Axial multiple foci of radially polarized hollow Gaussian beam

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Axial multiple foci patterns of radially polarized hollow Gaussian beam (HGB) with radial wavefront distribution is investigated theoretically. The wavefront phase distribution is cosine function of radial coordinate. Simulation results show that the multiple foci patterns can be adjusted considerably by the beam order of HGB and cosine parameter that indicates the phase change degree. The foci number fluctuates on increasing cosine parameter for certain beam order. And when the beam order is small, there occur five foci in focal region, and the cases are more frequently than that under the condition of higher beam order. Gradient force distributions are also given to show that the multiple foci of radially polarized HCB may be applied to construct tunable optical traps.

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Recently, hollow Gaussian beams (HGBs) have attracted much attention and become one focused point in the field of optical physics[1−11]. It was found that the area of the dark region across the HGBs can be easily controlled by proper choice of the beam parameters[1]. The propagation of HGBs through paraxial systems has been investigated in detailed[5,6]. The far-field intensity distribution and M^2 factor of hollow Gaussian beams were also studied by Deng *et al.*[7,8]. Zheng derived the analytical formulae for the Fractional Fourier transform for HGBs based on the definition in the cylindrical coordinate system[9]. Recently, the nonparaxial propagation of vectorial HGBs in free space was studied based on the vectorial Raleigh–Sommerfeld diffraction integral[10], and the analytical vectorial structure of HGB was investigated in the far field based on the vector plane wave spectrum and the method of stationary phase[11].

On the other hand, focusing properties of laser beams have attracted many researchers for several decades[12−16]. Single-beam optical tweezers are used for individual particle trapping. To extend the application, creation of multiple trapping sites has been investigated for the manipulation of more than one particle[17]. Stacking of particles by use of standing waves, Bessel beams, and Gaussian beams has been reported[18−20]. These light stacks are continuous, and the particles inside the stack cannot be manipulated, nor can the stack be moved vertically. Although a moving conveyor belt using angular Doppler effects that can trap and deliver atoms at intervals has been reported, its trapping is not stable in all three dimensions[21]. Zhao *et al.* proposed a design for producing a conveyable quasi-periodic optical chain that could stably trap and deliver multiple individual particles in three dimensions at different planes near the focus[22]. Method of generating multiple on-axis spherical spots in a 4pi focusing system with a radially polarized beam was also proposed[23]. Valle *et al.* have theoretically and experimentally demonstrated the generation of multiple coaxial foci by means of pupil plane phase masks[24].

In our knowledge, the focusing properties of the radially polarized HGBs with cosine phase wavefront were not studied. In fact, the polarization and phase wavefront are very important characteristics to alter propagating and focusing properties of beams[25−28]. This letter is aimed at studying focal shift in radially polarized HGBs with cosine phase wavefront by vector diffraction theory.

In the focusing system we investigated, incident beam is radially polarized HGB whose value of transverse optical field is same as that of the scalar HGB, and its polarization distribution turns on radial symmetry. Wavefront phase distribution of this kind of radially polarized HGB is cosine function of radial coordinate. Therefore, in the cylindrical coordinate system $(r, \varphi, 0)$ the field distribution $\mathbf{E}(r, \varphi, 0)$ of the radially polarized HGB at its waist plane is written as

$$\mathbf{E}_0(r,\varphi,0) = E_0(r,\varphi,0)\cdot \boldsymbol{n}_r, \qquad (1)$$

where n_r is the radial unit vector of polarized direction. Term $E_0(r, \varphi, 0)$ is optical field value distribution and can be written in the form[1]

$$\begin{aligned} E_0(r,\varphi,0) =& D\left(\frac{r^2}{\omega_0^2}\right)^n \exp\left(-\frac{r^2}{\omega_0^2}\right), \\ &\cdot \exp(\mathrm{i}\phi), \quad n = 0,1,2,\cdots, \end{aligned} \qquad (2)$$

where D is a constant, ω_0 is the waist width of the Gaussian beam, ϕ denotes wavefront phase distribution, and n is the order of HGB. When $n = 0$ and $\phi = 0$, Eq. (2) reduces to that for a fundamental Gaussian beam with beam waist size ω_0. Using the same analysis method as that in Refs. [25, 26], the electric field in focal region of radially polarized HGB is

$$\mathbf{E}\left(r,\varphi,z\right) = E_r \boldsymbol{e}_r + E_z \boldsymbol{e}_z, \qquad (3)$$

where $\boldsymbol{e}_r$ and $\boldsymbol{e}_z$ are the unit vectors in the radial and propagating directions, respectively. E_r and E_z are amplitudes of the two orthogonal components and can be expressed as[26]

$$E_r(r,z) = A\int_0^\alpha \sqrt{\cos\theta}\exp(\mathrm{i}\phi)E_0\sin(2\theta) \cdot J_1(kr\sin\theta)\exp(\mathrm{i}kz\cos\theta)\mathrm{d}\theta, \qquad (4)$$

$$E_z(r,z) = 2\mathrm{i}A\int_0^\alpha \sqrt{\cos\theta}\exp(\mathrm{i}\phi)E_0\sin^2(\theta) \cdot J_0(kr\sin\theta)\exp(\mathrm{i}kz\cos\theta)\mathrm{d}\theta, \qquad (5)$$

where r and z are the radial and longitude coordinates of observation point in focal region, respectively. k is wave number. θ is the tangential angle with respect to the z axis. $\alpha = \arcsin(\mathrm{NA})$ is convergence angle corresponding to the radius of incident optical aperture, where NA is numerical aperture. Here the wavefront phase distribution is cosine function of radial coordinate and is written in the form of

$$\phi = \pi\cos\left(C\frac{\theta}{\alpha}\pi\right), \qquad (6)$$

where C is called cosine parameter that indicates the phase change degree. In order to make focusing properties clear and simplify calculation process, after simple derivation, Eqs. (4) and (5) can be rewritten as[29]

$$E_r(r,z) = AD\int_0^\alpha \sqrt{\cos\theta}\left[\frac{\sin^2(\theta)}{w^2\mathrm{NA}^2}\right]^n \cdot\exp\left[-\frac{\sin^2(\theta)}{w^2\mathrm{NA}^2}\right]\exp\left[\mathrm{i}\pi\cos\left(C\frac{\theta}{\alpha}\pi\right)\right] \cdot\sin(2\theta)J_1(kr\sin\theta)\exp(\mathrm{i}kz\cos\theta)\mathrm{d}\theta, \qquad (7)$$

$$E_z(r,z) = 2\mathrm{i}AD\int_0^\alpha \sqrt{\cos\theta}\left[\frac{\sin^2(\theta)}{w^2\mathrm{NA}^2}\right]^n \cdot\exp\left[-\frac{\sin^2(\theta)}{w^2\mathrm{NA}^2}\right]\exp\left[\mathrm{i}\pi\cos\left(C\frac{\theta}{\alpha}\pi\right)\right] \cdot\sin^2(\theta)J_0(kr\sin\theta)\exp(\mathrm{i}kz\cos\theta)\mathrm{d}\theta. \qquad (8)$$

The optical intensity in focal region is proportional to the modulus square of Eq. (3). Basing on the above equations, focusing properties of radially polarized HGB can be investigated theoretically.

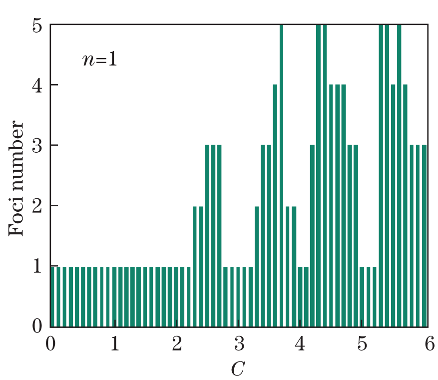

Fig. 1. Dependence of foci number on increasing C under condition of n=1.

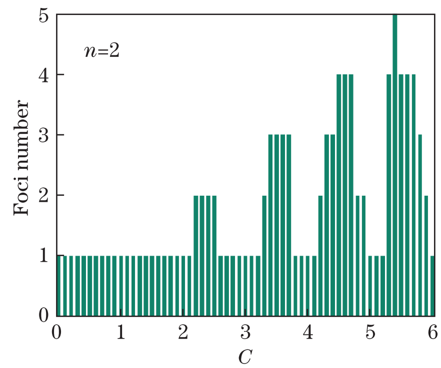

Fig. 2. Dependence of foci number on increasing C under condition of n=2.

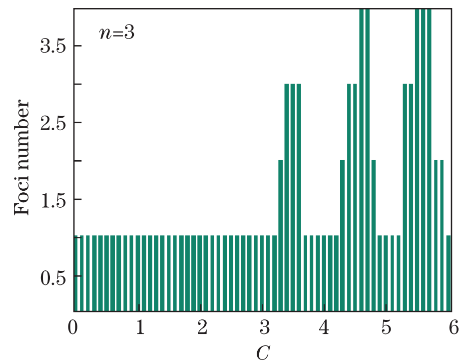

Fig. 3. Dependence of foci number on increasing C under condition of n=3.

In this letter, we focused on the axial multiple foci patterns of radially polarized hollow Gaussian beams with radial wavefront distribution. It should be noted that one focal peak is defined as one intensity peak whose intensity value is bigger than 50% value of maximum intensity in focal region. In addition, the intensity difference between the local intensity minimum and smaller peak of adjacent two intensity peaks is bigger than 5% value of maximum intensity in focal region. Without loss of validity and generality, the numerical aperture NA $= 0.95$, and $w = 1$. Figure 1 illustrates the dependence of foci number on increasing cosine parameter C under condition of beam order $n = 1$. We can see that there is only one focus for small C. By increasing C, there occur two foci in focal region, and then the number of foci fluctuates rapidly. And more foci may come into being for higher C, there are several conditions under which five foci appear.

In order to get insight into the effect of the beam order on the foci number evolution, the dependence of foci number on increasing cosine parameter C under condition of beam order $n = 2$ is given in Fig. 2. It can be seen by comparing Fig. 2 with Fig. 1 that the foci number also fluctuates on increasing cosine parameter C. However, under the condition of $n = 2$, the possibility for five foci is smaller than that for $n = 1$, namely, the smaller the beam order is, the lower possibility the multiple foci appear.

To confirm this evolution principle, the dependence of

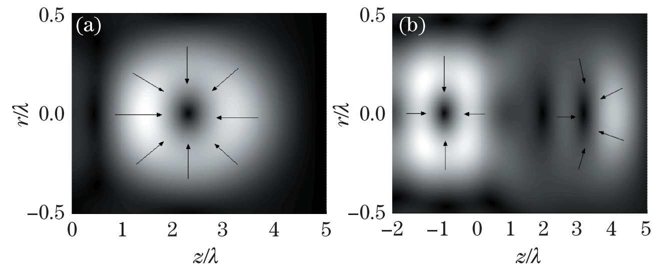

Fig. 4. Gradient force distributions under condition of (a) C=2, n=2 and (b) C=2.4, n=2. Arrows indicate the force direction.

foci number on increasing cosine parameter C under condition of beam order $n = 3$ is illustrated in Fig. 3. There is no five-peak focal pattern on increasing C, and the condition number for four foci still decreases. When the beam order is small, there occur five foci in focal region, and the cases are more frequently than that under the condition of higher beam order.

In optical trapping systems, it is usually deemed that the forces exerted on the particle in light field consist of two kinds of forces, one is the optical gradient force, which plays a crucial role in constructing optical trap and its intensity is proportional to the optical intensity gradient; the other kind of force is scattering force, which usually has complex forms because this kind of force is related to the properties of the trapped particles, and whose intensity is proportional to the optical intensity[30]. Therefore, tunable optical intensity distribution in focal region means that the controllable optical trap may occur. Gradient force points in the direction of the gradient of the light intensity if the diffractive index of particles is bigger than that of surrounding medium. Figure 4 illustrates the gradient force distributions for two cases under condition of higher diffractive index of particles in focal region. Arrows indicate the force direction. One optical trap evolves into two traps though the second trap is weak. Therefore, the focusing properties of radially polarized HCB may be used to construct optical traps.

In conclusion, axial multiple foci patterns of radially polarized hollow Gaussian beams with radial wavefront distribution is investigated theoretically. Simulation results show that the multiple foci patterns can be adjusted considerably by the beam order of HGB and cosine parameter that indicates the phase change degree. And foci number fluctuates on increasing cosine parameter for certain beam order. When the beam order is small, there occur five foci in focal region, and the cases are more frequently than that under the condition of higher beam order. Gradient force distributions are also given to show that radially polarized HCB may be used in optical tweezers technology.

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