Polarization aberration of optical systems in imaging polarimetry

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Polarization aberration of optical systems in imaging polarimetry affects the polarization detection accuracy, especially in wide field of view and large relative aperture systems. The polarization aberration of the imaging lens in imaging polarimetry is demonstrated and analyzed through the way of polarization ray tracing. The impact of polarization aberration on the polarization detection accuracy of imaging polarimetry is also discussed. The variation of Stokes parameters as functions of the field of view and the relative aperture is achieved. The polarization aberration can be reduced and calibrated at different field of view and the relative aperture of the optical systems, and the correct polarization information of the object can be derived.

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Imaging polarimetry extends the capability of conventional imaging by providing polarization information about a scene. It has emerged as an important technology in a variety of fields, such as remote sensing applications[1], astronomy and space exploration[2], bioscience and medicine[3], and so on. Calibration of this instrument is extremely important because orientation alignment errors of the polarizing elements and polarization aberration of optical systems can induce errors in the resultant measured polarization parameters. Peterson *et al.* analytically characterized systematic errors of a particular imaging polarimeter including corrected detector response uncertainty and spatial co-registration error[4]. Boger *et al.* presented an error evaluation template to itemize and quantify sources of error in polarimetric instruments[5].

However, the polarization effect of the optical systems on the polarization measurement uncertainty is lack of study. The polarization aberration of optical systems is critical in understanding imaging polarimeter performance. Chipman *et al.* proposed the polarization aberration of optical systems to analyze instrumental polarization[6] and described the polarization in terms of Jones matrix[7]. Nevertheless, it is more commonly utilized to represent the polarization information in terms of the Stokes vector for imaging polarimetry.

Apart from the previous studies, polarization characteristics of the imaging lens are described for imaging polarimeter in this letter. We analyze the polarization effect of the optical systems and numerically simulate the variation of state of polarization (SOP) of the light passed though the lens via the method of the polarization ray tracing in terms of Stokes vector.

The imaging polarimeter is consisted of imaging lens, polarization state analyzer (PSA) and the detectors, as shown in Fig. 1. We extend the intensity distribution function to SOP distribution function. The SOP is represented in terms of Stokes vector, and the Stokes vector of imaging plane is described as

$$\mathbf{S}_{\mathrm{i}}(x',y') = \int_s \mathbf{M}_{\mathrm{pe}}(x',x,y',y) \times \mathbf{M}_{\mathrm{op}}(x',x,y',y) \times \mathbf{S}_{\mathrm{o}}(x,y)\mathrm{d}x\mathrm{d}y, \quad (1)$$

where $\mathbf{M}_{\mathrm{pe}}$ and $\mathbf{M}_{\mathrm{op}}$ are respectively the Mueller matrix of the PSA and the imaging lens and $\mathbf{S}_{\mathrm{o}}(x,y)$ is the Stokes vector of the scene. (x, y) is the object plane coordinates, and (x', y') is the corresponding image plane coordinates.

The SOP of the object can be characterized in terms of Stokes vector in the image plane with an imaging polarmeter. Polarization detection accuracy is associated with instrument error including polarization aberration of imaging lens. The polarization characteristics of imaging lens result from non-normal incidence at its optical interfaces. To achieve the variation of the SOP of the ray, polarization ray tracing is used. We describe the ray passing through the each surface.

The coordinate system assumed for Stokes vectors at each interface q are determined as shown as Fig. 2, which are as same as Jones matrix description[7], three basis vectors for the coordinate at each interface are $\mathbf{R}_{\mathrm{q}}$, $\mathbf{P}_{\mathrm{q}}$, $\mathbf{Q}_{\mathrm{q}}$ with

$$\mathbf{P}_{\mathrm{q}} = \frac{\mathbf{R}_{\mathrm{q}} \times \mathbf{N}_{\mathrm{q}}}{|\mathbf{R}_{\mathrm{q}} \times \mathbf{N}_{\mathrm{q}}|}, \quad (2)$$

$$\mathbf{Q}_{\mathrm{q}} = \mathbf{R}_{\mathrm{q}} \times \mathbf{P}_{\mathrm{q}}, \quad (3)$$

where "×" denotes a cross product, $\mathbf{R}_{\mathrm{q}}$ is parallel to the geometrical ray, and $\mathbf{N}_{\mathrm{q}}$ is the surface normal. The

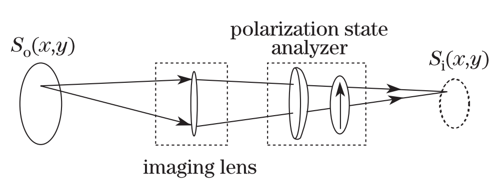

Fig. 1. Structural diagram of imaging polarimeter.

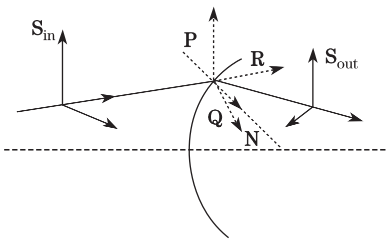

Fig. 2. Calculation of the polarization along a ray through an optical interface.

Stokes vector of light is written as a four-element column vector

$$\mathbf{S}_{\rm q}=\begin{pmatrix} S_{0\rm q}\\ S_{1\rm q}\\ S_{2\rm q}\\ S_{3\rm q}\end{pmatrix}=\begin{pmatrix} I_{\rm Pq}+I_{\rm Qq}\\ I_{\rm Pq}-I_{\rm Qq}\\ I_{45^\circ \rm q}-I_{-45^\circ \rm q}\\ I_{\rm Lq}+I_{\rm Rq}\end{pmatrix}, \quad (4)$$

where $I_{\rm Pq}$ and $I_{\rm Qq}$ are the intensity of horizontal and vertical linear polarization, respectively, $I_{45^\circ \rm q}$ and $I_{-45^\circ \rm q}$ are the intensities of 45° and −45° linear polarization, respectively, and $I_{\rm Lq}$ and $I_{\rm Rq}$ are the intensities of right and left circular polarization respectively. The Stokes vectors for successive surfaces are related by projection of the basis states for interface q+1 on the basis states for interface q:

$$\mathbf{S}_{\rm q+1}=\mathbf{M}_{\rm R}(\theta_{\rm q})\mathbf{S}_{\rm q}, \quad (5)$$

where $\mathbf{M}_{\rm R}(\theta_{\rm q})$ is rotation matrix, which can be described as

$$\mathbf{M}_{\rm R}(\theta_{\rm q})=\begin{pmatrix} 1 & 0 & 0 & 0\\ 0 & \cos\theta_{\rm q} & -\sin\theta_{\rm q} & 0\\ 0 & \sin\theta_{\rm q} & \cos\theta_{\rm q} & 0\\ 0 & 0 & 0 & 1\end{pmatrix}, \quad (6)$$

$\theta_{\rm q}$ is the angle between old and new axes, described as

$$\theta_{\rm q}=\arctan\left(\frac{\boldsymbol{P}_{\rm q+1}\cdot\boldsymbol{Q}_{\rm q}}{\boldsymbol{P}_{\rm q+1}\cdot\boldsymbol{P}_{\rm q}}\right). \quad (7)$$

Consider a ray incident at an interface between two homogeneous and isotropic mediums. The Mueller matrix $\mathbf{M}(i_{\rm q})$ relates the Stokes vectors at the interface,

$$\mathbf{S}'_{\rm q}=\mathbf{M}(i_{\rm q})\mathbf{S}_{\rm q}, \quad (8)$$

where $i_{\rm q}$ is the angle of incidence, "′" denotes quantities after the interface, the Mueller matrix $\mathbf{M}(i_{\rm q})$ at interface q is described as[8]

$$\mathbf{M}(i_{\rm q})=\begin{pmatrix} \frac{T_{\rm Pq}(i_{\rm q})+T_{\rm Qq}(i_{\rm q})}{2} & \frac{T_{\rm Pq}(i_{\rm q})-T_{\rm Qq}(i_{\rm q})}{2} & 0 & 0\\ \frac{T_{\rm Pq}(i_{\rm q})-T_{\rm Qq}(i_{\rm q})}{2} & \frac{T_{\rm Pq}(i_{\rm q})+T_{\rm Qq}(i_{\rm q})}{2} & 0 & 0\\ 0 & 0 & \sqrt{T_{\rm Pq}(i_{\rm q})\cdot T_{\rm Qq}(i_{\rm q})} & 0\\ 0 & 0 & 0 & \sqrt{T_{\rm Pq}(i_{\rm q})\cdot T_{\rm Qq}(i_{\rm q})}\end{pmatrix}, \quad (9)$$

$$i_{\rm q}=\arcsin\left|\mathbf{P}_{\rm q}\times\mathbf{Q}_{\rm q}\right|,$$

where $T_{\rm Pq}(i_{\rm q})$ and $T_{\rm Qq}(i_{\rm q})$ are Fresnel intensity transmission (or reflection) coefficients for perpendicular (P) and parallel (Q) to the plane of incidence, respectively.

To calculate the polarization by each surface in a system, we rotate to the $P_{\rm q}-Q_{\rm q}$ coordinates of the next surface. The Mueller matrix at interface q is written as

$$\mathbf{M}_{\rm q}(i_{\rm q},\theta_{\rm q})=\mathbf{M}_{\rm q}(i_{\rm q})\cdot\mathbf{M}_{\rm R}(\theta_{\rm q}). \quad (10)$$

If a ray is incident on a system of surfaces, the polarization along the ray is achieved by cascading the effect of each surface. The Mueller matrix of the whole optical system for each possible ray path at the P_1-Q_1 coordinates can be written as

$$\mathbf{M}(\mathbf{P}_1,\mathbf{Q}_1)=\prod_{\rm q=N}^{1}\mathbf{M}_{\rm q}(i_{\rm q},\theta_{\rm q}), \quad (11)$$

$$\mathbf{S}'_{\rm out}=\mathbf{M}(\mathbf{P}_1,\mathbf{Q}_1)\mathbf{S}_{\rm in}. \quad (12)$$

The coordinate systems of Stokes vector of incident ray are R_1-Q_1 coordinates, and the coordinate systems of Stokes vector of output ray $R_{\rm N}-Q_{\rm N}$ coordinates, in Eq. (12). If the coordinate system transformed to assumed $x-y$ coordinates, the Stokes vector should be multiplied by the rotation matrix. Polarization aberration of imaging lens can be analyzed by calculating Eq. (11).

For imaging lens, polarization aberrations are caused by weak polarization effects occurring when a ray incident to the interface. Each optical surface of the imaging lens behave as a diattenuator, whose diattenuation of P-polarization (perpendicular to the plane of incidence) and Q-polarization (parallel to the plane of incidence) is described as

$$Di=\frac{|T_{\rm Pq}(i_{\rm q})-T_{\rm Qq}(i_{\rm q})|}{T_{\rm Pq}(i_{\rm q})+T_{\rm Qq}(i_{\rm q})}. \quad (13)$$

We numerically analyze the diattenuation of refraction at an interface between glass and air as functions of incidence angle and reflective index, as shown in Fig. 3. The diattenuation increases as the refractive index and incident angle. The diattenuation is less than 0.07 when the incident angle is below 60° and would reach to 0.1 as the incident angle is up to 80°.

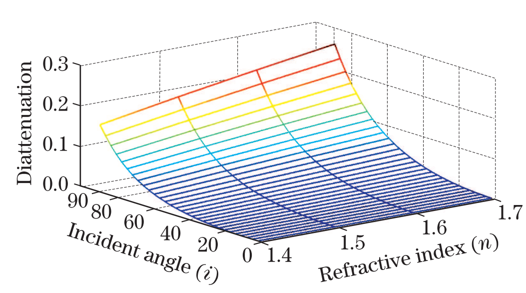

Fig. 3. Diattenuation of refraction interface of a lens and air.

To describe the polarization aberration of imaging lens, we calculate the SOP variation of the ray pass through the lens. From Eq. (1), the SOP of each image point is integral of all ray incident onto the point through the whole pupil.

Exact polarization ray tracing of imaging lens is difficult to achieve. We ignore the classical geometric aberrations to focus on the polarization aberration. We simplify the imaging lens to a typical spherical singlet lens, whose radii of front and back surface are 100 and −100 with glass refractive index of 1.5. Its effective focal length is 100 mm and relative aperture (aperture diameter/focal length, D/F) is range from 0.1 to 1. We numerically simulate the variation of Stokes parameters as functions of angle of view at several different relative apertures for unpolarized light (Fig. 4(a)), linear polarized light (Fig. 4(b)) and circle polarized light (Fig. 4(c)).

The variation of the Stokes parameters increases as the relative aperture and the angle of view. The fourth Stokes parameter S_3 which is the circularly polarized component varies little through a lens. The variation is still less than 0.01, when the relative aperture is up to 1 and angle of view is up to 60°. The second Stokes parameter S_1 and the third Stokes parameter S_2 describe the amount of linear polarization. Their variations mainly vary with the angle of view. Increase of the relative aperture of imaging lens can lead to increase of variation of S_2, and little increase of variation of S_1 and S_3.

We also analyze the degree of polarization as the functions of angle of view and relative aperture, as shown in Fig. 5. The degree of polarization will change to 0.92, when linear polarization light passed through the singlet with D/F of 1 at the angle of view of 60.

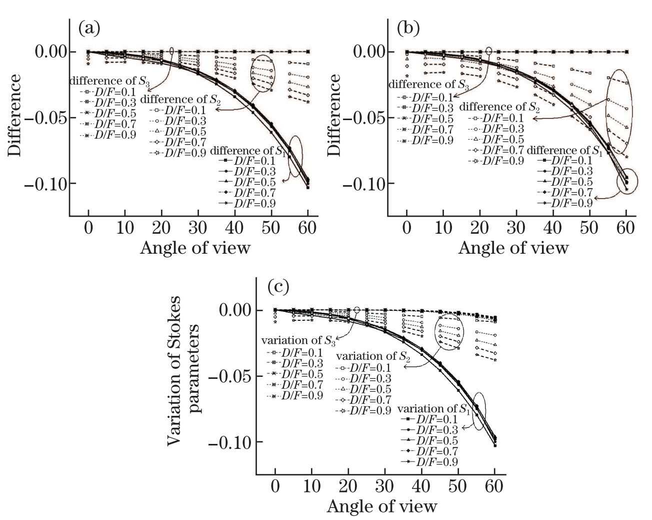

Fig. 4. Variation of polarization parameters through a singlet for incidence of (a) unpolarized light, (b) linear polarized light, and (c) circle polarized light.

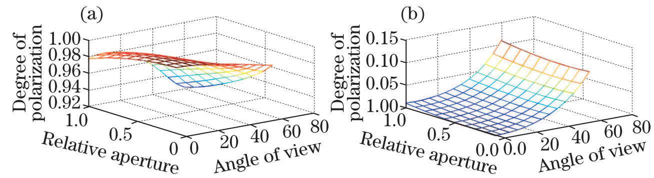

Fig. 5. Degree of polarization as functions of relative aperture and angle of view. (a) Incidence of linear polarized light; (b) incidence of unpolarized light.

Unpolarized light passed through the singlet with D/F of 1 becomes partially polarized, and the degree of polarization changes to 0.01 at the angle of view of 60.

In conclusion we analyze the effect of polarization aberration of imaging lens on the imaging polarization detection accuracy and numerically simulate the variation of Stokes parameters through a single lens. The results show that the variation of the Stokes parameters increases with the relative aperture and angle of view because of the incident angle of ray. When the incident light passes through imaging lens, the circularly polarized component varies less than the linear polarized component. The degree of polarization will change to 0.1 from 0 for incident unpolarized light and change to 0.92 from 1 for linear polarized light at the angle of view of 60°, through a single lens, whose relative aperture is between 0.1 to 1.0. Polarization aberration of imaging lens need be calibrated at different fields of view and corrected for highly accurate imaging polarimeter with large relative aperture and wide field. And the correct polarization information of the object can be derived.

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