# Pure－phase apodizer for generating double ring－shaped focuses 

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#### Abstract

We propose two design methods of concentric multi－belt pure phase apodizer and phase modulation func－ tions for generating two ring－shaped focuses by focusing the linearly polarized doughnut－shaped beam with a high numerical aperture objective．The phase modulation functions for odd and even belt are verified effectively by numerical simulation．The distance of the two focuses and the position of each focus can be controlled or separately adjusted by the use of liquid crystal spatial phase modulation．It could be useful in optical trapping，particle stacking，alignment and transportation．


OCIS codes：050．1970，120．5060，260．5460．
doi：10．3788／COL201210．S10501．

Tightly focused laser beams are of great inter－ est to the scientists in a variety of fields such as biomedical microscopic imaging ${ }^{[1-5]}$ ，optical trap－ ping and manipulation ${ }^{[6-10]}$ ，and the second harmonic generation ${ }^{[11-13]}$ ．The three－dimensional（3D）intensity distributions in the focal area have been extensively stud－ ied．It is well known that the intensity distribution de－ pends on the incident laser beam mode，the polariza－ tion orientation，the phase and amplitude modulation， and the numerical aperture of the objective．Increas－ ing focal depth is desired in many practical applications． Many methods have been proposed for achieving long focal depth such as inserting physical annular aperture at the incident pupil plane or obstructing the central part of incident light ${ }^{[14,15]}$ ，using shade mask in which focal depth is elongated by modulating amplitude trans－ mittance over the whole pupil aperture ${ }^{[16]}$ ，and applying pure phase apodizer ${ }^{[17,18]}$ ．The method for creating a needle of longitudinally polarized light by using binary optics has also been proposed ${ }^{[19]}$ ．This binary optics works as a special polarized filter enhancing the longi－ tudinal component．The needle of longitudinally polar－ ized light is non－diffracting，i．e．，it propagates without divergence over a long distance in free space．Another pattern of focal intensity distribution is a conveyable quasi－periodic optical chain that can stably trap and deliver multiple individual particles in three dimensions at different planes near the focus ${ }^{[20]}$ ．This 3D optical chain can be realized by a specially designed diffractive phase element for modulating incoming radially polarized beam．By controlling the phase modulation of the inci－ dent beam，one can manipulate the interference pattern to accelerate and transport trapped particles along the optical axis in an expected manner．A doughnut beam due to the helical phase distribution has been used in high－efficiency laser trapping ${ }^{[21-25]}$ and super－resolution optical microscopy ${ }^{[26]}$ ．Particularly，the use of a centrally obstructed doughnut beam focused by a high numerical aperture（NA）objective may lead to the 3D microstruc－ ture rotation by extending its focal depth．In this letter， we propose a novel design for producing a double longi－
tudinally polarized ring－like intensity distribution using multi－belt pure phase apodizer，incident linearly polar－ ized doughnut－shaped beam and high NA objective．A double focus intensity distribution may stably trap two micro－particles clearly separated and aligned along the optical axis．The trapped particles could be particles with lower refractive index than that of surrounding me－ dia．
According to the vector diffraction integral formulae， the electric field distribution in the focal region of a lin－ early polarized monochromatic doughnut beam along the $x$ direction focused by a high NA objective which satisfies the sine condition can be expressed as

$$
\begin{align*}
& E(r, \phi, z)=\frac{i}{\lambda} \int_{0}^{2 \pi} \int_{0}^{\alpha} \sqrt{\cos \theta} E_{0}(r, \varphi) \\
& \times \exp [-\mathrm{i} k r \sin \theta \cos (\varphi-\phi)] \exp (-\mathrm{i} k z \cos \theta) \\
& \times\left\{\left[\cos \theta+\sin ^{2} \varphi(1-\cos \theta)\right] \vec{i}\right. \\
& +\cos \varphi \sin \varphi(\cos \theta-1) \vec{j}+\cos \varphi \sin \theta \vec{k}\} \\
& \times \sin \theta \mathrm{d} \theta \mathrm{~d} \varphi \tag{1}
\end{align*}
$$

where $\theta$ is the converging semi angle，$\alpha=\arcsin (N A)$ is the maximum converging semi angle．The schematic diagram of focusing is illustrated in Fig．1．The elec－ tric field of a doughnut beam can be described by $E_{0}(\varphi)=\exp (\mathrm{il} l \varphi)$ ，where $\varphi$ is the polar coordinate in a plane perpendicular to the beam axis and $l$ is the topo－ logical charge．In Eq．（1）， $\exp (-\mathrm{i} k z \cos \theta)$ determines the axial position of focusing spot．If we introduce an additional function $\exp \left(-\mathrm{i} k z_{0} \cos \theta\right)$ into Eq．（1），then the axial position of focusing spot will be shifted by a distance $z_{0}$ ．The additional function can be introduced by inserting a pure phase apodizer at the entrance pupil． Furthermore，we may divide the apodizer into two parts， each of which has its own phase modulation function．So the focusing spot is expected to be split into two separate spots．

We propose two designed approaches for the division


Fig. 1. Schematic diagram for focusing linearly polarized doughnut beam, high NA objective, and multi-belt pure phase apodizer.
of the phase apodizer cross-section. The first one is called the equal area dividing method. Figure 2 shows an example of the pure phase apodizer with circularly symmetrical multi-belt structure. The width of each belt is designed so that each belt has an identical area. If the maximum pupil radius $R_{\text {max }}$ of the focusing objective with NA of 0.95 is 2 mm , the radius of the $n$th belt are $R_{n}=\sqrt{n / m} R_{\max }(n=1,2,3, \cdots, m)$. The phase modulation functions for odd belt and even belt numbered from center to the edge are $P(R)=\left(k z_{\text {odd }} \sqrt{1-(R / f)^{2}}\right)$ and $P(R)=\left(k z_{\text {even }} \sqrt{1-(R / f)^{2}}\right)$, respectively. Here $k=2 \pi / \lambda, R$ is the radial position, and $f$ denotes the focal distance. $z_{\text {odd }}$ and $z_{\text {even }}$ control the positions of shifted focal planes after phase modulation


Fig. 2. Pure phase apodizer. The number of phase modulation belts is 8 , and the width of the belts is so chosen that the area of each belt is equal with each other.


Fig. 3. Pure phase apodizer. The number of phase modulation belts is 20 , and the width of the belts is so chosen that the converging semi angle difference of two corresponding radii of each belt is equal with each other.
of corresponding belts. The second approach for designing the pure-phase apodizer is to divide the belt radius so that the each belt width $\Delta w$ has equal corresponding semi converging angle difference $\Delta \theta$, as shown in Fig. 1. The radius of the $n$th belt are $R_{n}=f \sin \left\{n / m\left[\arcsin \left(R_{\max } / f\right)\right]\right\}$, Fig. 3 is an example of such a pure phase apodizer which is composed of 20 belts. The odd belts and even belts have exactly the same phase modulation functions as in the first approach

When a circularly symmetrical pure phase apodizer is inserted at the entrance pupil plane, the complex amplitude function in the Eq. (1) may be expressed as $E_{0}(r, \varphi)=\exp [\mathrm{i} l \varphi+\mathrm{i} P(R)]$. Figure 4 shows the intensity contour of the optical field in the longitudinal plane which is on the optical axis and has an azimuthal angle of $\pi / 6$. The incident beam is a linearly polarized (polarized along the $x$ axis) doughnut-shaped beam described by $\exp (\mathrm{i} 2 \varphi)$. The pure phase apodizer designed by first approach is positioned at the pupil of the high NA objective (NA $=0.95, R_{\max }=2 \mathrm{~mm}$ ). It is seen that, on the axis, the intensity of the beam is zero and two ring-shaped focuses are constructed. The distance between two focuses is $Z=\left|z_{\text {even }}-z_{\text {odd }}\right|$. Particles with refractive indexes higher than that of surrounding medium can be separately trapped in ring-shaped beam, and form two layers of multiple particles. Particles with a refractive index lower than surrounding medium can also be stably trapped and stacked in the dark region at a distance of $Z / 2$ from the high-index particles which are trapped in the ring-shaped beam. By changing $z_{\text {even }}$ or $z_{\text {odd }}$ separately, the trapped particles can be transported along the axial direction together or individually. The changes of $z_{\text {even }}$ or/and $z_{\text {odd }}$ mean the changes of the phase modulations. This dynamical phase modulation can be achieved by the techniques such as liquid crystal spatial light modulation.
Figure 5 is the intensity contour distribution of the optical fields in the longitudinal plane simulated with a pure phase apodizer composed of 20 concentric belts in which each belt has equal corresponding converging angle difference as shown in Fig. 3. From Fig. 5, we can see that the basic structures of two ring-shaped focuses which are similar to that of Fig. 4. We conclude that two designed approaches of pure phase apodizers result in the same expected two ring-shaped focuses.


Fig. 4. Intensity contour distribution of the longitudinal plane on the optical axis. The pure phase apodizer is designed so that the area of each belt is equal with each other.


Fig. 5. Intensity contour distribution of the longitudinal plane on the optical axis. The pure phase apodizer is so designed that the converging semi angle difference of two corresponding radii of each belt is equal with each other.

In conclusion, we demonstrate two designed methods of pure phase apodizers for generating two ring-shaped focuses by focusing linearly polarized doughnut-shaped beam with a high NA objective. The phase modulation functions for odd phase belts and even phase belts are proposed. The numerical simulation is given to show the realization of two ring-shaped focuses which can be dynamically changed by liquid crystal spatial light modulation. These separated two ring-shaped focuses could be a powerful tool for multiple particle trapping, stacking, alignment, and transportation along the optical axis. These manipulation techniques may also have potential applications in fields such as unraveling DNA condensation and tying molecular knot without adjacent interactions ${ }^{[6,7,27]}$.

This work was supported by the National Natural Science Foundation of China (No. 11074105), the Natural Science Foundation of Shandong Province (No. ZR2010AM038) and the Research Program on Upgrading and Transformation of Large and Precision Instrument of Shandong Province (No. 2010GJC20808-16).

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