# Velocity estimation of an airplane through a single satellite image

**Zhuxin Zhao (赵竹新)[1*], Gongjian Wen (文贡坚)[1], Bingwei Hui (回丙伟)[1], and Deren Li (李德仁)[2]**

[1] *ATR Key Laboratory, National University of Defense Technology, Changsha 410073, China*

[2] *State Key Laboratory of Information Engineering in Surveying, Mapping and Remote Sensing, Wuhan University, Wuhan 430079, China*

* *Corresponding author: zzxwan2000@gmail.com*

Received July 11, 2011; accepted August 31, 2011; posted online October 24, 2011

The motion information of a moving target can be recorded in a single image by a push-broom satellite. A push-broom satellite image is composed of many image lines sensed at different time instants. A method to estimate the velocity of a flying airplane from a single image based on the imagery model of the linear push-broom sensor is proposed. Some key points on the high-resolution image of the plane are chosen to determine the velocity (speed and direction). The performance of the method is tested and verified by experiments using a WorldView-1 image.

*OCIS codes:* 100.2960, 280.4788, 120.0280.

*doi: 10.3788/COL201210.031001.*

The motion information of moving targets gathered through remote sensing technology is important in many fields, such as traffic management, border surveillance, and military applications. Traditionally, this information is acquired via radar data and image sequences[1]. However, estimation of the motion from a single image is desirable, especially in cases when the images obtained are insufficient. Although somewhat difficult, recent developments in high-resolution remote sensing satellites offer this possibility. Satellites such as QuickBird, WorldView-1, and WorldView-2 can achieve submeter-level resolution[2,3]. Etaya *et al.* noted the time lag between the image acquisition of the panchromatic (PAN) and multispectral (MS) sensors on QuickBird[4], and detected the moving targets. Their work was followed by several investigations on the detection of the speed of the vehicle on the ground[5−12]. These techniques require the image to be obtained by multiple sensors (at least two), and might fail if the image is acquired by only one sensor.

The linear push-broom sensor possesses a distinctive imagery mode. Unlike the matrix array sensor, it obtains images by composing sequential image lines collected at continuous time instants. When a moving target is scanned by the linear sensor, its contours are deformed because of the relative movement between the sensor and the target[13]. Therefore, the motion information of the target can be recorded in push-broom images, making estimation from a single image possible.

In this letter, a new method is proposed to estimate the velocity of high-speed target, such as flying airplane, through a single push-broom satellite image. The method is based on the imagery model of the push-broom sensor. Several key points are chosen from the image of the airplane, and the imagery relationships of these key points are explored to determine the speed and the direction of the flying airplane. The proposed method is tested on a WorldView-1 satellite image.

Most high-resolution satellites utilize a linear push-broom sensor to collect images. Thus, the imagery model of the push-broom must first be described. Orun *et al.* proposed a model primarily based on photogrammetric collinearity equations[14]. The photogrammetric collinearity equation model is a camera modeling system originally developed for perspective imaging. They extended the model to linear push-broom images.

Two right-hand coordinate systems are used (Fig. 1). The image space coordinate $o-xyz$ takes the perspective center point of the sensor as its origin $o$. Its $x$ and $y$ axes are vertical and parallel to the linear sensor, respectively. The $z$ axis is the primary optical axis. The ground measurement coordinate $O-XYZ$ is the measurement coordinate. The position and attitude of the linear push-broom sensor and the motion parameters of the target are all defined in the measurement coordinate.

Suppose an object point $A(X,Y,Z)$ is imaged at the point of coordinate $(0,y,-f)$ within the image coordinate, where $f$ is the focus length. According to the photogrammetric collinearity equations, the relation between the coordinates $O-XYZ$ and $o-xyz$ can be given as

$$x = 0 = -f\frac{a_1(X-X_0)+b_1(Y-Y_0)+c_1(Z-Z_0)}{a_3(X-X_0)+b_3(Y-Y_0)+c_3(Z-Z_0)}, \tag{1}$$

$$y = -f\frac{a_2(X-X_0)+b_2(Y-Y_0)+c_2(Z-Z_0)}{a_3(X-X_0)+b_3(Y-Y_0)+c_3(Z-Z_0)}, \tag{2}$$

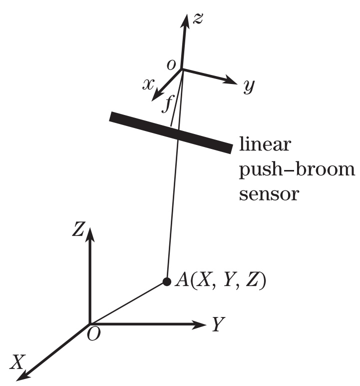


Fig. 1. Geometry of linear push-broom imagery.

 

where $X_0$, $Y_0$, $Z_0$ are the origins of the coordinate $o-xyz$ and $a_i$, $b_i$, $c_i (i = 1, 2, 3)$ are the factors of the rotation matrix $\mathbf{R}$ between the coordinates $O-XYZ$ and $o-xyz$.

$$\mathbf{R} = \mathbf{R}_\varphi \mathbf{R}_\omega \mathbf{R}_\kappa = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}. \quad (3)$$

Here, $\varphi$, $\omega$, $\kappa$ are the three angles rotated between the two coordinates used and $\mathbf{R}_\varphi$, $\mathbf{R}_\omega$, $\mathbf{R}_\kappa$ are their respective rotation matrices.

Equation (1) can be simply given as

$$x = 0 = a_1(X - X_0) + b_1(Y - Y_0) + c_1(Z - Z_0). \quad (4)$$

The left side of Eq. (4) is always equal to 0 because of the characteristics of push-broom sensor. Equation (4) is also the function of the viewing plane $(o - xz)$. Equations (2) and (4) formulate the imagery model of the linear push-broom sensor. The model implies that only the space point on the viewing plane (i.e., the coordinate of the point) satisfies Eq. (4) and will be imaged by the sensor via Eq. (2).

The aim of this letter is to estimate the velocity of a flying airplane from a single push-broom satellite image. The spatial relationship of an airplane image being collected by the push-broom sensor is described in this section.

Most high-resolution remote satellites orbit hundreds of kilometers around the earth. Because the radius of the earth is approximately 6 400 km (Fig. 2(a)) and the time of the airplane scanned is very short, the curvature of the earth can be ignored. Thus, the ground is regarded as a plane in this problem. As shown in Fig. 2(b), when the flying airplane is scanned firstly by the linear push-broom sensor, the ground measurement coordinate $O - XYZ$ is defined by taking the nadir point of the satellite as origin $O$, the $Z$-axis parallel as the nadir radial line, and the $X$-axis parallel as the scanning direction. All parameters of the camera and direction and speed of the airplane are defined in the ground measurement coordinate $O - XYZ$. The image space coordinate $o - xyz$ is defined as above. The rotation matrix $\mathbf{R}$ between the two coordinates is given by Eq. (3).

Suppose the ground scanning speed of the satellite is $V_\mathrm{s}$, the nadir distance is $H_\mathrm{s}$, and the time that the airplane is firstly scanned is 0. At this time, the coordinate of the origin $o$ in $O - XYZ$ is (0, 0, $H_\mathrm{s}$). With time $t$ increasing, the coordinate at $X$ axis is $V_\mathrm{s}t$. Therefore, Eqs. (2) and (4) can be transformed as

$$0 = a_1(X - V_\mathrm{s}t) + b_1Y + c_1(Z - H_\mathrm{s}), \quad (5)$$

$$y = -f\frac{a_2(X - V_\mathrm{s}t) + b_2Y + c_2(Z - H_\mathrm{s})}{a_3(X - V_\mathrm{s}t) + b_3Y + c_3(Z - H_\mathrm{s})}, \quad (6)$$

where $a_i$, $b_i$, $c_i (i=1, 2, 3)$ are the factors of the rotation matrix $\mathbf{R}$. The moving target considered in this letter is an airplane with flight altitude of usually no more than 10 km, much less than the altitude of satellite orbits. Suppose $(X, Y, Z)$ is a point on the airplane, then $Z \ll H_\mathrm{s}$. Therefore, Eqs. (5) and (6) can be approximately given by

$$0 = a_1(X - V_\mathrm{s}t) + b_1Y - c_1H_\mathrm{s}, \quad (7)$$

$$y = -f\frac{a_2(X - V_\mathrm{s}t) + b_2Y - c_2H_s}{a_3(X - V_\mathrm{s}t) + b_3Y - c_3H_s}. \quad (8)$$

Our method for estimating the velocity of the airplane is based on the imagery model of the linear push-broom sensor. Several key points on the airplane are picked to establish a series of equations according to Eqs. (7) and (8).

Some geometric parameters of the airplane are necessary (Fig. 3(a)), such as the length of the airplane $L$, the length of the wing $H$, and the length from the head to the wing $l$. In most of the cases, the airplane can be considered as flying horizontally. The direction of the airplane is denoted by the unit vector $(m, n)$, and the speed is denoted by $v$. The velocity is denoted by $\mathbf{V}(V_x, V_y) = (mv, nv)$.

The key points used are shown in Figs. 3(a) and (b). Here, $A(X_A, Y_A, Z)$ is the position of the head of the airplane when it is firstly scanned and $B$, $C$, $D$ are the positions of the tail and two tips of the wings at this time. Supposing their coordinates are $B(X_B, Y_B, Z)$, $C(X_C, Y_C, Z)$, and $D(X_D, Y_D, Z)$, respectively, then

$$\begin{cases} X_B = X_A - Lm \\ Y_B = Y_A - Ln \\ X_C = X_A - lm + Hn \\ Y_C = Y_A - ln - Hm \\ X_D = X_A - lm - Hn \\ Y_D = Y_A - ln + Hm \end{cases}. \quad (9)$$

Suppose that the time $A$ scanned is 0, the times that $B$, $C$, $D$ are scanned are $t_B$, $t_C$, and $t_D$. Each key point is

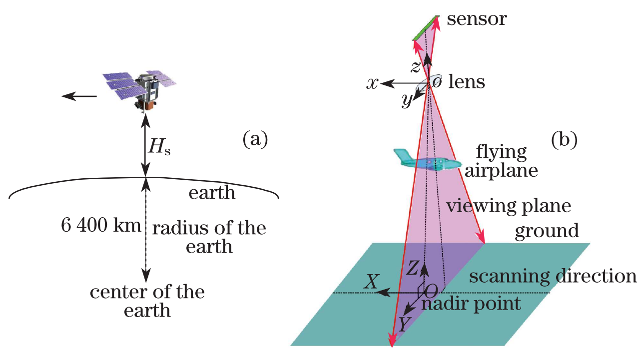


Fig. 2. (a) Remote satellite flying around the earth; (b) spatial relationship among the push-broom sensor, airplane, and ground.

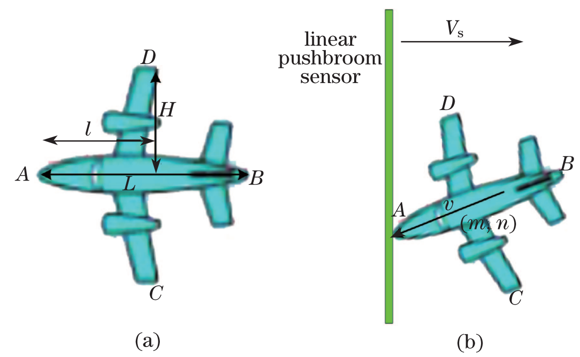


Fig. 3. (a) Geometry parameters of the airplane and key points; (b) relative movement between the airplane and the sensor.

able to establish the mathematical relationship with its corresponding point on the satellite image via Eqs. (7) and (8). The relation functions obtained according to Eq. (7) are

$$\begin{cases} a_1X_A + b_1Y_A - c_1H_\mathrm{s} = 0 \\ a_1(X_B + V_xt_B - V_\mathrm{s}t_B) + b_1(Y_B + V_yt_B) - c_1H_\mathrm{s} = 0 \\ a_1(X_C + V_xt_C - V_\mathrm{s}t_C) + b_1(Y_C + V_yt_C) - c_1H_\mathrm{s} = 0 \\ a_1(X_D + V_xt_D - V_\mathrm{s}t_D) + b_1(Y_D + V_yt_D) - c_1H_\mathrm{s} = 0 \end{cases}. \tag{10}$$

Together with Eq. (9), Eq. (10) can be simply rewritten as

$$\begin{cases} a_1(-Lm + V_xt_B - V_\mathrm{s}t_B) + b_1(-Ln + V_yt_B) = 0 \\ a_1(-lm + Hn + V_xt_C - V_\mathrm{s}t_C) + b_1(-ln - Hm+ \\ V_yt_C) = 0 \\ a_1(-lm - Hn + V_xt_D - V_\mathrm{s}t_D) + b_1(-ln + Hm+ \\ V_yt_D) = 0 \end{cases}, \tag{11}$$

and

$$\begin{cases} (a_1Lm + b_1Ln)t_C = t_B(a_1lm - a_1Hn + b_1ln + b_1Hm) \\ (a_1Lm + b_1Ln)t_D = t_B(a_1lm + a_1Hn + b_1ln - b_1Hm) \end{cases}. \tag{12}$$

Equation (12) includes only $m$ and $n$. Here, $(m, n)$ is unit vector; hence, $m^2 + n^2 = 1$. According to Figs. 2(b) and 3(b), if the direction of the airplane is close to the opposite direction of $X$ axis, then $m < 0$ (if the direction of airplane is close to the $X$ axis, the value of $m$ would be positive). Considering this restriction, the values of $m$ and $n$ can be obtained from Eq. (12). The direction of the flying airplane also can be denoted by azimuth angle $\varphi$, which is defined as

$$\varphi = \begin{cases} \dfrac{180}{\pi}\arctan\left(\dfrac{n}{m}\right), & m > 0, \\ \dfrac{180}{\pi}\left[\pi + \arctan\left(\dfrac{n}{m}\right)\right], & m < 0. \end{cases} \tag{13}$$

Because we know that $V_x = mv$, $V_y = nv$, then the value of $v$ can be calculated from Eq. (11) as

$$v = \frac{a_1V_\mathrm{s}t_B + b_1Ln + a_1Lm}{a_1mt_B + b_1nt_B}. \tag{14}$$

Here, $\mathbf{V}(V_x,V_y)$ is the velocity vector of the airplane. Thus, the attitude of the linear push-broom sensor, the geometric parameters of the airplane, and the time the key points are being scanned are necessary in the proposed method. Given that the scanning rate of the sensor is known, the time can be computed from the numbers of pixels at the scanning direction on the image.

A WorldView-1 satellite image is used to testify the proposed method. The WorldView-1 satellite sensor characteristics are listed in Table 1.

Figure 4 shows the satellite image and the four key points $A$, $B$, $C$, $D$ on the airplane. The positions of the key points on the image are listed in Table 2.

**Table 1. WorldView-1 Satellite Sensor Characteristics**

| | |
|---|---|
| Orbit Altitude $H_\mathrm{s}$ (km) | 496 |
| Resolution (m) | ⩾0.50 |
| Sensor Bands | Panchromatic only |
| Sensor Scanning Frequency (Hz) | 12 000 |
| Focus Length of the Lens $f$ (m) | 8 |
| Ground Scanning Speed $V_\mathrm{s}$ (m/s) | 6 583 |

**Table 2. Positions of the Key Points on the Image**

| Key Point | Column | Raw |
|---|---|---|
| $A$ | 11681 | 26251 |
| $B$ | 11703 | 26361 |
| $C$ | 11645 | 26337 |
| $D$ | 11741 | 26314 |

**Table 3. Characteristics of Boeing 777**

| | |
|---|---|
| Length of the Airplane $L$ (m) | 62.94 |
| Length of the Wing $H$ (m) | 30.465 |
| Length from the Head to the Wing $l$ (m) | 42.06 |

Based on the flight record, the airplane on the image is a Boeing 777 of the Boeing Company. The airplane characteristics of this particular airplane are listed in Table 3[15].

The attitude of the linear push-broom sensor can be obtained from the WorldView-1 ephemeris files. Given the parameters of the sensor and the airplane, the values of $m$ and $n$ are computed from Eq. (12). The value of $\varphi$ is then determined; $v$, $V_x$, $V_y$ can then be determined subsequently via Eq. (14).

The estimation values are finally given in Table 4.

From the satellite image information, the distance between the airplane and its destination, Beijing International Airport, is approximately 23 km. This estimation is reasonable when an airplane is preparing to land.

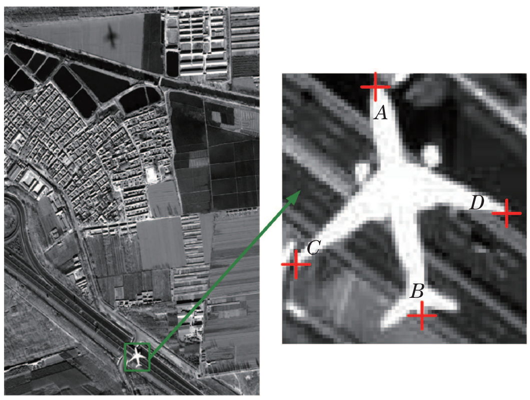


Fig. 4. Airplane in the WorldView-1 image and key points.

**Table 4. Estimation Values of the Experiment**

| | |
|---|---|
| Direction Vector of the Airplane $(m, n)$ | (−0.978, 0.211) |
| Azimuth Angle (deg.) | 167.83 |
| Velocity Vector of the Airplane $(V_x, V_y)$ (km/h) | (−463.3, 100.1) |
| Speed of the Airplane $v$ (km/h) | 474 |

**Table 5. Errors Caused by Sensor Attitude Error and Sensor Resolution**

| | Caused by Sensor Attitude Error | Caused by Sensor Resolution |
|---|---|---|
| Error with the Azimuth Angle (deg.) | ±0.8 | ±2.5 |
| Error with the Speed (km/h) | ±9.2 | ±38.7 |

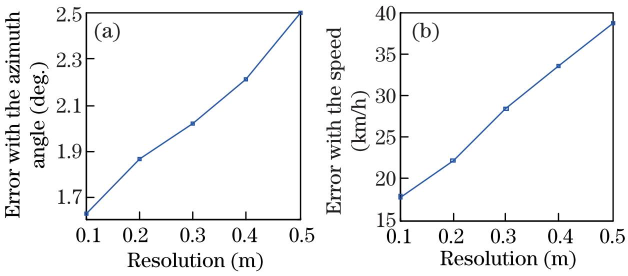


Fig. 5. Theoretical errors of (a) $\varphi$ and (b) $v$ versus the sensor resolution.

The computations of the $\varphi$ and $v$ are related to the sensor attitude factors $a_1$, $b_1$ and the times $t_B$, $t_C$, $t_D$ that the key points are scanned. Therefore, the accuracy of the velocity (speed and direction) are dependent on the satellite sensor attitude accuracy and the resolution of the satellite sensor.

The accuracy of the attitude of WorldView-1 satellite sensor may reach $10^{-2}$ degree level because of its prominent attitude control system. Using Eqs. (13) and (14), the errors of $\varphi$ and $v$ caused by the sensor attitude error are calculated (see Table 5).

The satellite sensor resolution affects the velocity estimation accuracy. In this letter, the time that the airplane is scanned is only approximately 1/100 s. The deformation caused by the relative movement measures only several pixels. Although the WorldView-1 satellite offers high-quality imagery (i.e., 50-cm resolution), keeping an accurate estimation of speed from such slight deformation remains a challenge. To solve the problem, four (or more) key points are chosen to make a bundle adjustment in determining $\varphi$ and $v$. Errors caused by the insufficiency of the sensor resolution are presented in Table 5.

Based on the error analysis above, the errors caused by the sensor resolution is the main source of the error. Considering the computed airplane speed of 474 km/h, the relative error is less than 10%. The errors of $\varphi$ and $v$ when the sensor resolution is changing are theoretically computed and illustrated in Fig. 5. The estimation errors of $\varphi$ and $v$ both decrease as the resolution improves. Therefore, the method would work better with the development of the satellite sensor.

In conclusion, a new method for estimating the speed and direction of a flying airplane from one push-broom satellite image is proposed. Compared with the other methods based on single image, the proposed method is more general because it is rooted from the imagery model of the linear push-broom sensor. The proposed method performs the estimation reasonably by exploring the relative movement during the push-broom scanning the flying airplane, even when the remote sensors only work on one kind of sensor bands (WorldView-1). In addition, the direction of the airplane is determined from the solution of $m$ and $n$; hence, it does not need to be near to the track of the satellite. Moreover, the exact positions of the key points on the image are unnecessary. The method is still able to reasonably estimate velocity if the pixel numbers covered by the different key points on the image are given (i.e., the time costs when the sensor scans from one key point to another is known).

However, the proposed method has limitations. It is mainly feasible for large and fast-moving targets, and the satellite sensor has high resolution demand. The geometric parameters of the airplane are required before the method can be used; hence, airplane recognition may be necessary. The method focuses on a horizontally flying airplane near the ground. The altitude of the airplane is relatively ignored compared with the altitude of the satellite. The motion in depth is always trivial; thus, the velocity can be regarded as a two-dimensional vector. The refined method can be further investigated for more rigorous mathematical relationships among the sensor, the moving target, and the earth to help improve the effectiveness of motion estimation.

This work was supported by the National Natural Science Foundation of China under Grant No. 60872153. The authors are also grateful for the help of ATR Key Laboratory.

## References

1. T. Jin, H. Jia, W. Hou, R. Yamamoto, N. Nagai, Y. Fujii, K. Maru, N. Ohta, and K. Shimada, Chin. Opt. Lett. **8,** 601 (2010).
2. DigitalGlobe Inc. QuickBird Imagery Products-Product Guide, Revision 4.7.1 (2007).
3. DigitalGlobe Inc. WorldView-1 Product Quick Reference Guide, Revision 3.31.09, Version 2 (2008).
4. M. Etaya, T. Sakata, H. Shimoda, and Y. Matsumae, J. Remote Sens. Soc. Jpn. **24,** 4 (2004).
5. Z. Xiong and Y. Zhang, in *Proceedings of ASPRS 2006 Annual Conference* (2006).
6. Y. Zhang and Z. Xiong, in *Proceedings of ISPRS Commission VII Mid-term Symposium* 397 (2006).
7. Z. Xiong and Y. Zhang, Photogramm. Eng. Remote Sens. **74,** 1401 (2008).
8. F. Yamazaki, W. Liu, and T. T. Vu, in *Proceedings of 5th International Workshop on Remote Sensing Application to Natural Hazards* (2007).
9. W. Liu and F. Yamazaki, in *Proceedings of Urban Remote Sensing Joint Event* 1 (2009).
10. W. Liu and F. Yamazaki, in *Proceedings of Safety, Reliability and Risk of Structures, Infrastructures and Engineering Systems* 1099 (2010).
11. G. Easson, S. DeLozier, and H. G. Momm, Remote Sens. **2,** 1331 (2010).
12. M. Pesaresi, K. H. Gutjahar, and E. Pagot, Int. J. Remote Sens. **29,** 1221 (2008).
13. L. Ai, F. Yuan, and Z. Ding, Chin. Opt. Lett. **6,** 505 (2008).
14. A. B. Orun and K. Natarajan, Photogrammetric Eng. & Remote Sens. **60,** 1431 (1994).
15. Boeing Company, Boeing 777-200/300 Airplane Characteristics for Airport Planning (1998).