Mobility of lattice solitons undergoing nonlocal diffusive nonlinearity in photorefractive $LiNbO_3$

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Received December 20, 2011; accepted February 27, 2012; posted online June 20, 2012

The influence of diffusive nonlinearity on mobility of photovoltaic lattice solitons is demonstrated. The dynamical evolution of collision between photovoltaic lattice solitons and nonlinear lattices are simulated numerically. The results show the lattice solitons with a transverse velocity have complicated behaviors and will not propagate with an oblique trajectory. When considering the diffusive nonlinearity, we find that diffusive nonlinearity can introduce a nonlinear chirped phase to lattice soliton and the lattice soliton with a special incident angle can become a "tilted soliton".

OCIS codes: 190.0190, 190.5330, 190.6135.

doi: 10.3788/COL201210.S11901.

Optical spatial solitons (OSSs) have been the object of intensive theoretical and experimental research during the last four decades[1,2], inasmuch as light-induced waveguides seem to entail large potentials for applications in novel generations of re-addressable and reconfigurable networks. When a very narrow optical beam induces (through self-focusing) a waveguide structure and guides itself in its own induced waveguide, the beam is called a spatial soliton. Thus far, lots of nonlinear effects have been found and used to form the OSS, for example, cubic nonlinearity(Kerr) effect[2], cubic-quintic competing nonlinearity[3], photorefractive effect[4−6], photoisomerization nonlinearity effect[7,8], etc. In all these kinds of OSSs, those forming in photorefractive and polymer material are particularly interesting, because they can be formed in the optical power level of μW (with intensity of $\mathrm{mW/cm^2}$).

In recent years, discrete and lattice solitons which are OSSs in periodic optical media (such as waveguide arrays and photonic lattices) have been the focus of considerable researches[9−11] due to a rich variety of functional operations of these solitons, e.g., blocking, routing, logic functions, and time gating. Many novel discrete solitons, such as diffraction managed solitons, discrete vector solitons, Floquet-Bloch solitons in homogeneous waveguide arrays, were predicted theoretically and subsequently verified in experiment[12−15]. Photonic Bloch oscillations and hybrid discrete solitons were found in inhomogeneous waveguide arrays[16,17]. Furthermore, some theoretical and experimental results have indicated the nonlinear periodic optical media can support some nonstationary solitons which are linearly unstable in homogeneous local media, such as high-order solitons, ring vortex solitons, dipole solitons, quadrupole solitons, necklacelike solitons[18−22]. At present, optically induced photonic lattices in photorefractive crystals which are dynamically adjustable, allowing real-time control of lattice spacing and potential well depth, provide an important experimental tool to form the photorefractive lattice solitons. The most popular experimental configuration for these solitons is based on uniaxial strontium barium niobate (SBN) photorefractive crystal, in which the lattice beam (LB) is *o*-ray while the soliton beam (SB) is *e*-ray polarized along the *c*-axis. The SB will feel nonlinearity arising from the photorefractive screening effect with a biased DC electric field along *c*-axis, while the LB remains linear propagation. These progresses open new applications for all-optical signal processing and switching. Recently, we propose a class of photorefractive lattice solitons induced by periodic background LB while the nonlinearity arises from the bulk photovoltaic photorefractive effect of SB and LB[23]. In this case, the transition of self-defocusing to self-focusing seen by photovoltaic lattice solitons can be realized by changing the wavelength of LB while it can be realized by reversing DC electric field for photorefractive screening lattice solitons. All these solitons are obtained with neglecting the diffusive effect. In fact, for photorefractive solitons in uniform media, the self-bending of these solitons is caused by diffusion effects in PR crystals and becomes an important effect when the beam size is in the range of the charge carrier diffusion length. In this letter, we study the mobility of photovoltaic lattice solitons in nonlinear lattices, especially, address the impact of diffusive effect on the mobility of these solitons.

Here we consider a signal beam propagating along direction z in a bulk photovoltaic photorefractive material. The direction x is the ferroelectric c axis and the beams are allowed to diffract only along this direction. Considering the photovoltaic effect of background illumination and the diffusion effect[24,25], under the condition of open-circuit, we can obtain the expression of the space-charge field E_{sc} based on the band-transport model developed by Kukhtarev *et al.*[26], as follows

$$E_{\mathrm{sc}}(x) = -E_{\mathrm{p}}\frac{S_{\mathrm{s}}I_{\mathrm{s}}+RS_{\mathrm{b}}I_{\mathrm{b}}}{S_{\mathrm{s}}I_{\mathrm{s}}+S_{\mathrm{b}}I_{\mathrm{b}}+\beta} - \frac{K_{\mathrm{B}}T}{q}\frac{\mathrm{d}\ln(S_{\mathrm{s}}I_{\mathrm{s}}+S_{\mathrm{b}}I_{\mathrm{b}}+\beta)}{\mathrm{d}x}, \qquad (1)$$

where S_{s} and S_{b} are the photoionization cross sections with respect to SB and LB; $E_{\mathrm{p}} = \kappa_{\mathrm{s}}\gamma N_{\mathrm{A}}/q\mu$, and $R = \kappa_{\mathrm{b}}/\kappa_{\mathrm{s}}$; β is the dark generation rate; γ is the recombination rate coefficient; N_{A} is the number density of the charged acceptors that compensate for the ionized donors; μ is the carrier mobility; κ_s and κ_b are the Glass constants with respect to SB and LB, respectively; q is the charge on the carrier; K_B is the Boltzman constant and T is the absolute temperature. I_s and I_b are the intensities of SB and LB, respectively. The periodic background LB is created by the interference pattern of two counterpropagating laser beams with ordinary polarization. The optically induced photonic lattice is essentially harmonic, having an intensity of distribution $I_b(x) = I_0 \cos^2(Kx)$, where $K = 2\pi n_o \cos\theta/\lambda_b$, λ_b the LB wavelength; θ is the angle between two wavevectors of the LBs and the x axis; n_o the refractive index of o-ray, and I_0 the maximum intensity of LB. It is clearly that the first and second terms of right hand of Eq. (1) correspond to the space-charge field excitated by photovoltaic and diffusion effects, respectively.

The refractive index change of SB is directly proportional to the space-charge field via the Pockels effect, which is

$$\Delta n(x) = -\frac{1}{2} n_e^3 \gamma_{\text{eff}} E_{\text{sc}}(x), \tag{2}$$

where n_e is the unperturbed refractive index of SB and γ_{eff} is the effective linear electro-optic coefficient.

The propagation dynamics of SB can be described by the following generalized nonlinear Schrödinger equation (NLSE) in the paraxial approximation and the slowly varying amplitude approximation:

$$\mathrm{i}2k\frac{\partial E}{\partial z} + \nabla_\perp^2 E + \frac{2k^2}{n_e}\Delta n \cdot E = 0, \tag{3}$$

where Δn is a real function describing the photo-induced index perturbation, $k = k_0 n_e$, and k_0 is the wavenumber of the SB in vacuum. The function E describes a complex envelope of the light field and $I_s = |E|^2$. In the following, we consider (1+1)-D case. So $\nabla_\perp^2 = \partial^2/\partial x^2$. Substituting Eqs. (1) and (2) into Eq. (3) and adopting scale transformation $\xi = z/kx_0^2, s = x/x_0$ (where x_0 is an arbitrary spatial scale), we obtain

$$\mathrm{i}\frac{\partial E}{\partial \xi} + \frac{1}{2}\frac{\partial^2 E}{\partial s^2} + \alpha\frac{S_s |E|^2 + RS_b I_b}{S_s |E|^2 + S_b I_b + \beta}E + \gamma_d \frac{\partial \ln(S_s |E|^2 + S_b I_b + \beta)}{\partial s}E = 0, \tag{4}$$

where $\alpha = k^2 x_0^2 n_e^2 \gamma_{\text{eff}} E_p/2$, $\gamma_d = k^2 x_0 n_e^2 \gamma_{\text{eff}} K_B T/(2q)$. γ_d stands for the strength of diffusion nonlinearity. Generally, for the case of ignoring the diffusion term of Eq. (4), we obtain

$$\mathrm{i}\frac{\partial E}{\partial \xi} + \frac{1}{2}\frac{\partial^2 E}{\partial s^2} + \alpha\frac{S_s |E|^2 + RS_b I_b}{S_s |E|^2 + S_b I_b + \beta}E = 0. \tag{5}$$

Ignoring the nonlinear term of Eq. (5) and comparing with standard Schrödinger equation in quantum mechanics, we can obtain the lattice potential $V(s)$ as follows

$$V(s) = -2\alpha\frac{R\cos^2(Kx_0 s)}{\cos^2(Kx_0 s) + I_d/I_0}. \tag{6}$$

Generally, $I_d(= \beta/S_s) << I_0$ is valid, so the potential strength of lattice are determined by $\alpha \cdot R$. The photovoltaic lattice solitons in the first (semi-infinite) band gap of linear periodic lattice of Eq. (5) were obtained in Ref. [23] when the working medium is photorefractive iron-doped lithium niobate ($LiNbO_3$) crystal and $R = 2.6$. Two photovoltaic lattice solitons corresponding to $\Gamma = -5$ and $\Gamma = -6.2$ with $I_0 = 0.1$ mW/cm^2 are displayed in Fig. 1. The dynamical evolution of the FS with $\Gamma = -6.2$ is shown in Fig. 1(b).

In uniform media, as is well known, the soliton having a transverse velocity will propagate with an oblique trajectory due to the Galilean transformation invariance of NLSE[27]. For lattice solitons, due to the absence of translational invariance along the s direction associated with the periodic potential, the Galilean transformation invariance is invalid. So the mobility of lattice solitons must be affected by nonlinear lattices even though the diffusive effect is not considered. We simulate Eq. (5) by Split-Step Fourier scheme with initial condition given by $E = U(s)\sqrt{I_0}\exp(\mathrm{i}\theta s)$, where $U(s)$ is the amplitude of soliton and θ stands for the collision angle (or the soliton transverse velocity). Unless stated otherwise, $U(s)$ corresponding to $\Gamma = -5$ displayed in Fig. 1(a) is chosen in our simulations.

In Fig. 2, we show the dynamical evolutions of photovoltaic lattice solitons with different θ. When $\theta = 1$, the soliton propagate with periodic oscillation in lattice potential well as shown in Fig. 2(a). This indicates the collision between soliton and lattice is almost elastic collision. The soliton almost like a particle restricted in lattice potential well. If the soliton is viewed as a particle, we can image an intuitive picture that when we increase θ which means increase the transverse velocity of soliton, it will traverse the hump of lattice potential and lose its transverse momentum simultaneously. When the transverse momentum of soliton is not large enough, it will be trapped in lattice potential well. Our simulations indicate the effective particle model will not be valid when the θ is large enough. When $\theta = 2$, the soliton undergoes scattering by the hump of nonlinear lattice and a part of energy of soliton couple into neighboring potential well as shown in Fig. 2(b). It is obvious that the period oscillation is disappeared and the collision is not elastic. When we increase θ, the soliton can transverse more humps of nonlinear lattice as shown in Fig. 2(c). In this case, the soliton is also robust even though there are some losses of radiation at each collision between the soliton and lattice.

Next, we study the effect of potential strength on interactions between photovoltaic lattice solitons and nonlinear lattices. For this purpose, we have repeated the above calculations with a lower potential value. In this case, we choose the value of $R = 2.0$ which can be

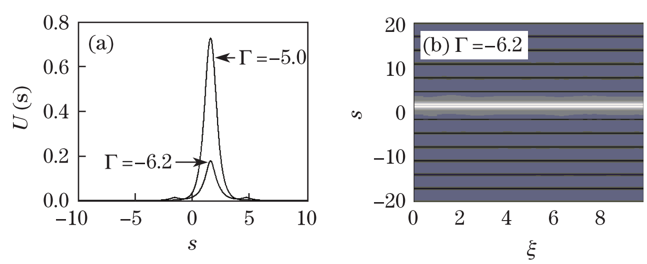

Fig. 1. (a) Photovoltaic lattice soliton solutions with $\Gamma = -6.2$ and $\Gamma = -5$; (b) the dynamical evolution of the lattice soliton corresponding to $\Gamma = -6.2$.

obtained by choose the wavelength of LB $\lambda_b = 594$ nm. The results are shown in Figs. 2(d)–(f). Comparing Fig. 2(a) with Fig. 2(d), one can see that oscillation frequency of soliton decreases with the decrease of lattice potential. In addition, as the decrease of lattice potential, the lattice has weaker blocking for tilted soliton as shown in Figs. 2(e) and (f). We would like to note that when θ are opposite numbers corresponding to aforementioned examples, the evolution results of photovoltaic lattice solitons will be symmetric with Fig. 2 about the $s = 0$ axis.

As is well known, the nonlocal diffusion nonlinearity of photorefractive crystal becomes significant for narrow light beams and results in the self-bending of soliton. In the following, we study the dynamical behavior of lattice soliton undergoing diffusion nonlinearity.

We directly simulate Eq. (4) by split-step Fourier scheme, with the diffusion nonlinear part treated in two ways: finite-difference scheme and pseudo-spectral method. The initial condition given by $E = U(s)\sqrt{I_0}\exp(\mathrm{i}\theta s)$, where $U(s)$ corresponds to $\Gamma = -5$ displayed in Fig. 1(a), and $\gamma_\mathrm{d} = 1$ is chosen in our simulations. In Fig. 3, we show the dynamical evolutions of photovoltaic lattice solitons with different θ. For small tilted angle cases ($\theta = \pm 1$), the soliton is still restricted by lattice potential well, however the periodic oscillation of soliton disappear. Furthermore, the evolution patterns with respect to $\theta = \pm 1$ are not symmetric with regard to the $s = 0$ axis as shown in Figs. 3(a) and (d) because the diffusive nonlinearity is asymmetric. The evident asymmetric behavior are shown in Figs. 3(b) and (e) and Figs. 3(c) and (f) where $\theta = \pm 2$ and ± 3.

An interesting case is observed in Fig. 3(f) which is corresponding to $\theta = 3$. In this case, the lattice soliton is like a "tilted (or traveling) soliton" which moves across the lattices undistorted[28,29]. The existence of tilted soliton is a novel phenomenon, because the moving

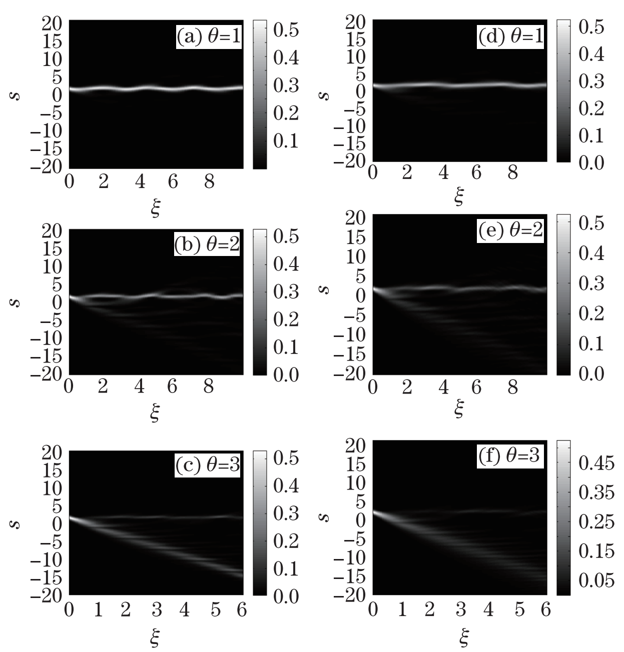

Fig. 2. Dynamical evolutions of photovoltaic lattice solitons with different θ and different lattice potentials. (a)–(c) $R = 2.6$; (d)–(f) $R = 2.0$.

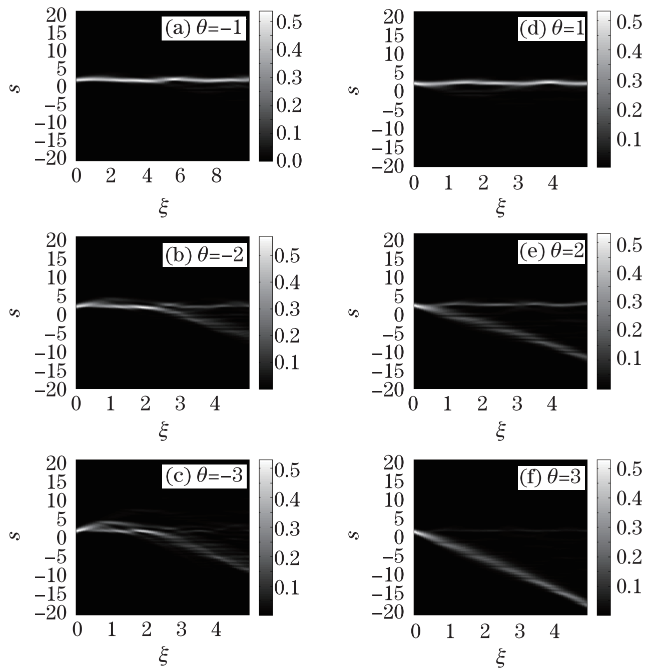

Fig. 3. Dynamical evolutions of photovoltaic lattice solitons undergoing diffusion nonlinearity with different θ. (a) $\theta = -1$; (b) $\theta = -2$; (c) $\theta = -3$; (d) $\theta = 1$; (e) $\theta = 2$; (f) $\theta = 3$.

soliton will lose its kinetic energy due to the presence of the lattice potential. Although there is a long-standing debate on the existence of exact tilted soliton in the discrete NLSE, it is generally believed that tilted soliton should possess a nontrivial nonlinear chirped phase and the motion of tilted soliton is supported by the phase gradient. Comparing Fig. 2(f) with Fig. 3(f), we can see the diffusive nonlinearity has evident contribution on the mobility of lattice soliton. We believe that the diffusive nonlinearity introduces a chirped phase to lattice soliton and the soliton becomes more robust.

In conclusion, we demonstrate theoretically the influence of diffusive nonlinearity on mobility of photovoltaic lattice solitons based on Kukhtarev model. Due to the presence of periodic lattice potential, the Galilean transformation invariance of nonlinear Schrödinger equation is invalid. The dynamical evolution of collision between photovoltaic lattice solitons and nonlinear lattices are simulated numerically. The results show the lattice solitons having a transverse velocity have complicated behaviors and will not propagate with an oblique trajectory. When we consider the diffusive nonlinearity, it is found that diffusive nonlinearity can introduce a chirped phase to lattice soliton and the lattice soliton with a special incident angle can becomes a "tilted soliton".

This work was supported by the Xin Miao Science Foundation of Guangzhou University (No. ZBZ1-1001) and the Foundation for Distinguished Young Talents in Higher Education of Guangdong Province, China (No. LYM11098).

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