\tau_{1,0}|$), fast light is obtain. In the case of $a_2 > 1$, $a_2|\tau_{1,0}| < 1 < a_1a_2$ and $t_1 < t_1^{(\mathrm{cr})}$, large $\mathrm{d}\phi_2^{(\mathrm{eff})}/\mathrm{d}\phi_2$ and transmittance about 1 can be achieved. As shown in Fig. 3, $\mathrm{d}\phi_2^{(\mathrm{eff})}/\mathrm{d}\phi_2$ increases with a_2 when $a_2|\tau_{1,0}| < 1$, and decreases with a_2 when $a_2|\tau_{1,0}| > 1$, and the transmittance at resonance always increases with a_2.

To realize large advancement and ideal transmittance ($\tau_{2,0}^2 \approx 1$) simultaneously, we select the CDRR parameters as t_1=0.9993, a_1=0.992, a_2=1.169, and t_2=0.8. Other parameters, including ring radius, waveguide length, and effective refractive index, are the same as the CDRR calculated in Fig. 2. An advancement of 2.3614 ns for above input pulse is obtained, as shown in Fig. 4.

In conclusion, categorizing the characteristics of CDRRs is considerably complicated due to the variety of their possible responses under different parameter conditions. By getting derivations of dispersion and transmittance with respect to each coupling coefficient and amplitude transmission coefficient respectively, we show that the disciplinarians of dispersion and filtering characteristics of CDRRs varying with different parameters can be summarized under different conditions. The group velocity controls are optimized. For CDRRs with

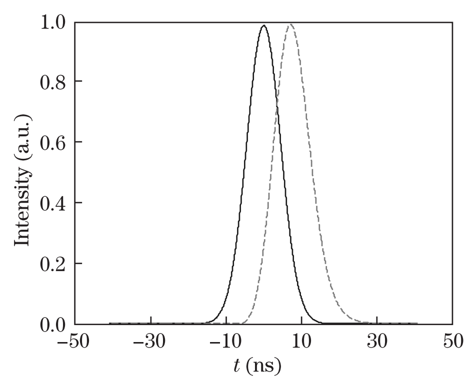

Fig. 2. Pulse delay by a CDRR. The solid and dashed curves indicate the pulses passing the straight waveguide only and output from the CDRR, respectively. t_1=0.9995, a_1=1.0001, a_2=0.99, and t_2=0.78.

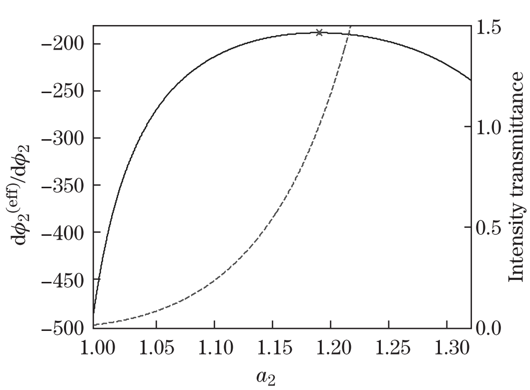

Fig. 3. Dispersive response (solid curves) and transmittance (dashed curves) of the CDRR versus a_2. The cross indicates the position of $a_2|\tau_{1,0}| = 1$. t_1=0.9993, a_1=0.992, and t_2=0.8.

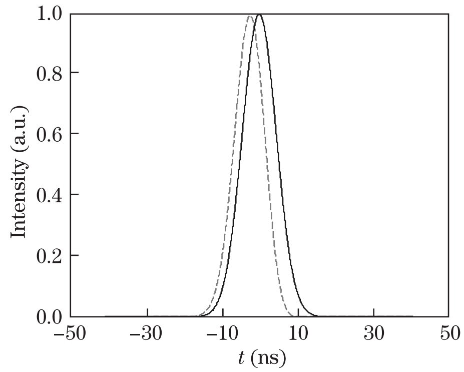

Fig. 4. Pulse advancement by a CDRR. The solid and dashed curves indicate the pulses passing the straight waveguide only and output from the CDRR, respectively. t_1=0.9993, a_1=0.992, a_2=1.169, and t_2=0.8.

ring radius of 300 μm, delay of 7.2042 ns is obtained when there is gain in the first ring and loss in the second, and advancement of 2.3614 ns is obtained when there is loss in the first ring and gain in the second. In addition to these large advancement and delay the peak transmittances of the pulses are ideally about 1. It is shown that by introducing gain, obvious group velocity control with ideal transmittance can be realized by CDRRs.

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