## Step-size selection for split-step based nonlinear compensation with coherent detection in 112-Gb/s 16-QAM transmission

Chien-Yu Lin<sup>1,2\*</sup>, Rameez Asif<sup>1,2</sup>, Michael Holtmannspoetter<sup>1,2</sup>, and Bernhard Schmauss<sup>1,2</sup>

<sup>1</sup>University of Erlangen-Nuremberg, Chair for High Frequency Technology, Cauerstrasse 9, D-91058 Erlangen, Germany <sup>2</sup>Erlangen Graduate School in Advanced Optical Technologies (SAOT), Paul-Gordan-Strasse 6, D-91052 Erlangen, Germany \*Corresponding author: sichilin@lhft.eei.uni-erlangen.de

Received October 12, 2011; accepted December 12, 2011; posted online January 12, 2012

Non-uniform step-size distribution is implemented for split-step based nonlinear compensation in singlechannel 112-Gb/s 16 quadrature amplitude modulation (QAM) transmission. Numerical simulations of the system including a  $20 \times 80$  km uncompensated link are performed using logarithmic step size distribution to compensate signal distortions. 50% of reduction in number of steps with respect to using constant step sizes is observed. The performance is further improved by optimizing nonlinear calculating position (NLCP) in case of using constant step sizes while NLCP optimization becomes unnecessary when using logarithmic step sizes, which reduces the computational effort due to uniformly distributed nonlinear phase for all successive steps.

OCIS codes: 060.1660, 060.5060, 060.4370. doi: 10.3788/COL201210.020605.

Nonlinear compensation by using digital signal processing (DSP) has become an active research topic for longhaul high-speed coherent transmission systems [1-3]. In a system with uncompensated transmission link, simple one-stage nonlinear compensation is not sufficient and the interaction of fiber dispersion and nonlinearity needs to be considered very carefully in digital processing algorithms. This can be effectively achieved by numerically propagating the received distorted signal through a virtual fiber with negative attenuation, dispersion and nonlinearity coefficients, i.e. by solving an inverse nonlinear Schrödinger equation (NLSE) using the split-step Fourier method (SSFM). The amplitude and phase of the transmitted optical signal thus can be reconstructed at the receiver side. This inverse process is often called digital backward propagation (DBP), which has shown a great potential in nonlinear compensation and has been applied to various optical transmission systems<sup>[4]</sup>. Recent investigations even focus on improving the computational efficiency of DBP by shifting the nonlin-earity calculating position  $(NLCP)^{[5,6]}$ , filtering<sup>[7,8]</sup> or employing a weighted averaging operation in the non-linear operator<sup>[9]</sup>. Results in Refs. [7,9] have proven to allow for combining subsequent spans in one split step. In such cases, the step size remains the same and can be increased up to 4 times the fiber span length.

On the other hand, step-size distribution plays a very important role in solving NLSE when modeling the signal propagation in fibers<sup>[10]</sup>. Alternatively non-uniform step-size distribution, where the step size decreases as power increases, has also been proposed to enhance the accuracy in estimation of signal distortions compared with constant step-size distribution. Especially in wavelength-division multiplexing (WDM) systems, Bosco *et al.*<sup>[11,12]</sup> used logarithmic step sizes for SSFM in the forward propagating simulations and the artifacts caused by numerical simulations was successfully suppressed. Our previous contribution has implemented logarithmic step sizes for DBP compensation in 1.12-Tb/s dual-polarization quadrature phase shift keying (DP-QPSK) WDM transmission<sup>[13]</sup>. The results show 1-dB improvement in Q by just applying logarithmic step sizes without optimizing NLCP and transmission parameters.

In this letter, we further analyze the influence of shifting NLCP on the performance of logarithmic step-size based DBP. By numerical simulations, a logarithmic distribution of step sizes is applied in a single-channel 16 quadrature amplitude nodulation (QAM) system with bit rate of 112 Gb/s over a  $20 \times 80$  km link of standard single mode fiber (SSMF) without in-line dispersion compensation. In order to enhance the accuracy in compensating nonlinearity, one DBP stage has been used to compensate for one propagation span. Each DBP stage includes at least two SSFM steps for implementing logarithmic step-size distribution but only one SSFM step for constant step-size distribution. The nonlinearity is calculated at different positions in each successive SSFM step, which can be modeled as symmetric, asymmetric, and the modified [6] schemes. The results of using both logarithmic and constant step sizes regarding variation in NLCP are compared, showing the corresponding potential to improve DBP performance.

In our investigation, numerical simulations of a singlechannel 16-QAM transmission system with bit rate of 112 Gb/s (28 Gbaud) have been performed with commercial software. This achieves to 100-Gb/s net data rate when a forward-error-correction (FEC) overhead of 7% and Ethernet overhead of 4% are used. At the receiver, an ideal laser source as local oscillator combined with an optical hybrid is used to convert the received optical signal down to base-band electrical signal and separate the I and Q components. Figure 1(a) shows the structure of applied homodyne coherent receiver including a DSP module which is implemented in matlab environment. The base-band electrical signal was sampled at 2 symbol rate by analog-to-digital converters (ADCs) and laser phase noise was neglected. In the DSP module, the digitized signal was firstly compensated by 20 DBP stages which was equal to the number of propagation spans during fiber transmission. As the signal distortion is fully compensated by DBP compensation, no extra finite-impulse-response (FIR) filter is needed for chromatic compensation. Each DBP stage can be performed with changing step-size distribution as well as varying NLCP by adapting the SSFM algorithms. Figure 1(b) illustrates different SSFM algorithms used in this study, taking an example of using 4 split steps in one DBP stage to compensate for one transmission span. In the constant step-size scheme, step size remains the same for all steps, while in the logarithmic step-size scheme, step-size decreases with increasing power. This basic principle is well known from the adaptive step-size methods for implementing signal propagation in optical fibers. Throughout this study we used logarithmic stepsize distribution according to Ref. [12].

The solid arrows in Fig. 1(b) depict the positions for calculating the nonlinear phase. In the symmetric scheme, NLCP is located in the middle of each step. In the asymmetric scheme, NLCP is located at the end of each step. In the modified scheme, NLCP is shifted between the middle and the end of each step and the position is optimized to achieve the best performance<sup>[6]</sup>. In all schemes, the nonlinear phase was calculated by  $\phi_{\rm NL}$  $= \gamma_{\text{DBP}} \cdot P \cdot L_{\text{eff}}$ , where the nonlinear coefficient for DBP  $\gamma_{\text{DBP}}$  was optimized to obtain the best performance and the nonlinear step size was determined by the effective length  $L_{\rm eff}$  of each step. All the algorithms were implemented for DBP compensation to recover the signal distortion in a single-channel 16-QAM transmission system with bit rate of 112 Gb/s. In this simulation model, we used a  $20 \times 80$  single mode fiber (SMF) link without any inline dispersion compensating fiber (DCF). SMF has the propagation parameters: attenuation coefficient  $\alpha = 0.2 \text{ dB/km}$ , dispersion coefficient  $D = 16 \text{ ps/(nm \cdot km)}$ and nonlinear coefficient  $\gamma_{\rm SMF}=1.2 \text{ km}^{-1} \text{W}^{-1}$ . The EDFA noise figure has been set to 4 dB.

Figure 2 compares the performance of all SSFM algo-



Fig. 1. (Color online) (a) Receiver structure with DBP compensation, and (b) schemes of SSFM algorithms for DBP compensation. S: symmetric-SSFM, A: asymmetric-SSFM, and M: modified-SSFM. The red-dashed curves show the power dependence along per-span length.



Fig. 2. BER of all SSFM algorithms with varying number of steps per DBP stage for (a) 3 and (b) 6 dBm.

rithms with varying number of steps per DBP stage. In our results, bit error ratio (BER), calculated from error vector magnitude (EVM) of received symbols<sup>[14]</sup>, was used for performance evaluation of received 16-QAM signals. Also various launch powers are compared. For both launch powers the logarithmic distribution of step sizes enables improved DBP compensation performance compared to using constant step sizes. This advantage arises especially at smaller number of steps (less than 8 steps per DBP stage with launch power=6 dBm). As the number of steps per DBP stage increases, BER stops decreasing and all the SSFM algorithms approach the minimum possible BER. Using logarithmic step sizes does not outperform the conventional methods when applying large number of steps per DBP stage. When smaller number of steps is used, for both logarithmic and constant step sizes, the modified SSFM scheme, which optimizes the NLCP, shows better performance than symmetric SSFM and asymmetric SSFM, where the NLCP is fixed. This coincides with the results in Ref. [6]. However, the improvement given by modified SSFM becomes less significant when logarithmic step sizes is used, which means the NLCP optimization reveals less importance and it is already sufficient to calculate the nonlinearity at the end of each step if logarithmic step sizes are used. On the other hand, at higher launch powers, the overall BER increases and the saturation of BER reduction happens toward larger number of steps.

Figure 3 shows the required number of steps per DBP stage to reach  $BER=10^{-3}$  at various launch powers for different SSFM algorithms. It is obvious that more steps are required for higher launch powers. Using logarithmic distribution of step sizes requires reduced number

of steps to reach a certain BER than using uniform distribution of step sizes. At launch power of 3 dBm, the use of logarithmic step sizes reduces 50% in number of steps per DBP stage with respect to using the asymmetric SSFM scheme with constant step sizes, and 33% in number of steps per DBP stage with respect to using the symmetric and modified SSFM schemes with constant step sizes. This improvement can be achieved based on minimizing the overall nonlinear phase shift by enlarging the step size at lower power to equalize the nonlinear phase calculated in every step along each complete DBP stage.

Uniformly-distributed nonlinear phase for all successive steps can be verified by multiplication of  $L_{\rm eff}$  and average power in each step resulting in a constant value (0.024 rad when 3-dBm launch power is applied). Figure 4 compares constellation diagrams of received 16-QAM signals at 3 dBm compensated by DBP with 2 steps per DBP stage, using constant step-size and logarithmic step-size distributions. In both cases the asymmetric SSFM has been applied.

In conclusion, we study logarithmic step sizes for DBP implementation and compare the performance with constant step sizes in a single-channel 16-QAM transmission



Fig. 3. Required number of steps per DBP stage at various launch powers for different SSFM algorithms.



Fig. 4. Constellation diagrams of received 16-QAM signals at 3 dBm after DBP compensation with (a) constant step-size and (b) logarithmic step-size methods.

system over a length of  $20 \times 80$  km at a bit rate of 112 Gb/s. The results reveal that use of logarithmic step sizes performs better than constant step sizes in case of applying the same number of steps, especially at smaller numbers of steps. Using logarithmic step sizes saves up to 50% in number of steps with respect to using constant step sizes. Besides, symmetric, asymmetric and modified SSFM schemes are applied for both logarithmic and constant step-size methods. By using logarithmic step sizes, the asymmetric scheme already performs nicely and optimizing nonlinear calculating position becomes less important in enhancing the DBP performance. Therefore the logarithmic step-size method is still a promising option in terms of improving DBP performance although more calculation efforts are needed compared with the existing multi-span DBP techniques where one DBP stage compensates for several propagation spans.

The authors gratefully acknowledge funding of the Erlangen Graduate School in Advanced Optical Technologies (SAOT) by the German National Science Foundation (DFG) in the framework of the excellence initiative.

## References

- K. P. Ho and J. M. Kahn, IEEE Photon. Technol. Lett. 22, 779 (2004).
- K. Kikuchi, M. Fukase, and S.-Y. Kim, in *Proceedings of* OFC 2007 OTuA2 (2007).
- J. C. Geyer, C. R. S. Fludger, T. Duthel, C. Schulien, and B. Schmauss, in *Proceedings of ECOC 2010* P3.02 (2010).
- D. Rafique, J. Zhao, and A. Ellis, Opt. Express **19**, 5219 (2011).
- D. S. Millar, S. Makovejs, C. Behrens, S. Hellerbrand, R. I. Killey, P. Bayvel, and S. J. Savory, J. Sel. Top. Quant. Electron. 16, 1217 (2010).
- C. Y. Lin, M. Holtmannspoetter, R. Asif, and B. Schmauss, in *Proceedings of ECOC 2010* P3.16 (2010).
- L. B. Du and A. J. Lowery, Opt. Express 18, 17075 (2010).
- E. Ip, N. Bai, and T. Wang, in *Proceedings of OFC 2011* OThF4 (2011).
- L. Li, Z. Tao, L. Dou, W. Yan, S. Oda, T. Tanimura, T. Hoshida, and J. C. Rasmussen, in *Proceedings of OFC* 2011 OWW3 (2011).
- O. V. Sinkin, R. Holzlohner, J. Zweck, and C. R. Menyuk, J. Lightwave Technol. 21, 61 (2003).
- G. Bosco, A. Carena, V. Curri, R. Gaudino, P. Poggiolini, and S. Benedetto, IEEE Photon. Technol. Lett. 12, 489 (2000).
- 12. M. Jaworski, J. Telecomm. Inf. Technol. 44 (2009).
- R. Asif, C. Y. Lin, M. Holtmannspoetter, and B. Schmauss, in *Proceedings of ICTON 2011* Tu.P.6 (2011).
- W. Webb and L. Hanzo, Modern Quadrature Amplitude Modulation (Pentech Press, London, 1994).