

# Vector projection method used for calibration of the polarimeter module

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To solve the problem of calibrating the polarimeter module, we design a vector projection (VP) method which is based on the theorem of vector projection and the principle of calibration. The calibration matrix can be calculated by VP method with the given data. The experimental result shows that, in comparison with the common iterative algorithm, VP method has very high computational efficiency and accuracy. The measured error of the degree of polarization (DOP) is less than 3%.

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Measuring the state of polarization (SOP) of light signal in fiber is one of the important issues in optical fiber communication system, in which we can get the degree of polarization (DOP) of the signal in fiber links to compensate PMD in the fiber, or get the SOP in the polarization stabilizers<sup>[1-7]</sup>, etc. In generally, a polarimeter includes optical part and electrical driver part (the four paths linear amplification, high-speed analog to digital converter, etc.). The outputs of the polarimeter are four voltages. In order to get Stokes vectors, a transformed matrix is needed. While for different input wavelengths, the transformed matrix has a little difference. So calibration of transformed matrix is an important but a complicated task. A fast and reliable calibration method should be established.

In this letter, we proposed a new method to calibrate the transformed matrix by using the data getting from a reference polarimeter, such as N7788BD (Agilent Technologies). This method is based on the vector projection algorithm.

In Fig. 1, we show the schematic of a polarimeter. The outputs of the polarimeter are four voltages, denoted by  $V_1, V_2, V_3, V_4$ . While  $S_0, S_1, S_2, S_3$  are the parameters we need. Assume the relation between them is linear, then, we have

$$\begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{pmatrix}. \quad (1)$$

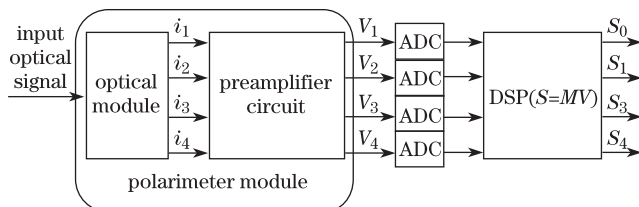


Fig. 1. Schematic of a polarimeter.

The matrix form of Eq. 1 (1) is

$$\mathbf{S} = \mathbf{M}\mathbf{V}, \quad (2)$$

where  $\mathbf{S} = (S_0, S_1, S_2, S_3)^T$  is called Stocks vector,  $\mathbf{V} = (V_1, V_2, V_3, V_4)^T$  is called voltage vector, which is obtained by detectors and

$$M = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix} = \begin{bmatrix} M(1, :) \\ M(2, :) \\ M(3, :) \\ M(4, :) \end{bmatrix} \quad (3)$$

is the transformed matrix. In this letter, our main task is to calibrate the transformed matrix, simplified as calibration matrix.

By a series of experiments, we can obtain a series of voltage vectors which are denoted by  $\mathbf{V}_i = (V_1^i, V_2^i, V_3^i, V_4^i)^T$ ,  $i = 1, \dots, n$ , and let  $V = [\mathbf{V}_1, \mathbf{V}_2, \dots, \mathbf{V}_n]$ . Let  $S = [\mathbf{S}_1, \mathbf{S}_2, \dots, \mathbf{S}_n]$  and the reference Stocks vectors obtained from the reference polarimeter which be denoted as  $\mathbf{S}_i^* = (S_0^{i*}, S_1^{i*}, S_2^{i*}, S_3^{i*})^T$ ,  $i = 1, 2, \dots, n$ , and  $S^* = [\mathbf{S}_1^*, \mathbf{S}_2^*, \dots, \mathbf{S}_n^*]$ . By these notations, Eq. (2) can be rewritten into a matrix form

$$S = MV. \quad (4)$$

Our purpose is to determine the calibration matrix  $M$ , so that  $S$  is as close as possible to the reference Stocks parameters  $S^*$ .

For simplicity, we use  $S^*(i, :)$ ,  $S(i, :)$ ,  $M(i, :)$ , and  $V(i, :)$ ,  $i = 1, 2, 3, 4$  to denote the row vector of  $S^*$ ,  $S$ ,  $M$ , and  $V$ , then Eq. (4) can also be rewritten as

$$\begin{bmatrix} S(1, :) \\ S(2, :) \\ S(3, :) \\ S(4, :) \end{bmatrix} = \begin{bmatrix} M(1, :) \\ M(2, :) \\ M(3, :) \\ M(4, :) \end{bmatrix} \begin{bmatrix} V(1, :) \\ V(2, :) \\ V(3, :) \\ V(4, :) \end{bmatrix}. \quad (5)$$

Then, we have

$$S(i, :)^T = V^T M(i, :)^T, \quad i = 1, 2, 3, 4. \quad (6)$$

Moreover, we denote  $R = S^* - S$  as the residual matrix and  $R(i, :)$  as the  $i$ th row of  $R$ .

Now, our problem can be changed as to find  $M(i, :)^T$ , such that  $S(i, :)^T$  as close as possible to  $S^*(i, :)^T$ , where  $i = 1, 2, 3, 4$ . Notice that  $S(i, :)^T$  is a vector in the range of the matrix  $V^T$ <sup>[9]</sup>, which is a linear subspace of  $\mathbf{R}^n$  and  $S^*(i, :)^T$  is a vector in  $\mathbf{R}^n$ , then by the idea of vector projection, the ‘‘closest’’ approximation to  $S^*(i, :)^T$  in the range of matrix  $V^T$ ,  $S(i, :)^T$ , is the vector which satisfies the condition

$$S(i, :)^T \perp R(i, :)^T, \quad i = 1, 2, 3, 4, \quad (7)$$

where ‘‘ $\perp$ ’’ means that  $S(i, :)^T$  is orthogonal to  $R(i, :)^T$ , or the following result holds

$$S(i, :)^T (S^*(i, :)^T - S(i, :)^T) = 0, \quad i = 1, 2, 3, 4. \quad (8)$$

In Fig. 2, we illustrate the idea of vector projection. By Eqs. (6) and (8), we have

$$M(i, :)^T (S^*(i, :)^T V^T - M(i, :)^T V V^T) = 0, \quad i = 1, 2, 3, 4. \quad (9)$$

To let Eq. (9) holds, we take

$$S^*(i, :)^T V^T - M(i, :)^T V V^T = 0, \quad i = 1, 2, 3, 4, \quad (10)$$

which is a sufficient condition for Eq. (9). By Eq. (10), under the assumption that  $V V^T$  is invertible, we have

$$M(i, :)^T = S^*(i, :)^T V^T (V V^T)^{-1}, \quad i = 1, 2, 3, 4. \quad (11)$$

If we written Eq. (11) in matrix form, we have

$$M = S^* V^T (V V^T)^{-1}, \quad (12)$$

and Eq. (12) gives the equation to determine the calibration matrix.

One thing should be pointed out that, although Eq. (12) need the assumption that  $V^T V$  should be invertible, this assumption seems not very sharp. Our examples show that this assumption can always be satisfied.

The setup of experiment to determine the calibration matrix is shown in Fig. 3. The laser light generated by continuous wave (CW) laser source passes through an adjustable attenuator, and input into the polarization analyzer, which is used as reference polarimeter. In the experiment, we set the polarization state of the polarization analyzer as  $n$  group of polarization states which are uniformly distributed on the Poincaré sphere. The output light of polarization analyzer is input to the polarimeter module. Here, the function of attenuator is to adjust the power of the polarimeter module input light, so that the 4 output voltages of polarimeter module are within the linear range of the amplifiers. The amplified 4 output voltages of polarimeter are changed into digital voltages by analog-to-digital converter (ADC) and are captured by digital signal processing (DSP). All outputs of the digital voltages are recorded in a matrix  $V = [\mathbf{V}_1, \mathbf{V}_2, \dots, \mathbf{V}_n]$ , where each column of  $V$  respects to one group of 4 voltages for one polarization state of the light. At the same

time, the polarization analyzer provide Stocks parameters of the light and the corresponding outputs are also be recorded in a matrix  $S^* = [\mathbf{S}_1^*, \mathbf{S}_2^*, \dots, \mathbf{S}_n^*]$ , where each column of  $S^*$  represents 4 Stocks parameters of one polarization state. By Eq. (12), we will obtain the calibration matrix  $M$ .

To test the efficiency of the vector projection based algorithm, we use an environment shown in Fig. 4 and program the algorithm into DSP. The function of attenuator is the same as in Fig. 3. The adjusted light generated by CW is inputted into the polarization scrambler (HP11896A). The scrambler is running under the speed is about 80 rad/s. The polarized light with rapidly changing polarization state is inputted into the polarimeter module and 4 output voltages are generated. The output voltages are captured by DSP and changed into Stocks parameters by Eq. (1), which is using the calibration matrix obtained by Eq. (12). By these Stocks parameters, we can calculate the DOP of the light by

$$f_{\text{DOP}} = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0}. \quad (13)$$

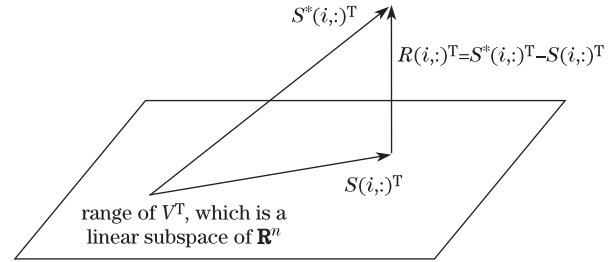


Fig. 2. Illustration of projection of vectors.

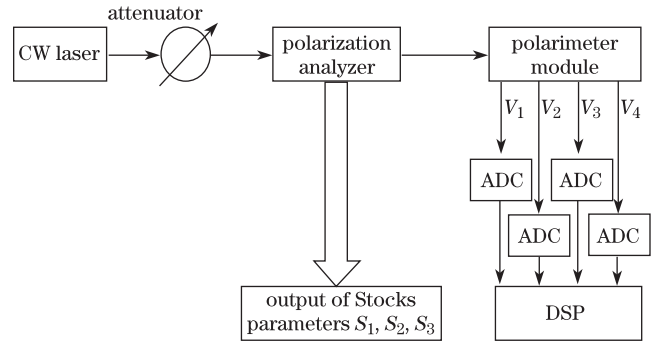


Fig. 3. Experimental setup to determine the calibration matrix.

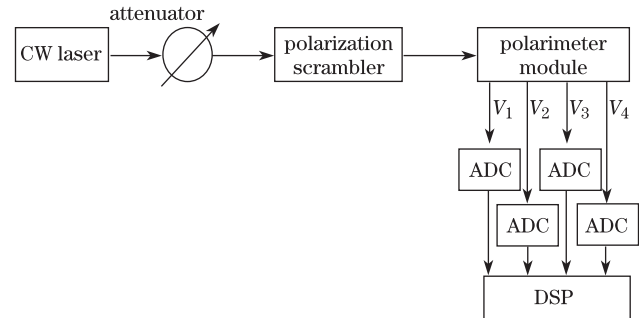


Fig. 4. Experimental setup to test the efficiency of vector projection based algorithm.

To the completely polarized light, we have  $f_{\text{DOP}} = 1$ , and  $f_{\text{DOP}} = 0$  represents to the unpolarized light. If  $0 < f_{\text{DOP}} < 1$ , the light is said to be partially polarized. Since the input light in our experiment are completely polarized and there are no noise introduced into the link system, then, the output light of polarization scrambler can also be treated as completely polarized light, and  $f_{\text{DOP}}$  still is 1 or close to be 1. The results of our experiments are shown in Figs. 5(a) and (b).

In Fig. 5(a), we show 10 000 groups of  $f_{\text{SOP}}$  and  $f_{\text{DOP}}$ ,

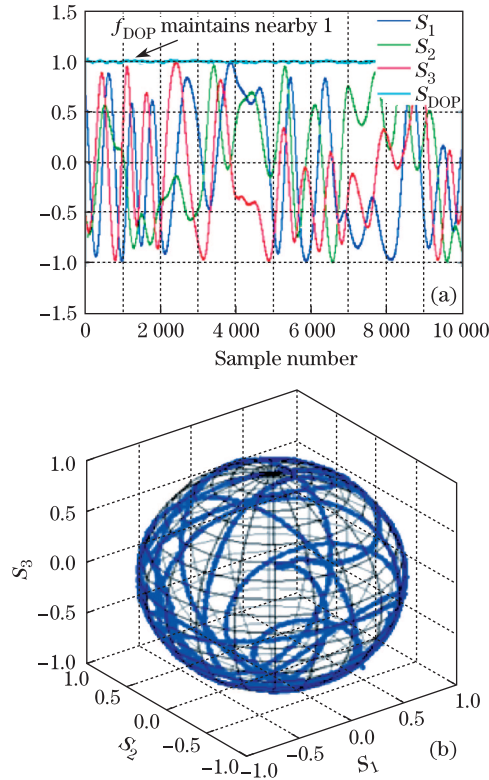


Fig. 5. (a) Experimental measured  $f_{\text{SOP}}$  and  $f_{\text{DOP}}$ ; (b) distribution of  $f_{\text{SOP}}$  in Poincaré sphere.

which are measured by polarimeter module and calculated by Eq. (1) with calibration matrix obtained by Eq. (12), respectively. In Fig. 5(b), we show the distribution of  $f_{\text{SOP}}$  on the Poincaré sphere. The experiment shows that, all  $f_{\text{DOP}}$  of sample points are around 1, without sharp oscillations and the relative error are no more than 3%. Except very few points, all calculated Stocks parameters,  $S_1$ ,  $S_2$  and  $S_3$ , are in the interval  $(-1, 1)$  and the polarization states are distributed on Poincaré sphere. This experiment shows that the calibration matrix obtained in the experiment in Fig. 4 has sufficient accuracy and can be used in practice.

In conclusion, we design a new method to determine the calibration matrix used by polarimeter, which is based on the idea of vector projection. Our experiments show that the calibration matrix obtained by the new method has enough accuracy for practice, the relative error of DOP value of polarization state is no more than 3%. Also, the process to obtain the calibration matrix is very fast. After obtain all data used in Eq. (12), only a few second are needed to find the calibration matrix.

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