# Polarization modulation in single－frequency He －Ne laser with an anisotropy feedback cavity 

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#### Abstract

The polarization state is modulated by tilting birefringence component placed in the feedback external cavity．The variation of the polarization state in one period of modulation is found to be similar to sine wave．The periods become increasingly smaller．The maximum of variation in one period decreases against the rotated angle．The experimental phenomenon is subjected to the change of optical path and secondary reflection．The phenomenon is analyzed theoretically based on geometrical optics and crystal optics． High－accuracy measurements of absolute and relative angles can be realized based on the experimental phenomenon．The angle resolution is 0.1 arcsec in theory．


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External optical feedback can dramatically affect the output intensity，coherence，and stability of laser ${ }^{[1,2]}$ ．Po－ larization control in laser by external optical feedback has attracted considerable interest ${ }^{[3-6]}$ ．Fei et al．found that the duty ratio of two eigenstates in one period of laser intensity modulation varied with different phases ${ }^{[7]}$ ．

In this letter，modulation in polarization is realized by tilting birefringence element in He －Ne laser external cav－ ity．New phenomena are found in our experiments．When the tilted angle of birefringence element is changed，duty ratio of polarization state varies and the maximum varia－ tions are modulated．We analyze this experimental phe－ nomenon theoretically based on geometrical optics and crystal optics．The results of theory analysis are consis－ tent with experimental results．The experimental phe－ nomenon can be used to measure the absolute angle and relative angle of rotating object．

Experiments are carried out on single－mode，linearly polarized $\mathrm{He}-\mathrm{Ne}$ laser．The experimental setup is shown in Fig．1．The wavelength is 632.8 nm ．The ratio of gaseous pressure in laser is $\mathrm{He}: \mathrm{Ne}=9: 1$ and $\mathrm{Ne}^{20}: \mathrm{Ne}^{22}=$ 1：1．

Here，$D_{1}$ and $D_{2}$ are photodetectors used to detect the laser intensity and the variations of laser polarization state，respectively；$M_{1}$ and $M_{2}$ are laser mirrors with re－ flectivities of $99.8 \%$ and $98.8 \%$ ，respectively；$M_{\mathrm{E}}$ is an external mirror with reflectivity of $15 \%$ used to reflect laser beams back into the laser； P is a polarizer used to separate different polarization state from laser；PZT is used to push and pull $M_{\mathrm{E}}$ ．The length of laser is 150 mm ．Together with $M_{2}$ and $\mathrm{S}, M_{\mathrm{E}}$ forms a birefringence external cavity 100 mm in length．

The length of external cavity is scanned by PZT，and the curves of intensity modulation are shown in Fig． 2. The tilted angle of birefringence element is $0.05^{\circ}$ in Fig． 2．The birefringence element is a quartz waveplate，and the optical axis is perpendicular to the incident surface of birefringence element．

From Fig．2，some phenomena that differ from the conventional laser feedback can be found．Firstly，dips on the laser intensity curve appear，and the laser inten－ sity curve is similar to sine curves in conventional laser feedback．Secondly，the polarization of laser hops at the dip point B ，and the intensities of two eigenstates are both modulated by the length of external cavity ${ }^{[7]}$ ．In the experiments，the duty ratios of two eigenstates in one period of intensity modulations are modulated by tilting birefringence element，in short，polarization is modulated in $\mathrm{He}-\mathrm{Ne}$ laser by tilting birefringence element．The duty ratio $D$ can be expressed as

$$
\begin{equation*}
D=\frac{l_{\mathrm{BC}}}{l_{\mathrm{AC}}} \tag{1}
\end{equation*}
$$



Fig．1．Experimental setup．W：glass windows，antireflective－ coated；S：birefringence elements；PZT：piezoelectric trans－ ducer．


Fig．2．Waveforms of laser intensity modulation and polariza－ tion flipping．


Fig. 3. Modulation of polarization by tilting birefringence element (a) waveform of duty ratio variation; (b) waveform of modulation periods; (c) maximum of duty ratio in one period; (d) minimum of duty ratio in one period.
where $l_{\mathrm{BC}}$ is the time difference between the points C and B , and $l_{\mathrm{AC}}$ is between the points C and A in Fig. 2.
In our experiments, the tilted angle of birefringence element is changed in a step of $0.07^{\circ}$, such as the dotted line in Fig. 1. When the tilted angle of the birefringence element is changed, the duty ratio of two eigenstates in one period of intensity modulation also changes. The curve of polarization modulation is shown in Fig. 3.

As shown in Fig. 3, the duty ratio of two eigenstates in one period of intensity modulation is modulated by tilting birefringence element. The maximum of variation in one period decreases against the tilted angle. The peak-to-peak value in one period is approximately 0.053 , and the period of variation becomes increasingly smaller when the tilted angle of birefringence element increases. The modulation period changes from $1.3^{\circ}$ to $0.7^{\circ}$ of the tilted angle.

The dynamics of polarization modulation is subjected to the change of optical path and the second reflection from inner surface of birefringence element. The two factors are then analyzed in detail.

The optical path changes when birefringence element is tilted. Two ways of tilting the birefringence element are analyzed in detail: (1) the tilting axis is parallel to ray axis of birefringence element, and (2) the tilting axis is perpendicular to the optical axis of birefringence element. The duty ratio can be given by

$$
\begin{align*}
D-D_{0}= & \frac{2 d}{\lambda}\left[\left(\frac{n_{\mathrm{e}}^{\prime}}{\cos \theta_{\mathrm{e}}}-\frac{n_{\mathrm{o}}^{\prime}}{\cos \theta_{\mathrm{o}}}\right)\right. \\
& \left.+\left(\tan \theta_{\mathrm{o}}-\tan \theta_{\mathrm{e}}\right) \sin \theta-\left(n_{\mathrm{e}}-n_{\mathrm{o}}\right)\right] \tag{2}
\end{align*}
$$

where $D$ represents the duty ratio when the tilted angle equals $\theta ; D_{0}$ represents the duty ratio when the tilted angle is zero; $d$ is the thickness of birefringence element; $\theta$ is the tilted angle; $\theta_{\mathrm{o}}$ and $\theta_{\mathrm{e}}$ are the angles of refraction of the o light and the e light, respectively, when the tilted angle is $\theta ; n_{\mathrm{o}}$ and $n_{\mathrm{e}}$ are refractive indices of the o light and the e light, respectively, when tilted angle is zero; $n_{\mathrm{o}}^{\prime}$ and $n_{\mathrm{e}}^{\prime}$ are refractive indices of the o light and the e light, respectively, when tilted angle is $\theta$. When tilt axis is parallel to ray axis or perpendicular to ray axis,
$n_{\mathrm{o}}^{\prime}$ and $n_{\mathrm{e}}^{\prime}$ can be given by
$n_{\mathrm{o}}^{\prime}=n_{\mathrm{o}}, n_{\mathrm{e}}^{\prime}=n_{\mathrm{e}}($ parallel $)$,
$n_{\mathrm{o}}^{\prime}=n_{\mathrm{o}}, n_{\mathrm{e}}^{\prime}=\sqrt{\frac{n_{\mathrm{o}}^{2} n_{\mathrm{e}}^{2}+\left(n_{\mathrm{o}}^{2}-n_{\mathrm{e}}^{2}\right) \sin ^{2} \theta}{n_{\mathrm{o}}^{2}}}$ (perpendicular).

The optical axis of birefringence element is perpendicular to the surface of paper. From Eq. (3), the relationship between duty ratio change and tilted angle is shown in Fig. 4.

As shown in Fig. 4, the duty ratio decreases when tilt axis is parallel to ray axis and the duty ratio increases when tilt axis is perpendicular to ray axis. The variation of duty ratio is proportional to the thickness of the birefringence element.

The secondary reflection from inner surface of birefringence element has a large effect on duty ratio ${ }^{[8]}$. From the refractive index ellipsoid, we can analyze the impact of second reflection from inner surface of birefringence element. The secondary reflection from inner surface of birefringence element is shown in Fig. 5.

Here, $\theta$ is the incidence angle, $\theta_{\mathrm{o}}$ is the refractive angle of the o light, and $\theta_{\mathrm{e}}$ is the refractive angle of the e light. The thickness of birefringence is equal to $d$.

From refractive index formula, the relationship between $\theta, \theta_{\mathrm{o}}$, and $\theta_{\mathrm{e}}$ are given by

$$
\begin{align*}
& n_{\mathrm{oo}} \sin \left(\theta_{\mathrm{o}}\right)=\sin \theta, \\
& n_{\mathrm{ee}} \sin \left(\theta_{\mathrm{e}}\right)=\sin \theta, \tag{4}
\end{align*}
$$

where $n_{\mathrm{oo}}$ and $n_{\mathrm{ee}}$ represent refractive indices of the o light and the e light with the tilted angle $\theta$, respectively.


Fig. 4. Variation of duty ratio corresponding to the tilted angle of birefringence element with different thickness. $d_{1}=0.4$ $\mathrm{mm}, d_{2}=0.8 \mathrm{~mm}, d_{3}=1.2 \mathrm{~mm}, d_{4}=1.6 \mathrm{~mm}$, and $d_{5}=2.0 \mathrm{~mm}$.


Fig. 5. Diagram of the second reflection from inner surface of birefringence element.

Based on refractive index ellipsoid, $n_{\text {oo }}$ and $n_{\text {ee }}$ can be given by

$$
\begin{align*}
& n_{\mathrm{oo}}=n_{\mathrm{o}}, \\
& n_{\mathrm{ee}}=\left[\frac{\cos ^{2}\left(\theta_{\mathrm{e}}\right)}{n_{\mathrm{e}}^{2}}+\frac{\sin ^{2}\left(\theta_{\mathrm{e}}\right)}{n_{\mathrm{o}}^{2}}\right]^{-\frac{1}{2}} . \tag{5}
\end{align*}
$$

The light through birefringence element will interfere with each other. The light field can be given by

$$
\begin{align*}
E_{\mathrm{oo}}= & E_{\mathrm{o}} t_{\mathrm{oa}-\mathrm{b}} t_{\mathrm{ob}-\mathrm{a}} \exp \left[\mathrm{i} k \frac{n_{\mathrm{oo}} d}{\cos \left(\theta_{\mathrm{o}}\right)}\right] \\
& \left\{1+r_{\mathrm{ob}-\mathrm{a}} \exp \left[2 \mathrm{i} k n_{\mathrm{oo}} d \cos \left(\theta_{\mathrm{o}}\right)\right]\right\} \\
= & E_{\mathrm{oo}}^{\prime} \cos \left(\phi_{\mathrm{o}}\right)+\mathrm{i} E_{\mathrm{oo}}^{\prime} \sin \left(\varphi_{\mathrm{o}}\right) \\
E_{\mathrm{ee}}= & E_{\mathrm{e}} t_{\mathrm{ea}-\mathrm{b}} t_{\mathrm{eb}-\mathrm{a}} \exp \left[\mathrm{i} k \frac{n_{\mathrm{ee}} d}{\cos \left(\theta_{\mathrm{e}}\right)}\right] \\
& \left\{1+r_{\mathrm{eb}-\mathrm{a}} \exp \left[2 \mathrm{i} k n_{\mathrm{ee}} d \cos \left(\theta_{\mathrm{e}}\right)\right]\right\} \\
= & E_{\mathrm{ee}}^{\prime} \cos \left(\varphi_{\mathrm{e}}\right)+\mathrm{i} E_{\mathrm{ee}}^{\prime} \sin \left(\varphi_{\mathrm{e}}\right) \tag{6}
\end{align*}
$$

where $E_{\text {oo }}$ and $E_{\text {ee }}$ are the electric vectors of the o light and the e light, respectively, through birefringence element; $E_{\mathrm{oo}}^{\prime}$ and $E_{\mathrm{ee}}^{\prime}$ are the amplitudes of the o light and the e light, respectively; $\varphi_{\mathrm{o}}$ and $\varphi_{\mathrm{e}}$ are the phases of the o light and the e light, respectively; $E_{\mathrm{o}}$ and $E_{\mathrm{e}}$ are the initial electric vectors of the o incident light and the e incident light, respectively; $t_{\mathrm{oa}-\mathrm{b}}$ and $t_{\mathrm{ob}-\mathrm{a}}$ are transmittances of the o light from air to birefringence and from birefringence to air, respectively; $t_{\mathrm{ea}-\mathrm{b}}$ and $t_{\mathrm{eb}-\mathrm{a}}$ are transmittances of the e light from air to birefringence and from birefringence to air, respectively; $r_{\mathrm{ob}-\mathrm{a}}$ and $r_{\mathrm{eb}-\mathrm{a}}$ are the reflectivities of the o light and the e light, respectively, from birefringence to air, $k=2 \pi / \lambda$.
Based on the Fresnel formal, we can obtain

$$
\begin{align*}
& r_{\mathrm{oa}-\mathrm{b}}=\frac{-\sin \left(\theta-\theta_{\mathrm{o}}\right)}{\sin \left(\theta+\theta_{\mathrm{o}}\right)}, r_{\mathrm{ea}-\mathrm{b}}=\frac{\tan \left(\theta-\theta_{\mathrm{e}}\right)}{\tan \left(\theta+\theta_{\mathrm{e}}\right)} \\
& r_{\mathrm{ob}-\mathrm{a}}=\frac{\sin \left(\theta-\theta_{\mathrm{o}}\right)}{\sin \left(\theta+\theta_{\mathrm{o}}\right)}, r_{\mathrm{eb}-\mathrm{a}}=\frac{-\tan \left(\theta-\theta_{\mathrm{e}}\right)}{\tan \left(\theta+\theta_{\mathrm{e}}\right)}, \\
& t_{\mathrm{oa}-\mathrm{b}}=\frac{2 \sin \left(\theta_{\mathrm{o}}\right) \cos \theta}{\sin \left(\theta+\theta_{\mathrm{o}}\right)}, t_{\mathrm{ea}-\mathrm{b}}=\frac{2 \sin \left(\theta_{\mathrm{e}}\right) \cos \theta}{\sin \left(\theta+\theta_{\mathrm{e}}\right) \cos \left(\theta-\theta_{\mathrm{e}}\right)}, \\
& t_{\mathrm{ob}-\mathrm{a}}=\frac{2 \sin \theta \cos \theta_{\mathrm{o}}}{\sin \left(\theta+\theta_{\mathrm{o}}\right)}, t_{\mathrm{eb}-\mathrm{a}}=\frac{2 \sin \theta \cos \theta_{\mathrm{e}}}{\sin \left(\theta+\theta_{\mathrm{e}}\right) \cos \left(\theta-\theta_{\mathrm{e}}\right)} \tag{7}
\end{align*}
$$

The relationship between duty ratio and phase difference of two principal optical axes is given by

$$
\begin{equation*}
D=\frac{\delta}{90}, \tag{8}
\end{equation*}
$$

where $\delta$ is phase difference of two principal optical axes, and $\delta$ equals $\varphi_{\mathrm{e}}-\varphi_{\mathrm{o}}$.

The variation of duty ratio because of second reflection is written as

$$
\begin{equation*}
D=\frac{\varphi_{\mathrm{e}}-\varphi_{\mathrm{o}}}{90} . \tag{9}
\end{equation*}
$$

The variation of duty ratio, induced by secondary reflection from inner surface of birefringence element is shown in Fig. 6.
In Fig. 6, the duty ratio varies when the tilted angle changes. The variation of duty ratio is similar to


Fig. 6. Duty ratio varying with the tilted angle of birefringence element.


Fig. 7. Variation of duty ratio corresponding to the tilted angle of birefringence element with synthesis affect.
the sine wave, and the maximum of variation in one period decreases with the tilted angle of birefringence element. The period of variation becomes increasingly smaller when tilted angle of birefringence element increases.

Both the optical path changes and the secondary reflection can affect duty ratio of birefringence. The synthetic effect can be expressed as

$$
\begin{align*}
D= & \frac{\varphi_{\mathrm{e}}-\varphi_{\mathrm{o}}}{90}+\frac{2 d}{\lambda}\left[\left(\frac{n_{\mathrm{ee}}}{\cos \theta_{\mathrm{e}}}-\frac{n_{\mathrm{o}}}{\cos \theta_{\mathrm{o}}}\right)\right. \\
& \left.+\left(\tan \theta_{\mathrm{o}}-\tan \theta_{\mathrm{e}}\right) \sin \theta-\left(n_{\mathrm{e}}-n_{\mathrm{o}}\right)\right] . \tag{10}
\end{align*}
$$

The variation of duty ratio corresponding to tilted angle from the synthesis effect is shown in Fig. 7.

In Fig. 7, the variation of duty ratio synthesis effect is similar to Fig. 3. Therefore, the experimental phenomenon can be explained with the change of optical path and second reflection.

Duty ratio is sensitive to tilted angle of birefringence element; therefore, very tiny angles can be measured based on this experimental result. When a rotating object is attached to the birefringence element, the tiny angle of the rotating object can be converted into the changes of duty ratio, from which we can conveniently obtain the angle information. In Fig. 3(a), the modulated periods become increasingly smaller. When the smallest modulated period is used to measure the angle change, the angle resolution can be analyzed. If the measurement resolution of duty ratio is 0.0000030091 , the angle reso-
lution is calculated as

$$
\begin{equation*}
0.48 \times 3600 /\left(\frac{0.612867-0.560869}{0.0000030091}\right)=0.1 \tag{11}
\end{equation*}
$$

where 0.48 is the smallest modulated period of tilted angle in Fig. 3(a), and 0.612867 and 0.560869 are the maximum and minimum of duty ratio in the smallest modulated period, respectively. The angle resolution of this method can reach 0.1 arcsec theoretically.

In conclusion, duty ratio of polarization state is modulated by tilting the birefringence element. The variation of duty ratio is similar to sine wave. The period of variation becomes increasingly smaller. The maximum of variation in one period decreases against the tilted angle. The experimental phenomenon is analyzed. The result of theoretical calculation is consistent with the experimental phenomena. Very tiny angles of rotating objects can be measured based on the experimental phenomena.

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