

# Real-time measurement of retardation of eighth-wave plate independent of fast-axis direction

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A real-time measurement method for the retardation of an eighth-wave plate is proposed. The collimated laser beam is split using a Glan Taylor polarizer with two side escape windows. The reflected sub-beam is detected using a detector, whereas the transmitted sub-beam passes through the quarter-wave plate and the eighth-wave plate of interest. Then, it is reflected by the mirror and passes reversely through the eighth- and quarter-wave plates. Finally, it is analyzed using the Glan Taylor polarizer and detected using another detector. With two detection signals, the retardation is resolved and found to be independent of the fast-axis direction, initial intensity, and circuit parameters. In the experiment, a crystal quartz sample is measured at different fast-axis angles. The standard deviation of the retardation is  $0.9^\circ$ . The usefulness of the method is verified.

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An eighth-wave plate is widely applied in interferometric vibration measurement sensors<sup>[1]</sup>, electro-optically Q-switched lasers<sup>[2]</sup>, optical coherence tomography systems<sup>[3]</sup>, and so on. The eighth-wave plate is usually placed in the reflection light path. When a linear polarization beam traverses the eighth-wave plate twice with a reflector, it becomes a circularly polarized light. If the retardation of the eighth-wave plate has some errors, the polarization property of the beam could be influenced, and hence, the performance of the whole system could degrade. Some methods such as the retardation rotation method<sup>[4]</sup>, the polarizer rotation method<sup>[5]</sup>, the electro-optic modulation method<sup>[6]</sup>, the photoelastic modulation method<sup>[7,8]</sup>, the magneto-optic modulation method<sup>[9]</sup>, and the acousto-optic modulation method<sup>[10]</sup> can be used to measure the retardation of the eighth-wave plate. These methods measure the retardation only after the fast-axis direction is fixed. They cannot measure the retardation in real time because of the multiple sampling of the signal. To overcome these difficulties, we propose a method for measuring the retardation of an eighth-wave plate in real time independent of the fast-axis direction.

The schematic diagram for measuring the retardation of an eighth-wave plate is shown in Fig. 1. The collimated beam emitted from the laser is split using a Glan Taylor polarizer with two side escape windows to form two linearly polarized beams, i.e., an s and a p components, whose transmission directions are perpendicular to each other. The s component is the reference beam and detected by detector 1. The p component is the measurement beam and passes through the quarter-wave plate with a fast-axis azimuth of  $-45^\circ$  and, subsequently, the eighth-wave plate of interest. Then, it is reflected by the mirror and passes reversely through the eighth- and quarter-wave plates. Finally, it is analyzed via reflection using the Glan Taylor polarizer and detected by detector 2. Using the two intensities on detectors 1 and 2, the

retardation of the eighth-wave plate is obtained in real time independent of its fast-axis direction.

The intensities of the reference and measurement beams emitted from the Glan Taylor polarizer are respectively given by

$$I_1 = \alpha I_0, \quad (1)$$

$$I_1' = \beta I_0, \quad (2)$$

where  $I_0$  is the initial intensity, and  $\alpha$  and  $\beta$  are the transfer coefficients. The Jones vector of the measurement beam can be described as

$$\mathbf{E}_i = \sqrt{\beta I_0} \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \quad (3)$$

For an ideal quarter-wave plate, the Jones matrix can be given by

$$\mathbf{G}_{\frac{\lambda}{4}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}. \quad (4)$$

The Jones matrix of the eighth-wave plate to be measured is given by<sup>[11]</sup>

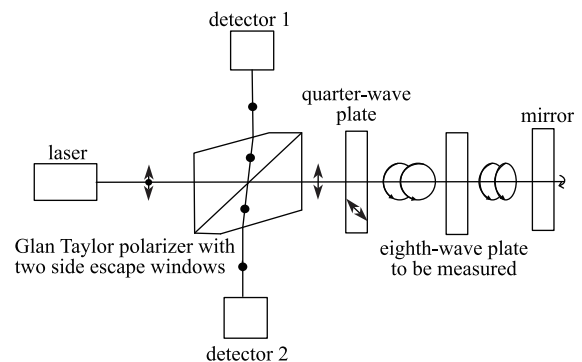


Fig. 1. Schematic diagram of real-time measurement for retardation of eighth-wave plate independent of fast-axis direction.

$$\mathbf{G}_{\frac{\lambda}{8}} = \begin{bmatrix} \cos \frac{\delta}{2} - i \sin \frac{\delta}{2} \cos(2\theta) & -i \sin \frac{\delta}{2} \sin(2\theta) \\ -i \sin \frac{\delta}{2} \sin(2\theta) & \cos \frac{\delta}{2} + i \sin \frac{\delta}{2} \cos(2\theta) \end{bmatrix}, \quad (5)$$

where  $\delta$  and  $\theta$  are the retardation and the fast-axis azimuth, respectively. The Jones matrix of the mirror is given by

$$\mathbf{G}_m = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (6)$$

As the beam is analyzed via reflection using the Glan Taylor polarizer, the Jones matrix of the polarizer can be described as

$$\mathbf{G}_A = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \quad (7)$$

Thus, the beam on detector 2 can be expressed as

$$\mathbf{E}_o = \sqrt{\gamma} \mathbf{G}_A \mathbf{G}_{\frac{\lambda}{4}} \mathbf{G}_{\frac{\lambda}{8}} \mathbf{G}_m \mathbf{G}_{\frac{\lambda}{8}} \mathbf{G}_{\frac{\lambda}{4}} \mathbf{E}_i = \sqrt{\beta\gamma} I_0 \begin{bmatrix} 0 \\ i \cos \delta \end{bmatrix}, \quad (8)$$

where  $\gamma$  is the reflection coefficient of the mirror. The intensity on detector 2 is

$$I_2 = \mathbf{E}_o \mathbf{E}_o^* = \frac{1}{2} \beta\gamma I_0 (1 + \cos 2\delta). \quad (9)$$

Using Eqs. (1) and (9), we can obtain the retardation  $\delta$  from

$$\cos 2\delta = \frac{2\alpha I_2}{\beta\gamma I_1} - 1. \quad (10)$$

Therefore, the measurement is immune to the fluctuation of the initial intensity.

To calibrate the value of  $\alpha/(\beta\gamma)$ , the calibration intensities  $I_{10}$  on detector 1 and  $I_{20}$  on detector 2 in the initial state with no eighth-wave plate can be respectively expressed as

$$I_{10} = \alpha I_0, \quad (11)$$

$$I_{20} = \beta\gamma I_0. \quad (12)$$

Using Eqs. (11) and (12), the value of  $\alpha/(\beta\gamma)$  can be obtained as

$$\frac{\alpha}{\beta\gamma} = \frac{I_{10}}{I_{20}}. \quad (13)$$

Using Eqs. (10) and (13),

$$\cos 2\delta = \frac{2I_{10}I_2}{I_{20}I_1} - 1. \quad (14)$$

The retardation is given by

$$\delta = \frac{1}{2} \arccos \left( \frac{2I_{10}I_2}{I_{20}I_1} - 1 \right). \quad (15)$$

Thus, the measurement is immune to the difference between the circuit parameters.

It is obvious that the retardation of the eighth-wave plate is independent of the fast-axis direction. In the measurements, intensities  $I_1$  and  $I_2$  can be detected simultaneously. Thus, the retardation can be obtained in real time. The experiment results are independent of the intensity fluctuation of the light source and circuit

parameters. The cosine function is closely linear when its variable is usually defined in the range of  $45^\circ$ – $135^\circ$ . Thus, the retardation  $\delta$  of the eighth-wave plate in Eq. (15) can be resolved in the range of  $22.5^\circ$ – $67.5^\circ$  with a nonlinear measurement error of less than  $0.2\%$ <sup>[12]</sup>.

The experimental setup is illustrated in Fig. 1. The light source was a He–Ne laser whose wavelength was 632.8 nm. The Glan Taylor polarizer was made of two calcite prisms assembled with an air space. Its extinction ratio was better than  $10^{-5}$ . The quarter-wave plate was a zero-order crystal quartz wave plate with less than  $\lambda/500$  retardation tolerance and was mounted in precision manual rotation stages to adjust its fast-axis direction. As the measurement error has a square relationship with the quarter-wave plate's retardation error<sup>[13–15]</sup>, the influence of the quarter-wave plate's retardation error can be ignored. The sample to be measured was a crystal quartz wave plate, and the nominal value of its retardation was  $37.8^\circ$  with tolerance of  $\lambda/300$  ( $\lambda = 632.8$  nm). The sample was mounted in the precision rotation stage with a minimum rotating angle of  $5''$ . The mirror had a reflectivity of 4%. Detectors 1 and 2 were silicon PIN photodiodes.

The retardation was measured when the sample was rotated from  $0^\circ$  to  $360^\circ$  at  $7.5^\circ$  intervals to change its fast-axis direction. The measured retardation values are shown in Fig. 2. The results  $\delta$  are independent of the rotating angles, i.e., fast-axis direction. The average and standard deviation of  $\delta$  are  $37.3^\circ$  and  $0.9^\circ$ , respectively. The results are consistent with their nominal retardation value and retardation deviation.

In conclusion, the method for measuring the retardation of the eighth-wave plate is presented. This method measures the retardation of the eighth-wave plate in real time, and its measurement result is independent of the fast-axis direction. In addition, the retardation measurement is immune to the fluctuation of the initial intensity and the different circuit parameters. In the experiment, the measurement average at different fast-axis angles and its standard deviation of the sample are consistent with their nominal retardation value and deviation of retardation, respectively. The usefulness of the method is also verified.

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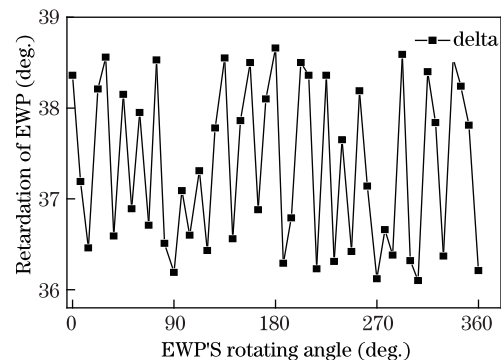


Fig. 2. Measured retardation values of eighth-wave plate. EWP: eighth-wave plate.

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