Two-wavelength sinusoidal phase-modulating interferometer for nanometer accuracy measurement

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A two-wavelength sinusoidal phase-modulating (SPM) laser diode (LD) interferometer for nanometer accuracy measurement is proposed. To eliminate the error caused by the intensity modulation, the SPM depth of the interference signal is chosen appropriately by varying the amplitude of the modulation current periodically. Then, the refine theory is induced to the measurement, and the two-wavelength interferometer (TWI) is combined with the single-wavelength LD interferometric technique to realize static displacement measurement with nanometer accuracy. Experimental results indicate that a static displacement measurement accuracy of 5 nm can be achieved over a range of 200 μm.

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High-accuracy non-contact measurement of displacement is required in many areas, such as information technology and micro-electro-mechanism systems (MEMS)[1−4]. The sinusoidal phase-modulating (SPM) interfeormetric technique based on laser diode (LD) has been proposed for conducting displacement measurement because of its unique features, such as high accuracy and simplicity[5−8]. However, the measurement range in this method is limited to half the wavelength due to 2π phase ambiguity[9]. For the extension of the measurement range, the two-wavelength interferometer (TWI) has been proposed as a means of eliminating phase ambiguity[10]. Sasaki *et al.* used sinusoidal signals with different frequencies to modulate the injection currents of the two LDs[10], and used the method described in Ref. [5] to get the phase of the two interference signals[8]. Then, the phase difference is used to calculate the displacement. With the influence of the intensity modulation of the light source, the measurement accuracy shown from the experiment result is not very high (the measurement error is approximately 6 μm). The conventional method for intensity modulation elimination[7] is not suitable for TWI, because the two interference signals detected by the photodiode (PD) cannot be separately distinguished. Suzuki *et al.* used photothermal modulation to suppress the intensity modulation in the SPM-LD interferometer[11]. However, in the two-wavelength SPM-LD interferometer, this method requires 4 LDs and can only reduce the intensity modulation effect by one order of magnitude[12].

In this letter, a two-wavelength SPM-LD interferometer for nanometer accuracy measurement is proposed. The SPM depth of the interference signals is chosen appropriately with the periodical variation of the modulation current amplitude; thus, the error caused by the intensity modulation can be eliminated during the interference signal processing with the optimized SPM depth. The measurement accuracy is improved to several submicrometers, thus matching the measurement range of the single-wavelength SPM-LD interferometer. Then, the refine theory is induced to the measurement to realize static displacement measurement with nanometer accuracy.

Figure 1 shows the two-wavelength SPM-LD interferometer for nanometer accuracy measurement. Two LDs were used as the light source. Laser beams radiated from the two LDs were coupled with coupler 1 and delivered into an all-fiber Fizeau interferometer after passing through the isolator and coupler 2. The light exiting from coupler 3 was collimated by collimator 1 and then sent to the object. The reflected light at the exit face of collimator 1 served as a reference wave, and the reflected light at the object surface served as an object wave. The reference and object waves were recombined in coupler 3 and interfered with each other. The resulting interference signal was detected by PD1. The PZT1 used for associated parameter determination was fixed on collimator 1. Moreover, a reference interferometer constructed with coupler 4, collimator 2, reference mirror, and PD2 was added into the optical path to compensate for the measurement error caused by the wavelength shift of LD.

When the injection current (IC) of LDi (i=1,2) is modulated by the sinusoidal signal $I_{\mathrm{m}i}(t) = a_i \cos\omega_i t (i = 1, 2)$, both the wavelength and the intensity of LD are modulated, which can be expressed as

$$\begin{array}{l} \lambda_i(t) = \lambda_{0i} + \beta_{0i} I_{\mathrm{m}i}(t) \\ g_i(t) = \beta'_i [I_{0i} + I_{\mathrm{m}i}(t)] \end{array} \quad i = 1, 2. \qquad (1)$$

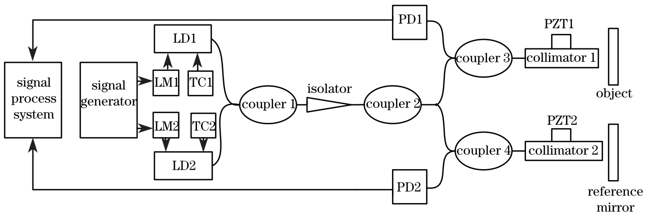

Fig. 1. Two-wavelength SPM-LD interferometer for nanometer accuracy measurement.

where λ_{0i} is the central wavelength of the two LDs, β_{0i} is the modulation coefficient of the wavelength, and β_i' is the modulation coefficient of the intensity. The interference signal detected by PD1 is expressed as

$$S(t) = \sum_{i=1}^{2} S_i(t), \tag{2}$$

where $S_i(t)\,(i = 1, 2)$ are the two signals generated by LD1 and LD2, respectively. The expression of $S_i(t)$ is given by

$$\begin{aligned} S_i(t) &= S_i(1 + \beta_i \cos\omega_i t)[S_{0i} + S_{1i}\cos(z_i \cos\omega_i t + \alpha_i)] \\ &= S_i(1 + \beta_i \cos\omega_i t)\{S_{0i} + S_{1i}\cos\alpha_i[J_0(z_i) \\ &\quad - 2J_2(z_i)\cos 2\omega_i t + \cdots] - S_{1i}\sin\alpha_i[2J_1(z_i)\cos\omega_i t \\ &\quad - 2J_3(z_i)\cos 3\omega_i t + \cdots]\}, \end{aligned} \tag{3}$$

where $S_i = \beta_i' I_{0i}\,(i = 1, 2)$ is the dc component of the output intensity of LD_i; $\beta_i = a_i/I_{0i}$ is a ratio of the amplitude of $I_{\text{m}i}(t)$ to I_{0i}; the term $S_i(1+\beta_i \cos\omega_i t)$ represents the intensity modulation of the LD_i; $z_i = 4\pi a_i \beta_{0i} L/\lambda_{0i}^2$ is the sinusoidal phase modulation depth; $\alpha_i = (4\pi/\lambda_{0i})L$ is the phase determined by the optical path difference (OPD) $2L$; $J_n(z_i)$ is the nth order Bessel function. With the signal processing system (SPS) containing band pass filter (BPF) and spectrum peak detecting function, the amplitudes of the 1st and 2nd order frequency components of $S_i(t)$ are obtained and expressed as

$$P_{1i} = C_i - K_i J_1(z_i)\sin\alpha_i + n_{1i}, \tag{4}$$

$$P_{2i} = -K_i J_2(z_i)\cos\alpha_i + n_{2i}, \tag{5}$$

where $C_i = \beta_i S_i S_{0i}$ is a fixed constant; $K_i = 2S_i S_{1i}$ is a proportional coefficient; n_{1i} and n_{2i} are interference terms caused by the intensity modulation, which are expressed as

$$n_{1i} = \beta_i S_i S_{1i}[J_0(z_i) - J_2(z_i)]\cos\alpha_i, \tag{6}$$

$$n_{2i} = -\beta_i S_i S_{1i}[J_1(z_i) - J_3(z_i)]\sin\alpha_i. \tag{7}$$

To calculate the accurate value of phase α_i, the two interference terms should be eliminated.

By calculating the value of the Bessel function, it can be concluded that $J_0(z)$ is equal to $J_2(z)$ when the SPM depth $z = 1.841$. In this condition, the term $n_{1i} = 0$[13], Eq. (8) can be expressed as

$$P_{\text{s}i} = C_{\text{s}i} - K_i J_1(z_{\text{s}})\sin\alpha_i, \tag{8}$$

where $z_{\text{s}} = 1.841$; $P_{\text{s}i}$ is the amplitude of the 1st order frequency component when $z_i = 1.841$; $C_{\text{s}i} = \beta_{\text{s}i} S_i S_{0i}$ is a fixed constant; and $\beta_{\text{s}i} = a_{\text{s}i}/I_{0i}$, $a_{\text{s}i} = z_{\text{s}}\lambda_{0i}^2/4\pi\beta_{0i}L$ is the modulation current amplitude of LDi when $z_i = 1.841$.

When the SPM depth z is 3.054, $J_1(z)$ is equal to $J_3(z)$. Under this condition, the term $n_{2i} = 0$, Eq. (9) can be expressed as

$$P_{\text{c}i} = -K_i J_2(z_{\text{c}})\cos\alpha_i, \tag{9}$$

where $z_{\text{c}} = 3.054$; $P_{\text{c}i}$ is the amplitude of the 2nd order frequency component when $z_i = 3.054$. In this set, the modulation current amplitude of LDi is $a_{\text{s}i} = z_{\text{s}}\lambda_{0i}^2/4\pi\beta_{0i}L$.

From Eqs. (8) and (9), we know that phase α_i can be calculated with Eq. (10) after the determination of $P_{\text{s}i}$ and $P_{\text{c}i}$. This is given by

$$\alpha_i = \arctan\left[\frac{J_2(z_{\text{c}})}{J_1(z_{\text{s}})} \cdot \frac{P_{\text{s}i} - C_{\text{s}i}}{P_{\text{c}i}}\right]. \tag{10}$$

Then, the phase difference $\alpha = \alpha_1 - \alpha_2$ is obtained, which can be used to calculate the displacement d_{c} expressed as

$$d_{\text{c}} = \frac{\Lambda}{4\pi}(\alpha - \alpha_0), \tag{11}$$

where $\Lambda = \lambda_{01}\lambda_{02}/(\lambda_{02} - \lambda_{01})$ is the synthetic wavelength; the phase difference α_0 is the phase corresponding to Λ when the object stays on the initial position. In Eq. (11), d_{c} is taken as a rough value of the displacement. The fine value of displacement can be calculated with the refine theory[14] and is given by

$$d = \frac{1}{2}m\lambda_{01} + \frac{\lambda_{01}}{4\pi}(\alpha_1 - \alpha_{10}), \tag{12}$$

where $m = \text{int}\left(\frac{2d_{\text{c}}}{\lambda_{01}}\right)$ is an integer satisfying the condition $m\lambda_{01} \leqslant 2d_{\text{c}} < (m+1)\lambda_{01}$; α_{10} is the phase corresponding to λ_{01} when the object stays on the initial position.

According to Fig. 2, for the detection of $P_{\text{s}i}$ and $P_{\text{c}i}$ the modulation current amplitude can be set to

$$a_i(t) = \begin{cases} a_{\text{s}i}, & n\Delta t \leqslant t < (n + 1/2)\Delta t \\ a_{\text{c}i}, & (n + 1/2)\Delta t \leqslant t < (n+1)\Delta t \end{cases}, \tag{13}$$

where Δt is the sampling time of the interference signal, which is also the alternating periodic of the modulation current.

Thus, the modulation current of LDi is given by

$$\begin{aligned} I_{\text{m}i}(t) &= a(t) \cdot \cos(\omega_i t + \theta) \\ &= \begin{cases} a_{\text{s}i}\cos(\omega_i t + \theta), & n\Delta t \leqslant t < (n + 1/2)\Delta t \\ a_{\text{c}i}\cos(\omega_i t + \theta), & (n + 1/2)\Delta t \leqslant t < (n+1)\Delta t \end{cases}. \end{aligned} \tag{14}$$

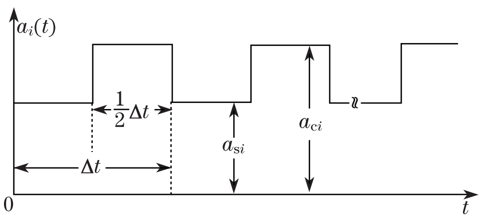

Fig. 2. Modulation current amplitude $a_i(t)$.

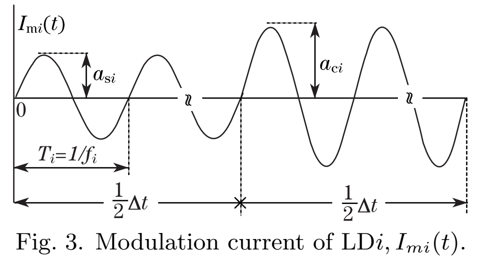

Fig. 3. Modulation current of LDi, $I_{mi}(t)$.

As shown in Fig. 3, in order to keep the continuity of the modulation current, the phase bias θ is set to $\theta = -\frac{\pi}{2}$, and the alternating periodic of the modulation current Δt should satisfy

$$\Delta t = \frac{2N}{f_i}. \quad (15)$$

When sampling the interference signal whose time length is Δt, the expression of the interference signal $S_i(t)(i = 1, 2)$ can be expressed as

$$S_i(t) = \begin{cases} S_i[1+\beta_{si}\cos(\omega_i t+\theta)]\left\{S_{0i}+S_{1i}\cos[z_s\cos(\omega_i t+\theta)+\alpha_{1i}]\right\}, & 0 \leqslant t < \frac{1}{2}\Delta t \\ S_i[1+\beta_{ci}\cos(\omega_i t+\theta)]\left\{S_{0i}+S_{1i}\cos[z_c\cos(\omega_i t+\theta)+\alpha_{1i}\right\}, & \frac{1}{2}\Delta t \leqslant t < \Delta t \end{cases}. \quad (16)$$

When $0 \leqslant t < \frac{1}{2}\Delta t$, the SPM depth of interference signal z_s=1.841, and the amplitude of the 1st order frequency component is P_{si}; when $\frac{1}{2}\Delta t \leqslant t < \Delta t$, the SPM depth of interference signal z_c=3.054, and the amplitude of the 2nd order frequency component is P_{ci}.

It is known from Eqs. (6) and (7) that in eliminating the influence caused by the interference terms n_{1i} and n_{2i}, the accurate values of modulation current amplitudes a_{si} and a_{ci} should be determined to make the SPM depth satisfy the equations: $z_s = 1.841$ and $z_c = 3.054$. Moreover, the fixed constant C_{si} should also be determined before calculating α_i with Eq. (10). The PZT fixed on the collimator is driven to perform the low-frequency vibration, of which the amplitude is larger than $\lambda_{0i}/8$ (Fig. 1). We turn on the LDi and then use the sinusoidal signal with the amplitude a'_i and frequency ω_i to modulate the injection current. By detecting the amplitudes of the 1st and 2nd order frequency components (P_{1i} and P_{2i}) in different times, the maximum and the minimum of P_{1i} and P_{2i} can be marked. It is known from Eqs. (4) and (5) that the expressions of the maximum and the minimum are

$$\begin{cases} [P_{1i}]_{\max} = C_i + \frac{1}{2}K_i\sqrt{\beta_i^2\left[J_0(z_i)-J_2(z_i)\right]^2+\left[2J_1(z_i)\right]^2} \\ [P_{1i}]_{\min} = C_i - \frac{1}{2}K_i\sqrt{\beta_i^2\left[J_0(z_i)-J_2(z_i)\right]^2+\left[2J_1(z_i)\right]^2} \\ [P_{2i}]_{\max} = \frac{1}{2}K_i\sqrt{\beta_i^2\left[J_1(z_i)-J_3(z_i)\right]^2+\left[2J_2(z_i)\right]^2} \\ [P_{2i}]_{\min} = -\frac{1}{2}K_i\sqrt{\beta_i^2\left[J_1(z_i)-J_3(z_i)\right]^2+\left[2J_2(z_i)\right]^2} \end{cases}. \quad (17)$$

It can be concluded from Eq. (17) that,

$$\frac{[P_{1i}]_{\max}-[P_{1i}]_{\min}}{[P_{2i}]_{\max}-[P_{2i}]_{\min}} = \frac{\sqrt{\beta_i^2\left[J_0(z_i)-J_2(z_i)\right]^2+\left[2J_1(z_i)\right]^2}}{\sqrt{\beta_i^2\left[J_1(z_i)-J_3(z_i)\right]^2+\left[2J_2(z_i)\right]^2}}. \quad (18)$$

In Eq. (18), the right side can be taken as a single-valued function of z_i; thus the SPM depth z'_i can be determined with the calculation result of $\frac{[P_{1i}]_{\max}-[P_{1i}]_{\min}}{[P_{2i}]_{\max}-[P_{2i}]_{\min}}$. Then, the parameter $\gamma_i = \lambda_{0i}^2/4\pi\beta_{0i}L = a_i/z_i$ can be calculated as the ratio of the set value of modulation current amplitude a'_i to the determined depth z'_i. Using the parameter γ_i, the modulation current amplitudes a_{si} and a_{ci} corresponding to the SPM depth $z_s = 1.841$ and $z_c = 3.054$, respectively, can be determined. By adjusting the modulation current amplitude to a_{si}, the parameter C_{si} can be determined by Eq. (19) after the calculation of $[P_{1i}]_{\max}$ and $[P_{1i}]_{\min}$ as

$$C_{si} = \frac{1}{2}\left([P_{1i}]_{\max}+[P_{1i}]_{\min}\right)\Big|_{z_i=z_s}. \quad (19)$$

Computer simulations were performed to demonstrate the validity of the proposed method. The position change of the object leads to subtle changes of the OPD $2L$. From the expression of SPM depth $z_i = 4\pi a\beta_{0i}L/\lambda_i^2$, we determine that the change of OPD leads to the change of z_i; in turn, this induces a phase measurement error φ_i. Figure 4 shows the distribution of the phase measurement error φ_i versus the phase of the OPD and the displacement that must be measured. The initial OPD between the object and collimator is set to $2L_0 = 50$ mm, and the range of the displacement d to measure is set from –100 to 100 μm. The phase error φ_i is smaller than 7.0×10^{-4} rad; thus the phase error φ_e corresponding to the synthetic-wavelength Λ is smaller than 1.4×10^{-3} rad (Fig. 4). When the wavelength of LDi and the synthetic-wavelength are set to $\lambda_i = 1\,310$ nm and Λ =400 μm, respectively, the rough measurement error becomes smaller than 45 nm, and the error of the measurement displacement calculated with the refine theory also becomes smaller than 0.08 nm. Thus, the influence of this system error to the measurement result can be ignored.

In the experiment, the light sources used in the interferometer shown in Fig. 1 were two LDs with temperature controllers (TCs). The maximum output powers of the two LDs were about 10 mW. The central wavelengths

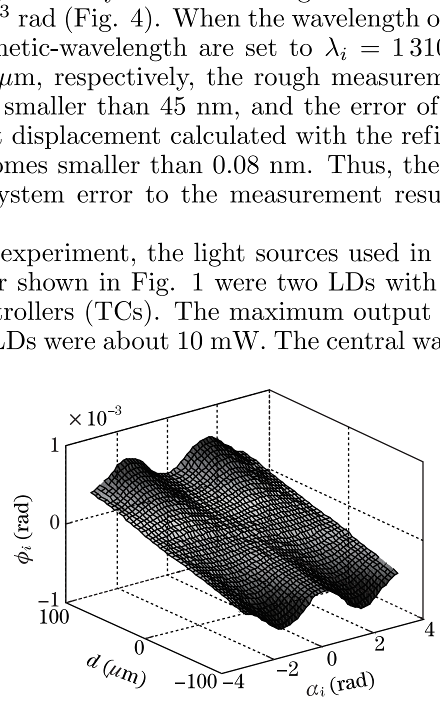

Fig. 4. Distribution of phase measurement error φ_i with different displacement d values, the initial OPD is set to $2L_0$ = 50 mm. When the range of d is set from –100 to 100 μm, the error φ_i is smaller than 7.0×10^{-4} rad.

of LD1 and LD2 were 1 312.95 and 1 308.76 nm, respectively. The frequencies of the modulated current of the two LDs were 9.0 and 5.6 kHz, respectively, and the alternating periodic of the modulation current was set to $\Delta t = 0.1$ s. The bandwidth of BPF in SPS was 100 Hz. The length of the measuring arm was set to approximately 25 mm; thus, the initial OPD between the object and the collimator1 was approximately 50 nm. By adjusting the position of the reference mirror, the length of the reference L_{r} became equal to L. By adjusting the micro-displacement stage, we changed the position of the object at equal interval of 10 μm and measured the displacement d of the object at multiple positions.

Figure 5 shows the interference signal $S(t)$ and its spectrum distribution when the object is fixed on a certain position. With the frequencies of the modulated current set at 9.0 and 5.6 kHz, respectively, the low order frequency components of $S(t)$ do not overlap with each other (Figs. 5(c) and (d)). Thus, $P_{\mathrm{s}i}$ and $P_{\mathrm{c}i}$ can be detected from $S(t)$ with BPF and spectrum peak detecting function.

Figure 6 shows the measurement result. The range of the displacement d to measure is –100-100 μm, the rough measurement error Δd_{c} is smaller than 0.3 μm (Figs. 6 (a) and (b)), which is smaller than a quarter wavelength of LD and can satisfy the requirement of the refine theory for single-wavelength interferometer. Then the displacement d is calculated with the phase α_1 and the rough measurement displacement d_{c} by the refine theory (Fig. 6 (c)). The measurement error is less than 5 nm (Fig. 6 (d)). This error is mainly caused by the noise induced by some devices such as the laser modulator, photodiode, and SPS. Moreover, some external disturbances such as mechanical vibration will also contribute to the error. In another hand, it is concluded from Fig. 6 that a displacement measurement accuracy of 5 nm is achieved over the range of –100 to 100 μm.

In conclusion, a two-wavelength SPM-LD interferometer for nanometer accuracy measurement is proposed. The SPM depth of the interference signals is optimized by adjusting the modulation current amplitude periodically. With the optimized SPM depth, the error caused

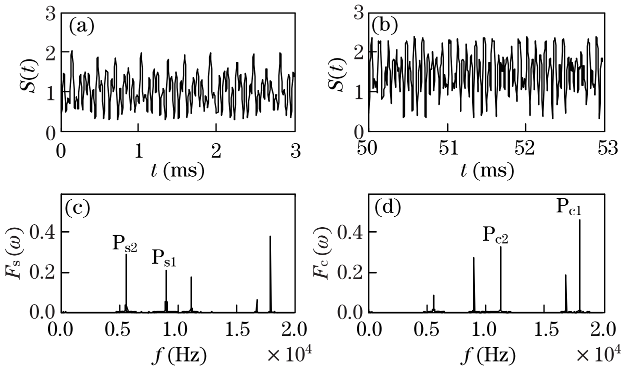

Fig. 5. Interference signal $S(t)$ and its spectrum distribution. (a) $S(t)$ intercepted during the first half alternating periodic, (b) $S(t)$ intercepted during the second half alternating periodic, (c) spectrum distribution $F_{\mathrm{s}}(\omega)$ of $S(t)$ in the first half alternating periodic, and (d) spectrum distribution $F_{\mathrm{c}}(\omega)$ of $S(t)$ in the second half alternating periodic.

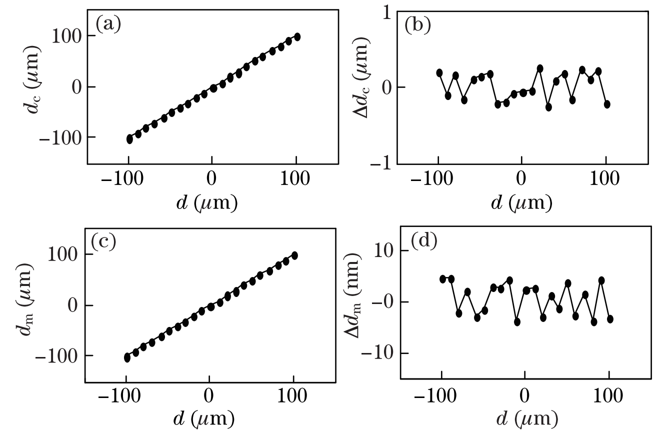

Fig. 6. Displacement measurement results. The range of the displacement d to measure is –100-100 μm. (a) Rough measurement displacement d_{c} and (b) its error Δd_{c}, and Δd_{c} is smaller than 0.3 μm; (c) measurement displacement d_{m} and (d) its error Δd_{m}, and Δd_{m} is smaller than 5 nm.

by the intensity modulation is eliminated during the interference signal processing. Thus, the TWI measurement accuracy improved to 0.3 μm, which matches the measurement range of the single-wavelength SPM-LD interferometer. Then, the refine theory is induced to the measurement to realize static displacement measurement with nanometer accuracy. Experimental results indicate that a static displacement measurement accuracy of 5 nm can be achieved over a range of 200 μm.

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References

1. N. A. Massie, Appl. Opt. **19,** 154 (1980).
2. R. Dändliker, R. Thalmann, and D. Prongué, Opt. Lett. **13,** 339 (1988).
3. O. Sasaki, K. Akiyama, and T. Suzuki, Appl. Opt. **41,** 3906 (2002).
4. J. Shang, S. Zhao, Y. He, W. Chen, and N. Jia, Chin. Opt. Lett. **9,** 081201 (2011).
5. O. Sasaki and K. Takahashi, Appl. Opt. **27,** 4139 (1988).
6. S. Song, X. Wang, X. Wang, F. Qian, and G. Chen, Chinese J. Lasers (in Chinese) **28,** 753 (2001).
7. X. Wang, X. Wang, Y. Liu, C. Zhang, and D. Yu, Opt. Laser Technol. **35,** 219 (2003).
8. Z. Li, X. Wang, Y. Liu, and Y. Bu, Chinese J. Lasers (in Chinese) **34,** 1267 (2007).
9. T. Suzuki, K. Kobayashi, and O. Sasaki, Appl. Opt. **39,** 2646 (2000).
10. O. Sasaki, H. Sasazaki, and T. Suzuki, Appl. Opt. **30,** 4040 (1991).
11. T. Suzuki, M. Matsuda, O. Sasaki, and T. Maruyama, Appl. Opt. **38,** 7069 (1999).
12. C. M. Klimcak and J. C. Camparo, J. Opt. Soc. Am. B **5,** 211 (1988).
13. Z. Li, X. Wang, P. Bu, B. Huang, and D. Zhen, Optik **121,** 799 (2009).
14. X. Sun, *Theory and Application of Interferometry using Laser Diode* (in Chinese) (National Defence Industrial Press, Beijing, 1998) pp 70-73.