Quantum limit in low-loss ring laser gyros

Zhiguo Wang (汪之国)*, Xingwu Long (龙兴武), and Fei Wang (王 飞)

Department of Opto-Electronic Engineering, College of Opto-Electronic Science and Engineering, National University of Defense Technology, Changsha 410073, China

**Corresponding author: maxborn@163.com*

Received September 23, 2011; accepted January 5, 2012; posted online March 15, 2012

Contrary to expectations, a measurement of the random walk in the ring laser gyro (RLG) as a function of laser power P shows that it is not consistent with the $P^{-1/2}$ rule. In the experiment, the random walk and laser power are tested and recorded at different discharge currents. The random walk decreases with increasing power, but with a rate much less than the theoretical value according to current literature. In order to solve the inconsistency above, we derive the expression for the random walk in RLGs based on laser theory. Theoretical analysis shows that, accumulating effects of lower energy level due to its limited lifetime lead to additional quantum noise from spontaneous emission. Results show that the random walk in the RLGs consists of two components. The former decreases with increasing power according to the $P^{-1/2}$ rule, whereas the other is power-independent. Thus far, the power-independent quantum limit has not appeared in the literature; therefore, the expressions for RLGs should be modified to describe the low-loss RLGs exactly, where the power-independent term takes a relatively larger proportion. The findings are significant to the further reduction of quantum limit in low-loss RLGs.

OCIS codes: 140.3560, 140.3370, 270.2500.

doi: 10.3788/COL201210.061404.

Ring laser gyros (RLGs) have been widely used in areas such as inertial technology, fundamental physics, and geophysics for their abilities to accurately measure angular rates with frequency difference between counter traveling wave modes[1−3]. In some applications, the random walk gives a limit to the RLG system, e.g., RLG rotating inertial navigation system[4], telescope pointing and tracking system[5], and ultralarge RLGs[6−8]. The ultimate limit to the random walk in RLGs is determined by quantum noise, which is also called the quantum limit[9,10]. The quantum limit is usually considered proportional to $P^{-1/2}$ in classical literature on RLGs, where P is laser power[9,10]. Dithering noise[11] is the main component of the random walk in the dithered RLGs; therefore, reducing their random walk with increasing power is rarely effective[12]. As for nondithered RLGs, such as ultralarge RLGs and differential RLGs (DILAG for short), the random walk decreases with increasing power according to the $P^{-1/2}$ rule[7,9]. We measured the random walk as a function of laser power for three DILAGs; however, the results showed enormous departure from the $P^{-1/2}$ rule. In order to explain the inconsistency between the experimental results and theory in current literature, we analyzed the experimental phenomena based on laser theory. Results show that the quantum limit in the low-loss RLGs is not proportional to $P^{-1/2}$. Therefore, expressions for quantum limit in the RLGs should be modified.

The DILAGs were tested according to “IEEE Stand Specification Format Guide and Test Procedure for Singular-axis Laser Gyros” and analyzed using the Allan variance method[13,14]. Discharge currents of the DILAGs were increased from 0.5 to 1.0 mA at a step of 0.1 mA. Random walk and laser power were obtained at each discharge current. Each DILAG had four mirrors. Laser output through one mirror was received by a PIN photodiode and converted to voltage with a transimpedance amplifier with bandwidth lower than 10 Hz. Therefore, the measured laser power was proportional to the straight output power of the DILAG, with a unit of volts (V). Experimental data for three DILAGs are shown in Table 1.

Let $x = 1/P$, $y = A^2$. According to the $P^{-1/2}$ rule, a_0 will approach zero if we use the equation $y = a_1x + a_0$ to make linear fitting for the data in Table 1. The actual results are shown in Fig. 1.

Contrary to expectations, a_0 was not close to zero, but was a relatively large constant. This phenomenon indicates a term independent of power in the random walk. The power-dependent term a_1x was less than a_0 in the range from 0.5 to 1.0 mA for our DILAGs. Therefore, the power-independent term is not negligible and serves as main component of the random walk at high power. Because a_0 was independent of power, the random walk of the DILAGs showed no remarked decrease with increasing power. As far as the authors know, current

Table 1. Random Walk Versus Output Power

	Current I(mA)	0.5	0.6	0.7	0.8	0.9	1.0
DILAG 1	Power P(V)	0.541	0.615	0.734	0.842	0.950	1.040
	Random Walk A ($\times 10^{-4}$ °/h$^{1/2}$)	5.582	5.257	5.268	4.966	4.950	4.843
DILAG 2	Power P (V)	0.458	0.604	0.721	0.831	0.943	1.039
	Random Walk A ($\times 10^{-4}$ °/h$^{1/2}$)	6.006	5.779	5.468	5.314	5.255	5.100
DILAG 3	Power P(V)	0.524	0.671	0.807	0.931	1.046	1.158
	Random Walk A ($\times 10^{-4}$ °/h$^{1/2}$)	5.460	5.144	5.035	4.966	4.929	4.885

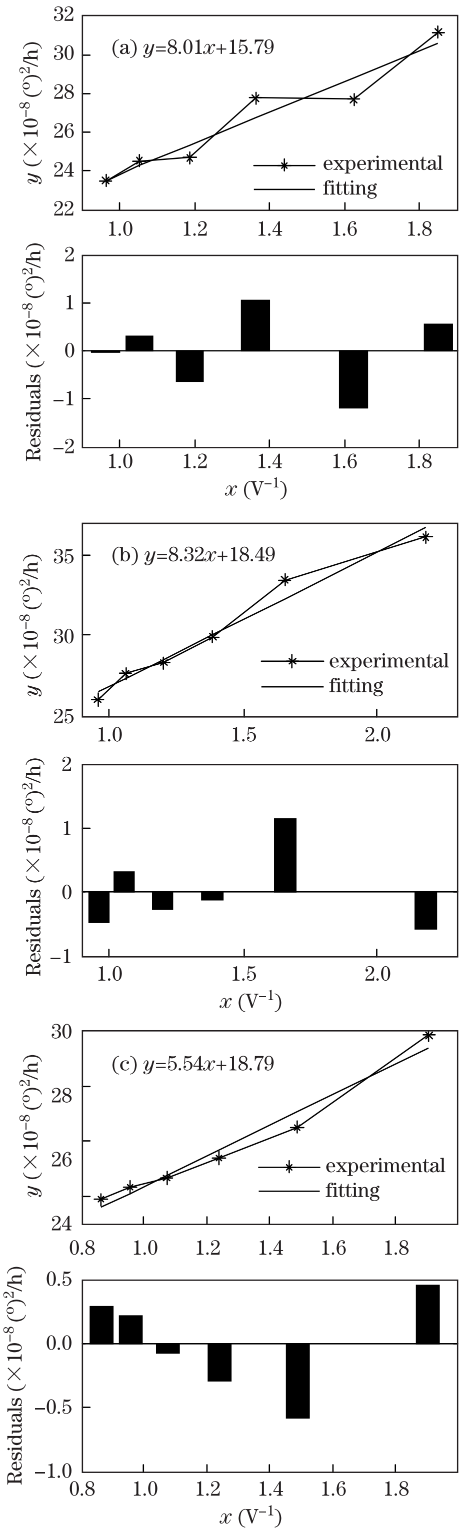

Fig. 1. Experimental results and fitting curves. (a) DILAG 1; (b) DILAG 2; (c) DILAG 3.

literature has not reported any theoretical analysis on the power-independent term of the random walk in the RLGs. Therefore, we can only derive the exact expressions for the random walk in the RLGs from the fundamental laser theory to find the origin of the power-independent term and further to propose methods to reduce it.

According to Refs. [15,16], the spectral linewidth of the field of a single mode laser is

$$\Delta\nu_{\mathrm{L}} = \frac{N_2}{(N_2 - N_1 g_2/g_1)} \frac{2\pi h\nu_0(\Delta\nu_{\mathrm{C}})^2}{P_{\mathrm{SE}}}, \tag{1}$$

where N_2 and N_1 are the populations of upper and lower energy level, respectively; g_2 and g_1 are the degeneracies of upper and lower energy levels, respectively; $h = 6.626 \times 10^{-34}$ J·s is the Planck constant; ν_0 is the laser frequency; $\Delta\nu_{\mathrm{C}}$ is the passive cavity bandwidth; P_{SE} is the stimulated emission power.

If the population of the lower energy level is negligible, Eq. (1) will be reduced to

$$\Delta\nu_{\mathrm{L}} = \frac{2\pi h\nu_0(\Delta\nu_{\mathrm{C}})^2}{P_{\mathrm{SE}}}, \tag{2}$$

which is the famous Schawlow-Townes linewidth[17].

When laser is running far above threshold, however, the lower level populates due to its limited lifetime. The upper level population increases to maintain a constant population inversion, which augments spontaneous emission power. As a result, the linewidth of the laser running far above threshold should be modified to[16]

$$\Delta\nu_{\mathrm{L}} = \frac{2\pi h\nu_0(\Delta\nu_{\mathrm{C}})^2}{P_{\mathrm{SE}}} \frac{\tau_2}{\tau_2 - \tau_1 g_2/g_1} + \frac{\Delta\nu_{\mathrm{C}} c\lambda_0^2}{2\pi\Delta\nu_{\mathrm{D}} V} \frac{\tau_1 g_2/g_1}{\tau_2 - \tau_1 g_2/g_1}, \tag{3}$$

where τ_2 and τ_1 are the lifetimes of upper and lower levels, respectively; $\Delta\nu_{\mathrm{D}}$ is the gain bandwidth; V is the mode volume; λ_0 is the wavelength.

The mirror transmission T is much less than the cavity loss δ in the DILAGs; therefore, output power is related to stimulated power by[9,10]

$$P_{\mathrm{out}} = P_{\mathrm{SE}} \frac{T}{\delta}. \tag{4}$$

Using Eqs. (3) and (4), we obtain

$$\Delta\nu_{\mathrm{L}} = \frac{\tau_2}{(\tau_2 - \tau_1 g_2/g_1)} \frac{2\pi h\nu_0(\Delta\nu_{\mathrm{C}})^2}{P_{\mathrm{out}}} \frac{T}{\delta} + \frac{c\Delta\nu_{\mathrm{C}}\lambda_0^2}{8\pi\Delta\nu_{\mathrm{D}} V} \frac{\tau_1}{(\tau_2 g_1/g_2 - \tau_1)}. \tag{5}$$

Measurement uncertainty $(\Delta\nu)_{\mathrm{RMS}}$ of the single mode laser frequency is[9,10]

$$(\Delta\nu)_{\mathrm{RMS}} = \frac{1}{2\sqrt{\pi}} \sqrt{\frac{\Delta\nu_{\mathrm{L}}}{\tau}}. \tag{6}$$

Angular rotation rate Ω measured by the DILAG is related to its differential frequency $\Delta\nu_{\mathrm{DF}}$ by[10]

$$\Omega = \frac{L\lambda_0}{8A} \Delta\nu_{\mathrm{DF}}, \tag{7}$$

where L is cavity length and A is area of the DILAG.

Measurement uncertainty of differential frequency $\Delta\nu_{\mathrm{DF}}$ is two times larger than $(\Delta\nu)_{\mathrm{RMS}}$ because four laser modes are used in obtaining $\Delta\nu_{\mathrm{DF}}$[9]. Substituting $\Delta\nu_{\mathrm{DF}}$ in Eq. (7) with $2(\Delta\nu)_{\mathrm{RMS}}$, we obtain measurement uncertainty of Ω:

$$(\Delta\Omega)_{\mathrm{RMS}} = \frac{L\lambda_0}{8A} \cdot 2(\Delta\nu)_{\mathrm{RMS}} = \frac{L\lambda_0}{8A} \sqrt{\frac{\Delta\nu_{\mathrm{L}}}{\pi\tau}}. \tag{8}$$

$(\Delta\Omega)_{\rm RMS}$ is related to the random walk coefficient S_Ω by [9,10]

$$(\Delta\Omega)_{\rm RMS} = \frac{S_\Omega}{\sqrt{\tau}}. \tag{9}$$

Comparing Eqs. (8) with (9) and using Eq. (5), we obtain the expression for the random walk coefficient S_Ω caused by quantum noise of spontaneous emission:

$$S_\Omega = \frac{L\lambda_0}{8A}\sqrt{\frac{\Delta\nu_{\rm L}}{\pi}} = \frac{L\lambda_0}{8\sqrt{\pi}A} \cdot\sqrt{\frac{\tau_2}{(\tau_2-\tau_1 g_2/g_1)}\frac{2\pi h\nu_0(\Delta\nu_C)^2}{P_{\rm out}}\frac{T}{\delta}+\frac{\tau_1}{(\tau_2 g_1/g_2-\tau_1)}\frac{c\Delta\nu_{\rm C}\lambda_0^2}{8\pi\Delta\nu_{\rm D}V}}. \tag{10}$$

Equation (10) indicates random walk caused by quantum noise consisting of two terms; one is power-dependent whereas the other is not.

We should note that the gain is assumed to be distributed in the whole ring path in deriving Eq. (3)[15]. In fact, the gain length is merely 3/10 of the whole ring path for our DILAGs. Therefore, the second term in the root of Eq. (10) should be multiplied by 0.3 and V should be the mode volume in the gain area.

If only the first term in the root of Eq. (10) is considered and the approximation $\tau_2 >> \tau_1$ is used, S_Ω will be reduced to $S_{\Omega1}$:

$$S_{\Omega1} = \frac{L\lambda_0}{8A}\sqrt{\frac{\Delta\nu_L}{\pi}} = \frac{c\lambda_0\sqrt{h\nu_0 T}}{8\pi\sqrt{2}A}\sqrt{\frac{\delta}{P_{\rm out}}}. \tag{11}$$

This is the expression used in classical literature on RLGs, such as in Refs. [2,9,10].

If only the second term in the root of Eq. (10) is considered, S_Ω will be reduced to $S_{\Omega2}$

$$S_{\Omega2} = \frac{c\lambda_0^2\sqrt{0.3}}{32\pi\sqrt{\pi}A}\sqrt{\frac{\tau_1}{(\tau_2 g_1/g_2-\tau_1)}\frac{L}{\Delta\nu_{\rm D}V}\cdot\delta}. \tag{12}$$

For the transition of 0.6328 μm line[15], $\tau_2/\tau_1 \approx 10$, $g_1/g_2 = 5/3$; therefore, we get

$$\frac{\tau_2}{(\tau_2-\tau_1 g_2/g_1)} = 1.064, \frac{\tau_1}{(\tau_2 g_1/g_2-\tau_1)} = 0.064.$$

Let us estimate the numerical values of $S_{\Omega1}$ and $S_{\Omega2}$ according to real parameters of the DILAGs.

We know light velocity is $c = 3\times10^8$ m/s, wavelength is λ_0=0.6328 μm, Planck constant is $h = 6.624\times10^{-34}$ J$\cdot$s, and laser frequency is $\nu_0 = c/\lambda_0$. Let gain bandwidth $\Delta\nu_{\rm D} = 10^9$ Hz, which is often used for 0.6328-μm transition in RLGs.

We can use the parameters in Ref. [9] to estimate random walk in high-loss DILAGs. The parameters are $A = 111.5\times10^{-4}$ m^2, $L = 0.5516$ m, $T = 0.00244$, $\delta = 0.012$, and $P_{\rm out} = 13$ μW (the reason T and δ are so large is very likely to be poor mirrors in the late 1970s). The mode volume is not given in Ref. [9]; thus, we use $V = 8.4\times10^{-8}$ m^3 and 0.3-m gain length as an estimate. With the numerical values above and Eqs. (11) and (12), we obtain the power-dependent random walk $S_{\Omega1} = 1.38\times10^{-3}$°/h$^{1/2}$, and the power-independent random walk $S_{\Omega2} = 3.41\times10^{-4}$°/h$^{1/2}$. Because $S_{\Omega2}$ is several times less than $S_{\Omega1}$ and $S_\Omega = \sqrt{S_{\Omega1}^2+S_{\Omega2}^2} \approx S_{\Omega1}[1+\frac{1}{2}(S_{\Omega2}/S_{\Omega1})^2] \approx S_{\Omega1}$, the DILAGs will show evident $P^{-1/2}$ rule.

For our low-loss DILAGs, the parameters are approximately $A = 25\times10^{-4}$ m^2, $L = 0.2$ m, $T = 10^{-4}$, $\delta = 1\times10^{-3}$, $V = 1.88\times10^{-8}$ m^3, and $P_{\rm out} = 10$ μW. We obtain the power-dependent random walk $S_{\Omega1} = 4.12\times10^{-4}$°/h$^{1/2}$, and the power-independent random walk $S_{\Omega2} = 4.23\times10^{-4}$°/h$^{1/2}$. Because $S_{\Omega1}$ is very close to $S_{\Omega2}$, reduction of $S_{\Omega1}$ through increasing power only has a slight effect to the whole random walk S_Ω.

Express Eq. (10) as

$$y = (S_\Omega)^2 = a_1\frac{1}{P_{\rm out}} + a_0 = a_1 x + a_0, \tag{13}$$

where $P_{\rm out}$ and S_Ω can be measured experimentally. The coefficients a_1 and a_0 can be obtained by using linear fitting method. Therefore, we can obtain the power-independent random walk $\sqrt{a_0}$.

With the results in Table 1 and Fig. 1, the power-dependent random walk and the power-independent random walk can be obtained, as shown in Table 2. The power-independent random walk is the main source of the whole random walk for our DILAGs normally running at 0.7 mA.

Most of current literature have considered only the power-dependent term of the quantum limit in the RLGs, likely due to the following reasons:

1) the power-dependent term is the main component of the random walk in the early age because of larger cavity loss due to lower technologic level;

2) the random walk from dithering noise is much larger than that from quantum noise in the dithered RLGs; therefore, their quantum limit is difficult to study. Because the DILAGs have an intracavity element, the cavity loss cannot be reduced to a lower level until a high technological level is reached.

We can reduce the random walk through increasing discharge current according to Eq. (11). However, the appearance of power-independent term results in a quantum limit larger than the power-dependent term for low-loss RLGs. If the limiting condition is constant power to insure sufficient signal-to-noise for the PIN photodetector, the random walk is proportional to $\sqrt{\delta}$ according to Eqs. (11) and (12). Therefore, the random walk can be reduced through reducing the cavity loss if the size of the DILAG is unaltered. In addition, the power-independent

Table 2. Power-dependent and Power-independent Random Walk

	Current I (mA)	0.5	0.6	0.7	0.8	0.9	1.0
DILAG 1	$S_{\Omega1}$($\times4^{-4}$ °/h$^{1/2}$)	3.848	3.609	3.303	3.084	2.904	2.775
	$S_{\Omega2}$($\times4^{-4}$ °/h$^{1/2}$)			3.974			
DILAG 2	$S_{\Omega1}$($\times4^{-4}$ °/h$^{1/2}$)	4.262	3.711	3.397	3.164	2.970	2.830
	$S_{\Omega2}$($\times4^{-4}$ °/h$^{1/2}$)			4.300			
DILAG 3	$S_{\Omega1}$($\times4^{-4}$ °/h$^{1/2}$)	3.252	2.873	2.620	2.439	2.301	2.187
	$S_{\Omega2}$($\times4^{-4}$ °/h$^{1/2}$)			4.334			

random walk can be reduced through increasing mode volume in the gain area.

Table 1 shows that the three DILAGs have some individual differences (different power and random walk coefficients at the same discharge current). These differences are mainly due to the errors produced in the process of cavity machining and aligning, which lead to different parameters of the DILAGs such as gain and loss.

In conclusion, in lasers running far above threshold, the population of upper level increases because of accumulating effects of lower energy level due to its limited lifetime, which augments spontaneous emission and thus quantum noise. As a result, the random walk of the RLGs consists of two components: the former decreases with increasing power according to the $P^{-1/2}$ rule, whereas the other is power-independent. The power-independent quantum limit has thus far not appeared in the literature. Therefore, the expressions for RLGs should be modified to describe the low-loss RLGs exactly, where the power-independent term takes a relatively larger proportion. In order to improve the ultimate quantum limit of the RLGs without increasing their sizes, the cavity loss should be reduced and the mode volume in the gain area should be augmented.

References

1. G. E. Stedman, Rep. Prog. Phys. **60,** 615 (1997).
2. M. Faucheux, D. Fayoux, and J. J. Roland, J. Opt. **19,** 101 (1988).
3. R. B. Hurst, R. W. Dunn, K. U. Schreiber, R. J. Thirkettle, and G. K. MacDonald, Appl. Opt. **43,** 2337 (2004).
4. X. Long, X. Yu, P. Zhang, Y. Wang, and J. Tang, J. Chin. Iner. Technol. (in Chinese) **18,** 149 (2010).
5. W. Schröer, H. Dahlmann, B. Huber, L. Schüse1e, F. Merkie, and M. Ravensbergen, Proc. SPIE **1585,** 98 (1991).
6. G. E. Stedman, R. B. Hurst, and K. U. Schreiber, Opt. Commun. **279,** 124 (2007).
7. C.P. Wyss, D. N. Wright, B. T. King, D. P. McLeod, S. J. Cooper, and G. E. Stedman, Opt. Commun. **174,** 181 (2000).
8. B. T. King, Appl. Opt. **39,** 6151 (2000).
9. T. A. Dorshner, H. A. Haus, M. Holz, I. W. Smith, and H. Statz, IEEE J. Quantum Electron. **QE16,** 1376 (1980).
10. W. W. Chow, J. Gea-Banacloche, L. M. Pedrotti, V. E. Sanders, W. Schleich, and M. O. Scully, Rev. Mod. Phys. **57,** 61 (1985).
11. J. E. Killpatric, Proc. SPIE **487,** 85 (1984).
12. R. Song, J. X. Tang, and J. Zhou, Laser J. (in Chinese) **31,** 36 (2010).
13. IEEE Std 647$^{\mathrm{TM}}$-2006, IEEE Stand Specification Format Guide and Test Procedure for Sing-Axis Laser Gyros, IEEE Aerospace and Electronic Systems Society, New York (2006).
14. L. C. Ng and D. J. Pines, J. Guidance **201,** 211 (1996).
15. A. Yariv, *Optical Electronics in Modern Communications* (Fifth Edition) (London, Oxford University Press, Inc 393-340) (1997).
16. A. Yariv and K. Vahala, IEEE J. Quantum Electron. **QE19,** 889 (1983).
17. A. L. Schawlow and C. H. Townes, Phys. Rev. **112,** 1940 (1958).