Equivalence of MTF of a turbid medium and radiative transfer field

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The equivalence of the modulation transfer function (MTF) of a turbid medium and the transmitted radiance from the medium under isotropic diffuse illumination is demonstrated. MTF of a turbid medium can be fully evaluated by numerically solving a radiative transfer problem in a plane parallel medium. MTF for a homogenous single layer turbid medium is investigated as illustration. General features of the MTF in the low and high spatial frequency domains are provided through their dependence on optical thickness, single scattering albedo, asymmetrical factor, and phase function type.

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Depending on its optical properties, multiple light scattering in a turbid medium to some degree, degrades imaging quality. Because the medium is generally considered as a component of the imaging system, the modulation transfer function (MTF) is usually utilized to describe its effect on imaging. MTF is the module of the more general optical transfer function (OTF). The quantitative knowledge of MTF of a turbid medium is important in designing an imaging optical system, imaging processing, and other related applications.

Although theoretical analysis, numerical simulation, and measurement in real fields have been employed, the general analytical solution for MTF has not been obtained. The currently and widely used MTF expression is the result of small-angle-approximation (SAA) analysis[1,2]. According to SAA result, MTF depends only on the scattering and absorption optical thickness. However numerical simulation and experimental results reveal that the MTF is affected by other factors such as those of the whole scattering properties of the turbid medium[3−9] and even instruments[10−14]. Moreover, the SAA approximation is valid only for a relatively narrow frequency range. Thus, it is necessary to obtain the MTF of a turbid medium in a general manner.

The imaging process in a turbid medium is a problem of multiple scattering and strong forward scattering. It is actually a specific problem of radiative transfer. The general feature of MTF of a turbid medium could be related to the imaging problem of light source at infinity. MTF of a turbid medium, as will be demonstrated, is equivalent to the transmitted radiance from the medium with isotropic diffuse illumination. Based on this theoretical foundation, the MTF of a turbid medium can be fully evaluated through the radiative transfer equation (RTE).

For incoherent imaging, the relation of optical intensities I and I_0 at positions $\vec{\rho}$ and $\vec{\rho}'$ in the image and object planes is obtained through the point spread founction (PSF) as[15]

$$I(\vec{\rho}) = \int P\left(\vec{\rho}' - \vec{\rho}\right) I_0(\vec{\rho}')\mathrm{d}\vec{\rho}'. \quad (1)$$

Taking the Fourier transform of the above equation, we obtain

$$\hat{I}(\vec{\nu}) = O_{\mathrm{TF}}(\vec{\nu})\hat{I}_0(\vec{\nu}), \quad (2)$$

where $\hat{I}(\vec{\nu})$ and $\hat{I}_0(\vec{\nu})$ are the spatial spectra of the intensities I and I_0 at spatial frequency $\vec{\nu}$, and O_{TF} is the OTF of the whole imaging system which relates to the PSF as

$$O_{\mathrm{TF}}(\vec{\nu}) = \int P(\vec{\rho}) \exp\left(-2\pi \mathrm{i}\vec{\nu}\cdot\vec{\rho}\right) \mathrm{d}\vec{\rho}. \quad (3)$$

If the quantitative values of the optical fields I and I_0 are known, OTF could be evaluated directly based on Eq. (2), i.e.,

$$O_{\mathrm{TF}} = \hat{I}(\vec{\nu})/\hat{I}_0(\vec{\nu}). \quad (4)$$

Consequently, MTF can be obtained directly by employing the module of the OTF.

Without the inclusion of an optical system, the MTF is solely intended for the medium. The use of a plane parallel random medium with a specific incident light intensity distribution to derive the outgoing light intensity could be considered as imaging in the infinite distance. This is a typical problem of radiative transfer[16].

The radiative transfer problem is illustrated in Fig. 1. The plane parallel medium consists of several homogenous layers. The optical properties of each layer are described by a series of parameters, such as optical thickness τ, single scattering albedo ω, scattering phase function P, etc. The incident light could be any kind of distribution, and if it is an axially symmetrical distribution, the outgoing light will be axially symmetrical. The Fourier transforms of both the source and the outgoing light fields become Hankel transforms, and the MTF obtained from Eq. (4) is a function of the polar angle.

Numerical evaluation of Eq. (4) may encounter several technical difficulties if the distribution of incident light field is not properly chosen. However, if the incident light is chosen as an isotropic diffuse field, the OTF can be related directly to the outgoing field distribution and thus, evaluation of Eq. (4) becomes unnecessary. For an

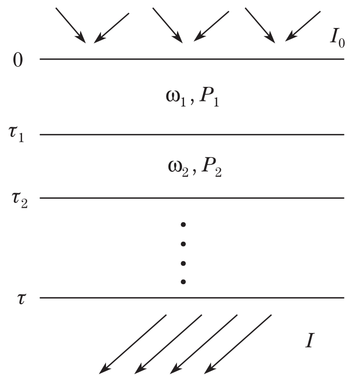

Fig. 1. Radiative transfer schematic for a plane parallel turbid medium.

isotropic incident source with unit intensity $I_0(\vec{\rho}\,') \equiv 1$, the image intensity can be expressed, based on Eq. (1), as

$$J(\vec{\rho}) = \int P\left(\vec{\rho} - \vec{\rho}\,'\right) \mathrm{d}\vec{\rho}\,'. \quad (5)$$

Since the PSF is the inverse Fourier transform of OTF,

$$P(\vec{\rho}) = \int O_{\mathrm{TF}}(\vec{\nu}) \exp\left(2\pi \mathrm{i}\vec{\nu}\cdot\vec{\rho}\right) \mathrm{d}\vec{\nu}. \quad (6)$$

Thus,

$$\begin{aligned} J(\vec{\rho}) &= \int P\left(\vec{\rho}\,' - \vec{\rho}\right) \mathrm{d}\vec{\rho}\,' \\ &= \int\int O_{\mathrm{TF}}(\vec{\nu}) \exp\left[-2\pi \mathrm{i}\vec{\nu}\cdot\left(\vec{\rho}\,' - \vec{\rho}\right)\right] \mathrm{d}\vec{\nu}\mathrm{d}\vec{\rho}\,'. \quad (7) \end{aligned}$$

On the other hand, from the definition of Dirac delta function, we derive

$$\begin{aligned} J(\vec{\rho}) &= \int J(\vec{\rho}\,')\delta\left(\vec{\rho}\,' - \vec{\rho}\right) \mathrm{d}\vec{\rho}\,' \\ &= \int\int J(\vec{\rho}\,') \exp\left[-2\pi \mathrm{i}\vec{\nu}\cdot\left(\vec{\rho}\,' - \vec{\rho}\right)\right] \mathrm{d}\vec{\nu}\mathrm{d}\vec{\rho}\,'. \quad (8) \end{aligned}$$

Comparing the two equations above, we obtain directly

$$O_{\mathrm{TF}}(\vec{\nu}) = J(\vec{\rho}\,'), \quad (9)$$

where the variable $\vec{\nu}$ takes an identical value of the variable $\vec{\rho}\,'$. Since $\vec{\nu}$ and $\vec{\rho}\,'$ are quantities in spatial frequency domain and spatial domain, respectively, they could take identical values only if they are unit-less. In this letter, it is to our advantage that we are dealing with a problem of imaging at infinite place with an isotropic incident source at infinity. In this case, the absolute positions in both the object and image planes are unnecessary and an angular representation could be used, i.e., $\vec{\rho}$ can be replaced by $\tan\theta$, where θ is the polar angle of $\vec{\rho}$, and the spatial frequency $\vec{\nu}$ can be replaced by the angular spatial frequency Ω. Since $\tan\theta$ is unit-less, the above requirement will be satisfied. Therefore, we have

$$O_{\mathrm{TF}}(\Omega) = J(\tan\theta). \quad (10)$$

In the small angle limit $\tan\theta \sim \theta$, when θ is taken the radian as its unit, the OTF takes 1/rad as its unit. Equation (9) or (10) is just the equivalence of MTF of a turbid medium in relation to the transmitted radiance from the medium with isotropic diffuse illumination.

Using this equivalence, full MTF in the whole angular spatial frequency range can be obtained through numerical solution of the radiative transfer problem. Thus, MTF of a single homogenous turbid layer is analyzed for illustration. The radiative transfer problem will be solved by the discrete ordinate method via the DISORT algorithm[17]. A Henyey-Greenstein phase function model $P_{\mathrm{HG}}(\Theta) = (1 - g^2)/(1 + g^2 - 2g\cos\Theta)^{3/2}$ with an adjustable asymmetric factor g is assigned to the turbid medium, where Θ is the scattering angle. The smaller the value of g is, the more scattered light intensity will be in the side directions. This model has often been used to simulate scattering phase function with a certain degree of scattering asymmetry.

Several general features of MTF are reflected in the following figures. The variations of MTFs for the optical thickness τ from 10^{-3} to 1 are plotted in Fig. 2, and the asymmetric factor g is set as 0.99. In Fig. 2(a), MTFs are plotted in a wide angular frequency interval and in Fig. 2(b), in a narrower interval (0, 100). For a relatively small optical thickness $\tau = 10^{-3}$, MTF is close to unity at lower frequency, and decreases with frequency until a critical frequency of approximately 5 000 1/rad is achieved. Beyond the critical frequency, the MTF is almost constant. Thus, only the behavior of MTF below the critical frequency is important. The larger the optical thickness is, the lower the critical frequency and the steeper the tendency to descend of the MTF with frequency below the critical value. In the case of the optical thickness being greater than 0.01, the critical frequency will be several ten 1/rad.

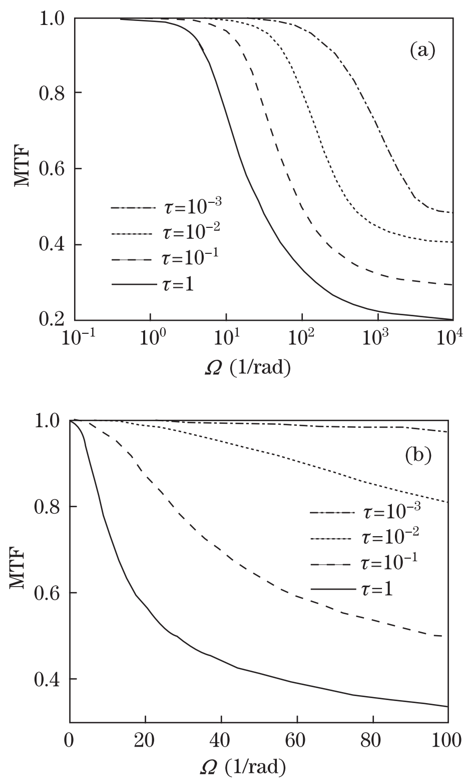

Fig. 2. MTF of a homogenous turbid medium with different optical thicknesses.

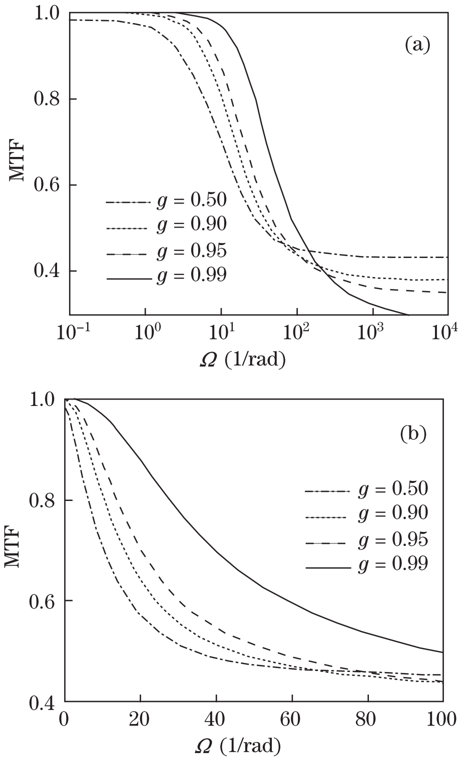

Fig. 3. MTF of a homogenous turbid medium with different asymmetric factors. Optical thickness is 0.1.

MTFs for four values of asymmetric factor g at the optical thickness $\tau = 0.1$ are plotted in Fig. 3. In Fig. 3(a), the MTFs are plotted in a wide angular frequency interval and in Fig. 3(b), in a narrower interval (0, 100). The effect of the scattering asymmetry produces different behaviors at lower and higher frequencies. Below the critical frequency, the smaller the asymmetric factor is, the steeper the tendency to descend of MTF in relation to frequency. Above the critical frequency, the smaller the asymmetric factor, the larger the value of MTF.

Although these results may provide important features about the MTF of a turbid medium, the application of the equivalence principle and the RTE computational procedure to the scenario directly to obtain the MTF for more complicated turbid media remains a better option. A practical medium may consist of a number of layers, or even has a reflecting bottom, which could be encountered in remote sensing in the atmosphere of the earth. However, the efficiency of the DISORT algorithm facilitates the acquisition of the MTF of any complicated medium particularly if quantitative optical information about the scattering particles are available.

In conclusion, based on the MTF of a turbid medium reported in this letter, and the theoretical solution of the MTF of a turbulent medium obtained previously[18], the MTFs of random media, both turbulent and turbid are completely solved quantitatively. Application of adaptive optics on turbulence compensation has achieved considerable progress[19,20]. However, no counterpart technology for the turbid media exists. The general MTF feature of the turbid medium may provide useful reference for developing a new kind of technology.

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