## Image coding based on integer wavelet and embedded optimal coefficient scaling

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Received January 3, 2012; accepted Feburary 21, 2012; posted online June 20, 2012

Integer wavelet transform (IWT) could offer the lower computational complexity and the less storage spending for image compression than the discrete wavelet transform (DWT). But the most coefficients of the IWT image have smaller dynamic change ranges than discrete wavelet transform. In this letter, an efficient and low-complexity coding algorithm called embedded optimal coefficient scaling (EOCS) is proposed. It optimizes the distribution of the wavelet coefficients in every threshold plane and provides an efficient embedded quadtree-partitioning scheme to encode the image. Experimental results show that the presented method cannot provide peak signal-to-noise ratio (PSNR) performance up about 2–6 dB better than set partitioning in hierarchical trees (SPIHT) without the OCSF scheme, but also support both efficiently lossy and lossless compression in single bitstream.

OCIS codes: 100.2000, 100.3008, 280.0280.

doi: 10.3788/COL201210.S11010.

The discrete wavelet transform (DWT) is widely used in image compression. Based on the DWT, many efficient coding algorithms were presented. For example, embedded zerotree wavelet (EZW) and set partitioning in hierarchical trees (SPIHT) algorithms based on the zerotree scheme were found in Refs. [1-3]. In Ref. [4] SPECK and EZBC were proposed with embedded zeroblock coding scheme. based on DWT, these algorithms are efficient. However, the main drawback of the DWT is that the wavelet coefficients are floating-point numbers, which increases the computational complexity and is not well suited for efficient lossless coding application. The lifting scheme (LS) presented by Sweldens<sup>[5]</sup> allows a low complexity and efficient implementation of the DWT. This allows new transforms to be used. One is the LSbased integer wavelet transform (IWT) scheme<sup>[6]</sup>.</sup>

Using reversible IWT for compression of image has three advantages. Firstly, through the use of appropriate techniques, a lossless decoding image can be reconstructed. This point is very significant for medical and remote sensing image processing. Secondly, IWT has lower computational complexity than DWT because of the LS. Finally, the use of IWT is also a means to reduce the memory demands of the compression algorithm as integers are used instead of real numbers<sup>[7]</sup>.

Although IWT is very interesting because of the previously cited advantages, its main drawback is that the most image coefficients after IWT has smaller dynamic change value and worse energy compaction than DWT, which would degrades the performances of the lossy coding<sup>[8,9]</sup>. In this letter, an efficient and lowcomplexity coding algorithm so-called embedded optimal coefficient scaling (EOCS) is presented, which could optimize the distribution of the wavelet image coefficients in every threshold plane by optimal coefficient scaling factor (OCSF). During encoding, a simple, efficient quadtree-partitioning scheme is proposed. Experimental results show that the EOCS algorithm can performance up about 2–6 dB better than SPIHT using 5/3 IWT without OCSF in the lossy image coding. The main drawback of the DWT is that the wavelet coefficients are floating-point numbers. In this case efficient lossless coding is not possible using DWT. The LS presented by Daubechies *et al.*<sup>[6,7]</sup> supports the low-complexity and efficient IWT scheme. Using IWT for compression of image can reduce the memory demands of the compression algorithm as integers are used instead of real numbers, which is very significant for medical and remote sensing image processing. Forward transforms of (5,3) and (6,14) were evaluated in Table 1. In LS, the integer wavelet transforms can be described through polyphase matrix using Euclidean algorithm as

$$P(z) = \prod_{i=1}^{m} \begin{bmatrix} 1 & s_i(z) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ t_i(z) & 1 \end{bmatrix} \begin{bmatrix} K & 0 \\ 0 & 1/K \end{bmatrix},$$
(1)

where P(z) and P(z) can be defined as analysis filters;  $s_i(z)$  and  $t_i(z)$  can be defined as Laurent polynomials. General interpolating biorthogonal integer wavelet transform (IB-IWT) can be described as

$$P(z) = \prod_{i=1}^{m} \begin{bmatrix} 1 & s(z) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ t(z) & 1 \end{bmatrix}.$$
 (2)

IB-IWT is the efficient and low complexity IWT for image compression. Several forward transform of IWTs can be found in Table 1.

In Table 1, we use the notation (x, y) to indicate that the underlying filter bank has lowpass and highpass analysis filters of lengths x and y, respectively. In the forward transform equations, the input signal, lowpass subband signal, and highpass subband signal are denoted as x[n], s[n], and d[n], respectively. For convenience, we also define the quantities  $s_0[n]=x[2n]$  and  $d_0[n]=x[2n+1]$ .

Although IWT can allow both lossy and lossless compression using single bitstream, its lossy compression performance is not as efficient as that of DWT. Experimental results show that using the IWT instead of the DWT

Table 1. Several Forward Transform of IWTs

Name $(x, y)$	Forward Transform of IWT
(5,3)	$\begin{cases} d[n] = d_0[n] - \lfloor 1/2(s_0[n+1] + s_0[n]) \rfloor \\ s[n] = s_0[n] + \lfloor 1/4(d[n] + d[n-1]) + 1/2 \rfloor \end{cases}$
(6,14)	$\begin{cases} d_1[n] = d_0[n] - s_0[n] \\ s[n] = s_0[n] + \lfloor 1/16(8d_1[n] + d_1[n-1] - d_1[n+1]) + 1/2 \rfloor \\ d[n] = d_1[n] + \lfloor 1/16(s[n+2] - s[n-2] + 6(s[n-1] - s[n+1]) + 1/2 \rfloor \end{cases}$
(13,7)	$\begin{cases} d[n] = d_0[n] + \lfloor 1/16((s_0[n+2] + s_0[n-1]) - 9(s_0[n+1] + s_0[n]) + 1/2) \rfloor \\ s[n] = s_0[n] + \lfloor 1/32(9(d[n] + d[n-1]) - (d[n+1] + d[n-2]) + 1/2 \rfloor \end{cases}$

degrades the peak signal-to-noise ratio (PSNR) performances of the lossy coding. For example, the PSNR value decreases 6 dB for Barbara image using the IWTbased SPIHT and decreases 5 dB for Lena image using the IWT-based SPIHT. Two main drawbacks are found to degrade the IWT lossy performance<sup>[1]</sup>. 1) The reversible IWT is not orthonormal, and the information content of each coefficient is no longer directly related to magnitude; this is particularly harmful for encoders with rate allocation based on bitplanes, such as EZW, SPIHT and SPECK coding scheme; 2) to guarantee all transform coefficients being integer values for integer inputs, mantissa-rounding operations are adopted. So the RB-IWTs are the nonlinear transform.

In Ref. [9], subbands scaling is introduced for ensuring optimum rate-distortion performance. However, subbands' scaling only solves the first problem. In this letter, we propose a OCSF K based on LS in Eq. (1). If we only consider the biorthogonal DWT based on the same LS, the OCSF can be computed using a method similar to the one described in Ref. [5]. However, the IWT is the nonlinear because of the rounding operations. The computation for OCSF is very difficult. We use many experiments to analyze the OCSF value for different IWTs. Experimental results show two significant conclusions for the OCSF: 1) the OCSF value of IWT is near that value of biorthogonal DWT; 2) if the lowpass and highpass analysis filters of the underlying filter bank are shorter, its OCSF is higher. When the lowpass and highpass analysis filters are longer, the OCSF will be smaller.

Figures 1 and 2 show the PSNR values comparison of lossy compression performance of 6/14 IWT at different K values for test image Barbara and Lena.



Fig. 1. PSNR value comparison of lossy compression performance of 6/14 IWT at different K values for Barbara.



Fig. 2. PSNR value comparison of lossy compression performance of 6/14 IWT at different K values for Lena.

We adopt the integer powers of two as the threshold of coefficient quantization. We say that a set  $\Omega$  of coefficients is significant with respect to n if

$$\max_{(i,j)\in\Omega} \{|c_{i,j}|\} \ge 2^n (n=0,1,2,3\cdots).$$
(3)

Table 2. OCSF of Several IWTs

IB-IWT	(5, 3)	(2, 10)	(6, 14)	(13, 7)
OCSF	1.412	1.417	1.406	1.385

Otherwise it is insignificant. We can write the significance of a set  $\Omega$  as

$$\Gamma_n(\Omega) = \begin{cases} 1, & \text{if } 2^n \leq \max_{(i,j) \in \Omega} |c_{i,j}| < 2^{n+1} \\ 0 & \text{else} \end{cases}$$
(4)

According to the definition of thresholds, the basic steps of embedded quadtree-partitioning scheme (EQPS) are presented as following. 1) Start with S as the root set and set all transform coefficients to S. Append set S to an array of insignificant sets and pixels, called the AISP; 2) the significance test is adopted for the same n to each of these sets S belonged to the AISP. If S is significant for threshold  $n_{\text{max}}$ , it is partitioned by the quadtree-partitioning scheme. Set S is spitted into four quadrant sets, collectively denoted O(S); 3) significant sets continue to be recursively split until all there are four pixels, whereupon, the significant ones are found and appended to a list of significant pixels, called the LSP and output its sign; 4) for the insignificant sets and pixels, their sizes and coordinates can be appended to the AISP; 5) for each $(i, j) \in LSP$ , except those included

in the last sorting pass, output the *n*th MSB of  $|c_{i,j}|$ ; 6) decrement *n* by 1, and go to step 3, until *n* is equal to 1.

Figure 3 shows the partitioning scheme of the EQPSbased coding algorithm for image coefficients after IWT.

The EQPS algorithm has three primary advantages for IWT-based coding. Firstly, the OCSF is proposed for improving energy compaction of IWT. Secondly, it sets all image transform coefficients as partitioning set in initialization and adopts simple quadtree partitioning and reduces the computational complexity. Finally, during initialization, an array called the AISP is used, so only are the first coefficient coordinates and sizes of insignificant sets are put into AISP, which reduce the coding complexity of the presented method.

Figure 4 introduces the block diagram of the EOCS algorithm.

In the experiments, we compared the IWT-based EOCS algorithm with the IWT-based SPIHT for the lossy coding. Table 3 shows that the encoding and decoding times comparison between the presented algorithm and IWT-based SPIHT for  $512 \times 512$  remote sensing image. The (5, 3) filter was used to decompose and synthesize the image. The Intel Pentium D personal computer and VC++6.0 were used. No arithmetic coding was used on the quadtree-partitioning scheme, significance test and any symbols produced by the presented SPIHT.

Figure 5 gives the decoding results of Barbara image using the proposed EOCS algorithm based on (6, 14) at 0.25 and 1.0 bpp.

Figure 6 gives the decoding results of remote sensing image using the proposed EOCS algorithm based on (6, 14) at 0.25 and 1.0 bpp.

In Table 4, the PSNR performances using the EOCS algorithm based on (5, 3), and (6, 14) without arithmetic coding for remote sensing image are shown and compared with the results of SPIHT using (5, 3) and (6, 14).

Table 4 shows the comparison of the lossy compression



Fig. 3. Partitioning scheme of the EQPS-based coding algorithm.



Fig. 4. Block diagram of the EOCS algorithm.

Table 3. Encoding and Decoding Times Comparison between the Presented Algorithm and IWT-based SPIHT for 512×512 Remote Sensing Image

	IWT-based EOCS		IWT-based SPIHT	
$\operatorname{Rate}(\operatorname{bpp})$	Encoding	Decoding	Encoding	Decoding
	(ms)	(ms)	(ms)	(ms)
0.25	33	24	46	38
0.5	60	43	76	65
1.0	113	85	141	118



Fig. 5. Decoding results of Barbara using EOCS algorithm based on (6, 14). (a) 0.25 bpp and (b) 1.0 bpp.



Fig. 6. Decoding results of remote sensing image using EOCS algorithm based on (6, 14). (a) 0.25 bpp and (b) 1.0 bpp.

Table 4.	Comparison of Different Lossy Coding
Methods	Using IWT for Remote Sensing Image

Rate	IWT-based	EOCS (PSNR	) IWT-based	SPIHT (PSNR)
(bpp)	(5, 3)(dB)	(6, 14)(dB)	(5, 3)(dB)	(6, 14)(dB)
0.25	21.91	22.07	19.09	18.67
0.5	23.74	23.95	20.53	19.89
1.0	26.82	27.12	24.06	23.07

performance between the IWT-based EOCS algorithm and the IWT-based SPIHT for remote sensing image image. In this comparison, (5, 3) and (6, 14) IWT are adopted at the 0.25, 0.5, and 1.0 bpp. The EOCS algorithm uses the OCSF for each IWT, and the SPIHT uses the common IWT, which the scaling factor K is 1. The presented method provides PSNR performance up about 2–6 dB better than SPIHT without scaling.

In conclusion, we propose an efficient and lowcomplexity image coding algorithm using IB-IWT based on EOCS. Two strategies-OCSF and EQPS are adopted. The presented method has four primary advantages. The OCSF of LS obtains the low computational complexity of IB-IWT. The OCSF improves the lossy compression performance of IB-IWT. The single quadtree partitioning scheme realizes the lower coding complexity than SPIHT and EBCOT that is adopted by JPEG2000. This new algorithm can support both lossless and lossy compression using a single bitstream efficiently.

This work was supported by the National Natural Science Foundation of China (Nos. 60602035 and 61071103) and the Foundation of State Key Laboratory of Remote Sensing Science (No. OFSLRSS201001).

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