Non-approximate method for designing annular field of two-mirror concentric system

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Annular field aberrations of a three-reflection concentric system, which are composed of two spherical mirrors, are analyzed. An annular field with a high level of aberration correction exists near the position where the principal ray is perpendicular to the object-image plane. Aberrations are determined by the object height and aperture angle. In this letter, the general expression of the system aberration is derived using the geometric method, and the non-approximate design method is proposed to calculate the radii of the annular fields that have minimum aberrations under different aperture angles. The closer to 0.5 (the ratio of the radius of convex mirror to the radius of concave mirror) is, the smaller the system aberration is. The examples analyzed by LABVIEW indicate that the annular field designed by the proposed method has the smallest aberration in a given system.

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Concentric optical system is constituted by a series of refracting or reflecting spherical surfaces that have the same center and separated by homogeneous media. Any plane containing the optical axis is symmetrical, and any ray traversing through the spherical center is the axis of the rotational symmetry of the system. Therefore, this system can eliminate high-order aberrations, and consequently, it can be widely applied in a remote-sensing system[1,2] as well as in the large-scale integrated circuit manufacturing industry[3]. Generally, the concentric optical system can be divided into systems with a single mirror[4,5], two mirrors, and three mirrors. In 1973, Offner proposed a kind of two-mirror three-reflection concentric system[6,7]. The author pointed out that an annular field was present in this concentric system, which could be considered to have zero aberration because the fifth-order aberration could be balanced by the third-order aberration[8]. In 1987, Charls[9] analyzed the concentric optical system in detail and proved that the concentric optical system had the following characteristics. Firstly, the product nd for any ray that traces such a system will maintain the same value throughout, where n is the refractive index of the space medium and d is the perpendicular distance of the ray from the spherical center of the space. Secondly, if the object and the image are in the air and the incident ray emerges as parallel, then the system will perform ideal imaging at a point located at the joint of the object and the system center. Thirdly, the distance of the incident and emergent rays from the center will be in the ratio of the object to the image space refractive indices. Fourthly, for a system consisting only of refracting surfaces or a system with an even number of reflections, the system will produce virtual images. The rays will be positioned on the same side of the system center. For a system with an odd number of reflections, real images will be produced, and the incident and reflected rays will be located on the opposite sides of the center. This is true for the Dyson system[4] and the Offner system[8]. By carefully analyzing the aberrations of the two-mirror three-reflection concentric system, we can determine that the convex mirror constitutes the aperture stop of the system. Spherical aberration is inherent in the system, but the spherical aberration can be eliminated by departing from the zero Petzval sum condition while maintaining mirror concentricity. Suzuki provided a quantitative analysis on the two-mirror three-reflection optical system[10], and pointed out that in the concentric condition, the system had no sagittal aberration. However, high-order field curvature and aberrations existed, and the annular field was located near the object point where the incident and emergent rays of the principle ray were parallel to each other. In spite of the excellent and interesting properties of this system, its characteristics are known only qualitatively. Offner failed to provide the formula of the radius of the annular field as well as the specific design method.

The two-mirror three-reflection concentric system comprises a concave mirror and a convex mirror. This system has the advantages of excellent image quality, compactness, easy installation, etc. (Fig. 1). In the late nineties, the Jet Laboratory of NASA developed this system as the new type of spectrometer[11]. In 2003, with the support of the Shanghai Committee of Science & Technology, we launched "A research on the imaging spectrometer with convex grating in the visible wavelength" (Grant No. 036105027), which achieved satisfying theoretical and experimental results[12].

This letter presents a non-approximate geometric optimization design method, which guarantees that we can accurately calculate the radius of the annular field with-aberration correction in a given two-mirror concentric system. Furthermore, optimizations in two different systems are also described. The results indicate that the

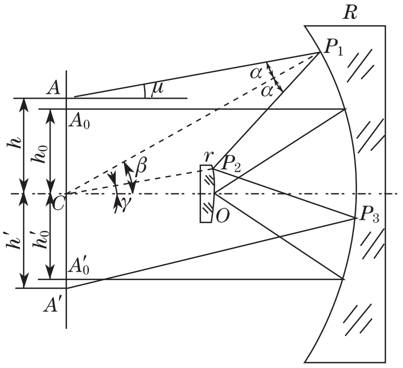

Fig. 1. Two-mirror concentric system.

closer to 0.5 (the ratio of the radius of convex mirror to the radius of concave mirror) is, the smaller the system aberration will be.

Use of mathematical formula facilitates the expression of the aberrations in the concentric optical system. Figure 1 shows a two-mirror concentric system that is constituted by a concave mirror and a convex mirror. Point C is the common spherical center on the common object-image plane $AA_0A'A'_0$, which is perpendicular to the optical axis. A and A_0 are the object points and A' and A'_0 are the image points. R is the radius of the concave mirror, and r is the radius of the convex mirror. The off-center distance of the object point is h, and the off-center distance of the image point is h'. The arbitrary ray in the aperture angle μ can be traced by

$$h = \frac{R\sin(\beta-\mu)}{\cos\mu}, \tag{1}$$

$$\frac{\sin(\beta-\mu)}{\sin(2\beta-\mu-\gamma)} = \frac{r}{R}, \tag{2}$$

$$h' = \frac{R\sin(\beta-\mu)}{\cos(\mu-2\gamma)}, \tag{3}$$

$$\Delta = h' - h, \tag{4}$$

where Δ is the aberration.

If the convex mirror constitutes the aperture stop and can be placed on the focal plane of the concave mirror, then the system can be considered as a telescope system because it is telecentric in relation to the object and the image spaces. Thus, the incident ray emerges in parallel. If the spherical aberration is not considered, then the optical system will produce an ideal image based on the second characteristic of the concentric system. In Fig. 2(a), only two incident rays emerge in parallel for a certain h in the aperture angle $\mu_{//}$ due to the presence of the spherical aberration. The rays satisfy $\Delta = h' - h = 0$ and meet the condition of ideal imagery, and their ray-to-center distances are $h\cos\mu_{//} = R\sqrt{1-\left(\frac{R}{2r}\right)^2}$. Figure 2(b) shows the principal ray that also satisfies $\Delta = h' - h = 0$ which is also symmetrical to the optical axis. Because of the symmetry, spherical aberration of the principal ray can be offset, which can bring the Petzval sum to zero. For the remaining incident rays, it is impossible to secure the perfect collimation of parallel pencils and the zero Petzval sum condition; thus, $h' \neq h$. To maintain good image quality, it is necessary to arrange the rays so that the departures from parallelism are minimized. In general, the three rays do not coincide with one another, and the principal ray is not perpendicular to the object-image plane. By changing the zero Petzval sum condition while keeping it concentric, we can achieve the ideal imaging of the principal ray wherein the incident ray is parallel to the emergent ray. The ray $A_0OA'_0$ in Fig. 1 with the aperture angle $\mu_{//} = 0$.

Moreover,

$$h_0 = h'_0 = R\sqrt{1-\left(\frac{R}{2r}\right)^2}. \tag{5}$$

In this system wherein R and r are given, there is only one A_0 position. Based on Eq. (5), r must satisfy $r > R/2$, which means that the radius of the convex mirror must be larger than the focal length of the concave mirror. The variation of the ray distance for a given aperture angle of rays from the center over the pencil will obtain the least value if the following conditions are fulfilled. Firstly, the object-image plane that contains the common center is perpendicular to the optical axis, and secondly, the plane is in a normal position in relation to the central rays of the imaging pencils, which are the principal $A_0OA'_0$. According to the first and second characteristics of the concentric optical system, the smaller the variation is, the smaller the non-parallelism will be. Thus, the aberration of the system becomes smaller.

For any ray that radiates from the object point A, the image A' can be traced only after the distance h and the aperture angle μ are given. Consequently, the aberration Δ can be accurately calculated. Thus, μ is another main parameter that influences the aberration of the entire system. For any ray with the aperture angle μ, the ray-to-center distance is $h\cos u$. According to the system symmetry, the emergent ray is no longer parallel to the incident ray. Generally, when the principal ray satisfies Eq. (5), i.e., the three rays overlap and the principal ray is perpendicular to the object-image plane, the aberrations caused by the rest of the rays will be smaller than those of the case wherein the principal ray does not satisfy Eq. (5). When μ is very small, $h\cos u \approx h$ can be obtained, and the incident ray is approximately parallel to the emergent ray. Thus, the aberration is slight, and a thin annular field in the system exists, i.e., the so-called annular field with minimum aberration[8]. If $R = 2r$ ($h = 0$), the telecentric optical system is formed because the stop is in the focal plane. The system will not meet the practice requirements because of the presence of high-order aberrations. We can determine from

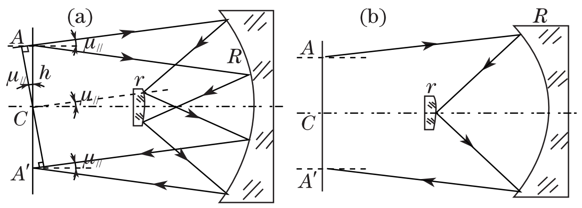

Fig. 2. Three special rays ($\Delta = h' - h = 0$). (a) Incident ray parallel to the emergent ray and (b) principal ray.

the expression of the wave aberration that field curvature is the sum of the even powers of off-center distances of the object and the image points. The aberration term that describes the variation of focus with the Nth power of the field radius is proportional to the (N–2)th-order pupil spherical aberration. Therefore, the spherical aberration introduced by a slight increase of the radius of the convex mirror, while maintaining the concentricity with the concave mirror, can balance the high level of field curvature. By changing the zero Petzval sum condition, the spherical aberration can be eliminated. Based on Eq. (5), $r > R/2$ must be satisfied. Only $r > R/2$ of the principal ray can produce the ideal image. Through the comprehensive analysis of the above parameters h, μ and the system aberrations, we can establish that the annular field is in the vicinity of h_0.

For a given system wherein radius r of the convex mirror and radius R of the concave mirror are known, we can ascertain that the radius of the annular field is in the vicinity of h_0 according to the above description. For a given h in the vicinity of h_0, the aberration Δ of each ray in the numerical aperture $n \sin \mu$ can be calculated using Eqs. (1)−(4) and by plotting the $\Delta - \mu$ curves. The area S between the curve and the axis μ, i.e., $S = \sum_i^{\infty} |\Delta_i|$, is the aberration in aperture angle μ. Search for h_j corresponding to the minimum area $S_{j\,\min}$ by scanning h in a proper range, where h_j is the optimal radius of the annular field.

For every specific system with given R and r, there is an annular field with a minimal aberration. Different systems correspond to different minimal aberrations. Thus, choosing the optimal radii of the concave and the convex mirrors is necessary. We can take $h = h_0$ because the radius of the annular field is in the vicinity of h_0. Subsequently:

$$\Delta = \left[\frac{\cos \mu}{\cos (\mu - 2\gamma)} - 1\right] R\sqrt{1 - \left(\frac{R}{2r}\right)^2}, \qquad (6)$$

where $r/R > 0.5$, and γ is determined by μ. Based on Eq. (6), it can be seen that the closer to 0.5 r/R is, the smaller aberration Δ will be. Moreover, h will also be smaller according to Eq. (5). Therefore, the two parameters of the system, the radius h of the annular field and the radius R of the concave mirror must be determined according to the actual requirements. After R and h are determined, the value of r should be derived because this can cause the value of r/R to become close to 0.5. Once this is achieved, aberrations will be reduced and the image quality will improve.

Furthermore, the distortion aberration of the symmetrical optical system is proportional to the third power of the off-center distance, and the distortion directions of the convex mirror and the concave mirror are opposite to each other. Therefore, another feature of the two-mirror concentric system is shown which is characterized with a very slight distortion.

Design systems with $R_1 = 222.3$ mm, $r_1 = 112.98$ mm, $R_2 = 100$ mm, and $r_2 = 55$ mm. Figure 3(b), which is exported from Fig. 3(a), shows that aberration curves vary by the aperture angle in the range of $\pm 5°$ in the two-mirror concentric system of $R_1 = 222.3$ mm, $r_1 = 112.98$ mm ($r/R = 0.5028$). The abscissa represents the aperture angle μ, and the ordinate represents the difference of the off-center distance of the object, which in turn, represents the aberration. We can see that the curve with $h_0 = 39.85$ mm (dashed line) intersects with the μ axis at one point. This corresponds to the case ($A_0OA'_0$) wherein the three special rays in Fig. 2 are coincident, as analyzed above. Furthermore, each of the other curves in Fig. 3(b) has three points of intersection with the axis μ. This means that there are only three rays which satisfy $h' = h$. Figure 4(b) is exported from Fig. 4(a). The solid line denotes the curve that shows the sum of aberrations under the difference of the object's off-center distance in the aperture angle of $\pm 5°$ by varying the object's off-center distance h from 39 to 41 mm. The smaller the sum is, the smaller the corresponding aberration will be. The minimum value of the area is $S_{1\,\min} = 0.000416249$ mm^2, and the corresponding off-center distance of the object is $h = 39.9242$ mm, which is regarded as the radius of the annular field with minimum aberration. The dashed line represents the case with an aperture angle of $\pm 10°$, and the minimum area is $S'_{1\,\min} = 0.00674649$ mm^2. The corresponding object's off-center distance is $h = 40.1485$ mm, i.e., it is the same system but with different aperture angles. The bigger the aperture angle is, the larger the aberration will be. $S'_{1\,\min}$ is 16 times larger than $S_{1\,\min}$. Therefore, the small aperture angle is preferred when the practice design requirements are considered. Figure 5(b), which is exported from Fig. 5(a), shows the aberration curve caused by varying the object's off-center distance h in the system of $R_2 = 100$ mm, $r_2 = 55$ mm ($r/R = 0.55$) with the aperture angle of $\pm 5°$. Its minimum area is $S_{2\,\min} = 0.00847257$ mm^2, and the corresponding off-center distance of the object point is $h = 41.7387$ mm. Consequently, we can obtain $S_{2\,\min} = 20 S_{1\,\min}$ with the same aperture angle, which not only indicates that the aberrations of the annular field with minimum aberration vary greatly in different systems, but also verifies that the closer to 0.5 the value of r/R is, the smaller the aberration of the annular field with minimum aberration will be.

The analysis and calculation cited above shows that although the image quality can be improved when r/R is close to 0.5, the radius of the annular field h will become smaller. Moreover, smaller aperture angle can reduce the aberration, but in this case, the radius of the annular field will become smaller. However, to satisfy the space camera of scanning and to conduct image stitching, there must be an annular field with a specific size which can

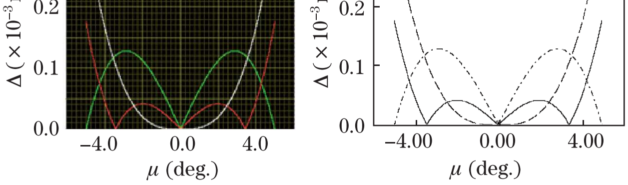

Fig. 3. System 1: aberration curve variations caused by aperture angle in three different off-center distances of object points.

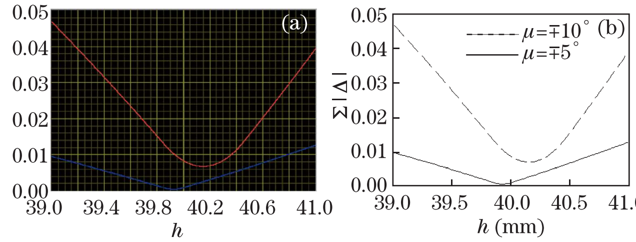

Fig. 4. System 1: aberration curve variations caused by off-center distances of object points in two different aperture angles.

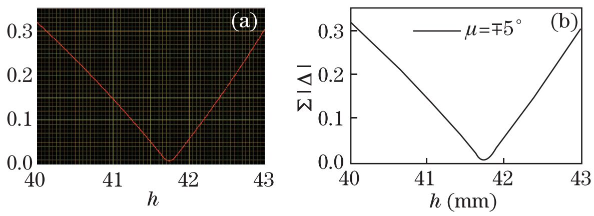

Fig. 5. System 2: aberration curve variations caused by off-center distances of object points in the aperture angle $\pm 5°$.

produce the straight slit in the annular field of view. Therefore, the concentric two-mirror system can be designed with minimum aberration by considering the size of the whole system, the radius of the annular field, and the size of the aperture angle.

In conclusion, the general characteristics of the concentric optical system and a detailed analysis of the aberration of the two-mirror three-reflection concentric system are presented. The so-called annular field with minimum aberration is the field of high quality, which can be obtained by introducing appropriate primary spherical aberration to balance the high-order field curvature. The three rays that can form the ideal image are analyzed. Results show that when the principal ray is perpendicular to the object-image plane, the three rays will overlap. In this situation, the variation of the ray distance from the center over the pencil will be of the least value, and the off-center distance of object h is the most important parameter that influences the aberration. Furthermore, the aberration is also influenced by aperture angles, and thus, the annular field with minimum aberration should be located in the vicinity of $h = h_0$. We derive the general expression of the system aberration through the ray tracing method. Moreover, we conduct quantitative calculation of the system and subsequently propose the design method to calculate accurately the aberration of system using the formula. The ratio of the radius of the convex mirror and the radius of the concave mirror should be greater than 0.5. To obtain a smaller aberration, the radius should be as close to 0.5 as possible. This can be achieved by considering the size of the system and the annular field. In this letter, detailed steps for optimal design are introduced, and the given examples indicate that a system with minimum aberration can be designed in a non-approximate manner under certain conditions. The radii of the annular field with minimum aberration can also be obtained using the method proposed in this letter.

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