Discussions of ground state in Lambda-type three-level system

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The effects of counter-rotating terms on ground states (GS) of a lambda-type three-level atomic system coupled with two fields are examined. The GS, which are dark states in rotating wave approximation (RWA), can be expressed by a simple formula including the excited states. When the coupling strength is less than 0.2 of the maximum energy splitting in the atomic system, the component coefficients of excited states in the GS can be described by the formula similar as that in a two-level system and have linear relationship with their coupling constant. Further increasing the coupling strength will increase the excited components in the GS more rapidly, very different from the two-level system.

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Electromagnetically induced transparency (EIT) is a phenomenon that a field can pass a medium without absorption even at the resonance condition[1−3]. It enables one to dramatically modify the optical properties of the medium and offers many new possibilities for nonlinear optics and quantum information science[4−7]. This phenomenon is usually modeled as a three-level atomic system, typically a lambda-type configuration, coupled with two laser fields (one is the probe field and the other is the coupling field). The physical interpretation underlying the cancellation of absorption in EIT are related to dark state[7]. Dark state is one state that is decoupled from the light fields. If the system is in dark state, the EIT will occur. This conclusion is made under the rotating wave approximation (RWA) in which the counter-rotating wave terms in the system Hamiltonian are discarded and the eigenstates which have no contributions from the excited states can be found. In experiments, a gas-phase atomic or molecular medium is usually chosen to realize EIT. In such systems, the coupling strength λ between the atom and fields is largely determined by the intrinsic dipole moment of the atom and is very small compared to the atomic transition frequency ω ($\lambda/\omega \sim 10^{-7} - 10^{-6}$)[8]. The weak coupling between the atom and the field makes sure that RWA is effective and provides a succinct way for explaining the EIT. Recent study[9] shows that ground state (GS) energy and wavefunction in a two-level system Jaynes-Cummings (JC) model are mainly decided by counter-rotating wave terms. When coupling constant becomes large the ground state deviates dark state obviously. In recent years, significant experimental progress in developing strong coupling systems has been made and much larger coupling with $\lambda/\omega \approx 10^{-3}$ can be produced in quantum-limited solid-state device systems[10−14]. The possibility of still larger coupling strengths than those possible with dipole coupling offers an opportunity to explore the effects of counter-rotating terms on EIT. In such regime of strong coupling strength, RWA is invalid and the counter-rotating wave terms should be considered.

In this letter, the effects of counter-rotating terms on the GS in a two-level JC model are briefly introduced firstly. Then the GS in a lambda-type atom coupled with two laser fields, including one probe field and one coupling field, is studied considering the counter-rotating terms. The results show that the effects of the counter-rotating wave terms on GS cannot be ignored when the coupling constant is more than 0.2 (if the maximum energy splitting in the atomic system is one unit).

Before we turn to the three-level Lambda system, we would like to discuss the two-level system firstly, which would help us to better understand the more complicated three-level system coupled with two laser fields. The interaction of a single-mode quantized field with a two-level atom can be described by the Hamiltonian

$$H^{(2)} = \epsilon_1|g\rangle\langle g| + \epsilon_2|e\rangle\langle e| + \omega a^\dagger a + \lambda(a^\dagger + a)(|e\rangle\langle g| + |g\rangle\langle e|), \quad (1)$$

where $|g\rangle$ and $|e\rangle$ are the atomic lower and upper levels, respectively. ϵ_1 and ϵ_2 are the energies for $|g\rangle$ and $|e\rangle$. ω is the frequency of the field, λ is the coupling constant between the field and the atom. For the sake of simplicity, $\hbar$ is taken to be 1. If we use the notation,

$$\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|, \quad \sigma_+ = |e\rangle\langle g|, \quad \sigma_- = |g\rangle\langle e|,$$

and ignore the constant $\frac{1}{2}(\epsilon_1 + \epsilon_2)$[15], the Hamiltonian in Eq. (1) will change into a familiar form,

$$H^{(2)} = \frac{1}{2}\epsilon\sigma_z + \omega a^\dagger a + \lambda(a^\dagger + a)(\sigma_+ + \sigma_-), \quad (2)$$

where $\epsilon = \epsilon_2 - \epsilon_1$ is the energy splitting between the upper state and the lower state. The corresponding rotating wave approximation is

$$H^{(2)}_{\rm RWA} = \frac{1}{2}\epsilon\sigma_z + \omega a^\dagger a + \lambda(a^\dagger\sigma_- + a\sigma_+). \quad (3)$$

For a three-level Lambda system characterized by atomic levels g_1, g_2, and e with energies $\epsilon_1^0 < \epsilon_2^0 < \epsilon_3^0$,

the Hamiltonian can be expressed as

$$\begin{aligned} H^{(3)} =& \epsilon_1^0|g_1\rangle\langle g_1| + \epsilon_2^0|g_2\rangle\langle g_2| + \epsilon_3^0|e\rangle\langle e| + \sum_{k=1}^{2}\omega_k a_k^\dagger a_k \\ &+ \sum_{k=1}^{2}\lambda_k^{(1)}(a_k^\dagger + a_k)(|e\rangle\langle g_1| + |g_1\rangle\langle e|) \\ &+ \sum_{k=1}^{2}\lambda_k^{(2)}(a_k^\dagger + a_k)(|e\rangle\langle g_2| + |g_2\rangle\langle e|), \end{aligned} \tag{4}$$

where ω_1 is the frequency of the probe field and ω_2 is the frequency of the coupling field. $\lambda_k^{(1)}$ and $\lambda_k^{(2)}$ are the coupling constants between the different levels with the fields. Since the lower two states $|g_1\rangle$ and $|g_2\rangle$ are forbidden transition by the laser field, there are no corresponding transition terms in Hamiltonian. In the following, we change the Hamiltonian into a more simple form as

$$\begin{aligned} &\epsilon_1^0|g_1\rangle\langle g_1| + \epsilon_2^0|g_2\rangle\langle g_2| + \epsilon_3^0|e\rangle\langle e| \\ =&\frac{1}{3}(\epsilon_3^0-\epsilon_1^0)(|e\rangle\langle e|-|g_1\rangle\langle g_1|)+\frac{1}{3}(\epsilon_3^0-\epsilon_2^0)(|e\rangle\langle e|-|g_2\rangle\langle g_2|) \\ &+ \frac{1}{3}(\epsilon_2^0 - \epsilon_1^0)(|g_2\rangle\langle g_2| - |g_1\rangle\langle g_1|) + \frac{1}{3}(\epsilon_1^0 + \epsilon_2^0 + \epsilon_3^0). \end{aligned} \tag{5}$$

Similarly as doing in the two-level system, we define

$$\begin{aligned} &\sigma_{1z} = |e\rangle\langle e| - |g_1\rangle\langle g_1|, \quad \sigma_{1+} = |e\rangle\langle g_1|, \quad \sigma_{1-} = |g_1\rangle\langle e|, \\ &\sigma_{2z} = |e\rangle\langle e| - |g_2\rangle\langle g_2|, \quad \sigma_{2+} = |e\rangle\langle g_2|, \quad \sigma_{2-} = |g_2\rangle\langle e|, \\ &\sigma_{3z} = |g_2\rangle\langle g_2| - |g_1\rangle\langle g_1|. \end{aligned}$$

Then the Hamiltonian in Eq. (4) has the familiar form,

$$\begin{aligned} H^{(3)} =& \frac{1}{3}(\epsilon_1\sigma_{1z} + \epsilon_2\sigma_{2z} + \epsilon_3\sigma_{3z}) + \sum_{k=1}^{2}\omega_k a_k^\dagger a_k \\ &+ \sum_{k=1}^{2}\lambda_k^{(1)}(a_k^\dagger + a_k)(\sigma_{1+} + \sigma_{1-}) \\ &+ \sum_{k=1}^{2}\lambda_k^{(2)}(a_k^\dagger + a_k)(\sigma_{2+} + \sigma_{2-}), \end{aligned} \tag{6}$$

where $\epsilon_1 = \epsilon_3^0 - \epsilon_1^0$, $\epsilon_2 = \epsilon_3^0 - \epsilon_2^0$, and $\epsilon_3 = \epsilon_2^0 - \epsilon_1^0$.

If RWA is considered, the Hamiltonian becomes

$$\begin{aligned} H_{\rm RWA}^{(3)} =& \frac{1}{3}(\epsilon_1\sigma_{1z} + \epsilon_2\sigma_{2z} + \epsilon_3\sigma_{3z}) + \sum_{k=1}^{2}\omega_k a_k^\dagger a_k \\ &+ \sum_{k=1}^{2}\lambda_k^{(1)}(a_k\sigma_{1+} + a_k^\dagger\sigma_{1-}) \\ &+ \sum_{k=1}^{2}\lambda_k^{(2)}(a_k\sigma_{2+} + a_k^\dagger\sigma_{2-}). \end{aligned} \tag{7}$$

For the two-level system, the energy eigenstates have the form,

$$|\psi\rangle = \sum\nolimits_n (c_n|g,n\rangle + d_n|e,n\rangle), \tag{8}$$

where $|g,n\rangle$ and $|e,n\rangle$ are the states in which the field has n photons and the atom is in the state $|g\rangle$ or $|e\rangle$.

For RWA, it is well known that analytic eigen solutions can be obtained for Hamiltonian in Eq. (3). The GS is $|g,0\rangle$, which is a dark state. When counter-rotating terms are considered (Hamiltonian in Eq. (2), $|g,0\rangle$ will not be the energy eigenstate any more. In this case, the GS can be approximated in the first order as (see Ref. [9])

$$|\psi\rangle = c_0|g,0\rangle + d_0|e,1\rangle, \tag{9}$$

where

$$c_0 = \frac{\epsilon+\omega}{\sqrt{(\epsilon+\omega)^2+\lambda^2}}, \quad d_0 = \frac{\lambda}{\sqrt{(\epsilon+\omega)^2+\lambda^2}}. \tag{10}$$

Since the GS is mainly decided by counter-rotating wave terms which are energy nonconserving, the two main components in GS are $|g,0\rangle$ and $|e,1\rangle$. It is seen that when λ increases, the nonzero phonon state will increase in the eigenvector. When λ approaches zero, the GS will become $|g,0\rangle$, and this corresponds rotating-wave approximation.

In the three-level system, the energy eigenstates can take the form,

$$\begin{aligned} |\psi\rangle = \sum_{n_1 n_2} &(c_{n_1 n_2}^{(1)}|g_1, n_1 n_2\rangle \\ &+ c_{n_1 n_2}^{(2)}|g_2, n_1 n_2\rangle + d_{n_1 n_2}|e, n_1 n_2\rangle), \end{aligned} \tag{11}$$

in which n_1 and n_2 represent the photon numbers in probe field and coupling field, respectively. $|g_1\rangle$, $|g_2\rangle$ and $|e\rangle$ are the atomic states. It is obvious that $|g_1,00\rangle$ and $|g_2,00\rangle$ are eigenvectors of Hamiltonian (see Eq. 7) in rotating wave approximation. Both of two states have zero photons in the laser fields. This means when the initial state is the linear combination of these two states, that is, $|\psi(t=0)\rangle = c_1|g_1,00\rangle + c_2|g_2,00\rangle$, it forms a dark state. If the atom is prepared in this state there is no possibility of excitation to the excited state $|e\rangle$. Usually $|g_1,00\rangle$ is the GS, and $|g_2,00\rangle$ is another metastable GS (MGS) of the system. When considering the counter-rotating wave terms, there is no analytic eigen solutions for Hamiltonian in Eq. (6) and each eigenstate will include the component of all of the bare atomic states. States that have no contribution from $|e\rangle$ can not be found and there is no dark state any more. Analyzing from the numerical results, we find that the GS and the MGS in which the state $|g_2,00\rangle$ is the main term mainly have four terms $|g_1,00\rangle$, $|g_2,00\rangle$, $|e,10\rangle$ and $|e,01\rangle$. Therefore, we can write these two states for the first order approximation in the form

$$|\psi\rangle = c_0|g_1,00\rangle + c_1|g_2,00\rangle + c_2|e,10\rangle + c_3|e,01\rangle. \tag{12}$$

Table 1 lists the energy and coefficient results of GS for different coupling constants calculated by numerical method and by in Eq. (12). For comparison, the results for the MGS are also given. Numerical method means that we calculate these two eigenstates according to Eq. (11). Since the expression in Eq. (11) has infinite terms, we have to omit higher photon number states and just keep finite terms. In calculation, the highest photon number in two fields are both chosen to be three.

Without loss of generality, we take all the coupling constants equal and consider the resonance with $\epsilon_1 = \omega_1 = 1$ and $\epsilon_2 = \omega_2 = 0.6$, just as in EIT experiment[2]. When the coupling strength is very small, in the regime of the most experimental situation, that is $\lambda^{(1)} = \lambda^{(2)} = 1.0^{-6}$, both states can be considered as dark states, just as doing in RWA. It is a very good approximation since the excited states almost have no contribution. Increasing the coupling constant, the energy of the system decreases and the main term $|g_1, 00\rangle$ in the GS and $|g_2, 00\rangle$ in the MGS also decrease their contribution to the eigenstates. The other terms gradually increase their weights in eigenstates. The contributions of excited states $|e, 10\rangle$ and $|e, 01\rangle$ increase roughly proportional to the coupling constant when $\lambda^{(1)} = \lambda^{(2)} \leqslant 0.1$. Although the coefficients of each states in the GS are difficult to express in an analytic way, compared with the two-level system, it is found that the coefficients c_2 and c_3 of the GS have the similar dependence on the energy splitting and the field frequency. c_2 is roughly proportional to $\lambda_1/(\epsilon_1 + \omega_1)$ and $c_3 \approx \lambda_2/(\epsilon_2 + \omega_2)$. When $\lambda^{(1)}$ and $\lambda^{(2)}$ increase to 0.2, c_2 and c_3 in the GS already have rather values more than 0.14. When $\lambda^{(1)}$ and $\lambda^{(2)}$ increase further, more and more other terms contribute to these two states. For the MGS, the energy will not have the second lowest energy as in the small coupling constant. The eigenstate in which $|g_2, 00\rangle$ is the main contribution will become the excited state. In this regime of coupling strength, the four components cannot describe these two eigenstates well and it calls for more components to express the eigenstates. In order to show this point more clearly, the square sum of four coefficients in GS and MGS are listed in Table 2

Table 1. Energy, the Coefficents of Four Main Components in Eigenstates of GS and MGS. Numerical Results Compared with Approximation Given by Eq. (12) under the Conditions $\epsilon_1 = 1$, $\epsilon_2 = 0.6$, $\epsilon_3 = 0.4$, $\omega_1 = 1$, and $\omega_2 = 0.6$ with Different Coupling Strengths

$\lambda^{(1)} = \lambda^{(2)} = 10^{-6}$	E	c_0	c_1	c_2	c_3
Numerical (GS)	−0.46666666666780	−0.99999999999968	−0.00000000000281	0.00000050000000	0.00000062500000
Approximation (GS)	−0.46666666666780	−0.99999999999968	−0.00000000000281	0.00000050000000	0.00000062500000
Numerical (MGS)	−0.06666666666813	0.00000000000365	−0.99999999999946	0.00000062500000	0.00000083333333
Approximation (MGS)	−0.06666666666813	0.00000000000365	-0.99999999999946	0.00000062500000	0.00000083333333
$\lambda^{(1)} = \lambda^{(2)} = 10^{-3}$	E	c_0	c_1	c_2	c_3
Numerical (GS)	−0.46666779167546	−0.99999967967658	−0.00000281251317	0.00050000426607	0.00062500416483
Approximation (GS)	−0.46666779166911	−0.99999967968231	−0.00000281249730	0.00050000096484	0.00062500111816
Numerical (MGS)	−0.06666812500774	0.00000364586398	−0.99999945744639	0.00062500412000	0.00083333615395
Approximation (MGS)	−0.06666812499310	0.00000364582740	−0.99999945746423	0.00062499681262	0.00083332883031
$\lambda^{(1)} = \lambda^{(2)} = 0.01$	E	c_0	c_1	c_2	c_3
Numerical (GS)	−0.46677925463650	−0.99996785948180	−0.00028138167751	0.00500426836665	0.00625416675299
Approximation (GS)	−0.46677919109511	−0.99996791688506	−0.00028122293464	0.00500096433377	0.00625111749673
Numerical (MGS)	−0.06681257734044	0.00036488992837	−0.99994555740481	0.00625412018155	0.00833615281970
Approximation (MGS)	−0.06681243102844	0.00036452396655	−0.99994573603058	0.00624681347366	0.00832883172817
$\lambda^{(1)} = \lambda^{(2)} = 0.1$	E	c_0	c_1	c_2	c_3
Numerical (GS)	−0.47885611813675	−0.99558067844234	−0.02944175868922	0.05449940206680	0.06685642157746
Approximation (GS)	−0.47815561293750	−0.99629084273119	−0.02781686377238	0.05091291743871	0.06355040218386
Numerical (MGS)	−0.08204244373590	0.03967523728079	−0.99247389188863	0.06660296515124	0.08599760793592
Approximation (MGS)	−0.08058043224937	0.03583833588171	−0.99446034692126	0.05939735018577	0.07896953129775
$\lambda^{(1)} = \lambda^{(2)} = 0.2$	E	c_0	c_1	c_2	c_3
Numerical (GS)	−0.53000110294021	−0.96074991130877	−0.13132750184075	0.14092387615499	0.16331889400799
Approximation (GS)	−0.51519480409053	−0.97988001722139	−0.10601732236405	0.10601732236405	0.13174143830901
Numerical (MGS)	−0.13760677518215	0.20146776129615	−0.93452013334987	0.15249678658836	0.17897801176034
Approximation (MGS)	−0.11519480409053	0.13480390226043	−0.97633622729385	0.10209499078960	0.13480390226043
$\lambda^{(1)} = \lambda^{(2)} = 0.3$	E	c_0	c_1	c_2	c_3
Numerical (GS)	−0.68498581241200	−0.80298215875686	−0.28352086483950	0.28337345913349	0.30098113736162
Approximation (GS)	−0.58302868604773	−0.94204537731691	−0.21228963079931	0.16363008751033	0.20176425400030
Numerical (MGS)	−0.23774968776544	0.53681563431686	−0.71828409649033	0.20392015806469	0.20570055271785
Approximation (MGS)	−0.15495137642234	0.26710787609399	−0.94310339076402	0.12012112247961	0.15741757459768

Table 2. Sum of the Square of the Four Coefficients in GS and the MGS Calculated by Numerical Method

λ	10^{-6}	10^{-3}	10^{-2}	10^{-1}	0.2	0.3
GS	1.00000000000000	0.99999999999565	0.99999995647592	0.99948767037421	0.98681990483009	0.89605459046993
MGS	1.00000000000000	0.99999999998565	0.99999985638135	0.99841009407149	0.96920553709140	0.88799921677150

under different coupling strengths calculated by numerical method. When λ increases from 10^{-6} to 0.3, the square sum decreases from 1 to less than 0.90. At the regime of strong coupling, the values predicted by the simple formulae for c_2 and c_3 are much less than the numerical values. In two-level system (see Ref. [9]), the formula in Eq. (10) can work in a much larger regime than in the three-level system. For example, when $\lambda = \epsilon = \omega = 1$, the departure from the numerical value is only 10%. In the case of three-level system, the increase of coupling strength results in much rapid increase of the coefficients of c_2 and c_3 in the GS.

In conclusion, the effects of counter-rotating wave terms on the GS in the Lambda-type system are discussed. When the coupling constant is much less than the frequency of resonance field as in the experimental environment at the present time, the GS, can be described by dark state well. Increasing the coupling strength, the GS will include the contribution from the excited state. When the coupling strength is less than 0.2, the component coefficients of $|e, 10\rangle$ and $|e, 01\rangle$ can be described by the formulae similar in the two-level system and has linear relationship with the corresponding coupling constant. Further increasing the coupling strength will increase the excited components in the GS more rapidly, very different from the two-level system.

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References

1. S. Harris, J. Field, and A. Imamoğlu, Phys. Rev. Lett. **64,** 1107 (1990).
2. K.-J. Boller, A. Imamolu, and S. E. Harris, Phys. Rev. Lett. **66,** 2593 (1991).
3. S. Harris, Phys. Today **50,** 36 (1997).
4. L. Hau, S. Harris, Z. Dutton, and C. Behroozi, Nature **397,** 594 (1999).
5. C. Liu, Z. Dutton, C. Behroozi, and L. Hau, Nature **409,** 490 (2001).
6. M. Lukin and A. Imamoglu, Nature **413,** 273 (2001).
7. M. Fleischhauer, A. Imamoglu, and J. Marangos, Rev. Mod. Phys. **77,** 633 (2005).
8. E. K. Irish, J. Gea-Banacloche, I. Martin, and K. C. Schwab, Phys. Rev. B **72,** 195410 (2005).
9. X. H. Li, K. L. Wang, and T. Liu, Chin. Phys. Lett. **26,** 044212 (2009).
10. P. Herskind, A. Dantan, J. Marler, M. Albert, and M. Drewsen, Nature Physics **5,** 494 (2009).
11. S. Gröblacher, K. Hammerer, M. Vanner, and M. Aspelmeyer, Nature **460,** 724 (2009).
12. Y. Colombe, T. Steinmetz, G. Dubois, F. Linke, D. Hunger, and J. Reichel, Nature **450,** 272 (2007).
13. A. Boozer, A. Boca, R. Miller, T. Northup, and H. Kimble, Phys. Rev. Lett. **97,** 83602 (2006).
14. J. Reithmaier, G. Sek, A. Löffler, C. Hofmann, S. Kuhn, S. Reitzenstein, L. Keldysh, V. Kulakovskii, T. Reinecke, and A. Forchel, Nature **432,** 197 (2004).
15. M. Scully and M. Suhail Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, 1997).