Characterizing topological charge of optical vortex using non-uniformly distributed multi-pinhole plate

Lei Shi (石　蕾), Linghao Tian (田凌浩), and Xianfeng Chen (陈险峰)*

Department of Physics, State Key Laboratory on Fiber Optic Local Area Communication Networks and Advanced Optical Communication Systems, Shanghai Jiao Tong University, Shanghai 200240, China

**Corresponding author: xfchen@sjtu.edu.cn*

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We propose an efficient method for characterizing the orbital angular momentum (OAM) of an optical vortex with a large topological charge (TC) through distinguishing the interference pattern of the non-uniformly-distributed multi-pinholes using fewer pinholes. This method overcomes the limit on large TC detection by multi-point interferometer and can be used to probe optical vortices with arbitrary sizes. In addition, it also has potential application in measuring light beam with OAM from astronomical sources.

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A light beam with a spatial distribution wave front, described by a phase cross section of $\exp(\mathrm{i}l\theta)$, where l is the topological charge (TC), carries orbital angular momentum (OAM) of $l\hbar$ per photon[1]. Considerable methods have been used to generate light beams with OAM. Some of these methods include the use of a pair of cylindrical lenses, spiral Fresnel zone plates, spiral phase plates, fork holograms, and some nonlinear optical processes[2−6]. A vortex beam with OAM has been widely used in various fields, ranging from optical manipulation to quantum information processing and cryptography to astronomical applications[7−12]. Phase distribution is required to achieve a better understanding of the vortex beam. Determining the TC value of light is important for the extensive applications and in understanding the nature of light. Various mechanisms have been proposed for characterizing the TC value of an optical vortex, such as the Shack-Hartmann wave front sensor[13] and the Mach–Zehnder interferometer with a Dove prism placed on each arm and double-slit interference[14,15]. Direct measurement of the phase in the visible regime is not possible; thus, a more commonly used method is to interfere the measured wave front with its mirror image or with a plane wave front, where the interference patterns reveal the TC value of the measured optical vortex[4,16]. A computer-generated hologram with a pinhole has also been used to determine TC[17]. However, such a hologram can only test one state at a time, and more sophisticated holograms have to be made for the test of multiple states[18]. In 2008, Berkhout *et al.* proposed a uniformly distributed multi-pinhole interferometer to probe the OAM state by distinguishing the interference pattern[19]. This method samples the phase information of a finite number of points in the field, thus, it can be used to characterize light beams with arbitrary size, which is especially suitable for charactering the TC of vortex beams with a large beam size after good propagation distance in astrophysics. In addition, the Fourier transform of the intensity pattern in a uniformly-distributed multi-pinhole interference also reveals the phase value at the pinholes from which the vortex TC can also be determined[20]. Several probing methods have also been formulated based on this theory[15,20−23], making it much more meaningful and fundamental. However, this method has a blind spot, wherein the interference patterns will repeat when the TC of a vortex is larger than the number of pinholes. The interference patterns of the optical vertices with different TCs possibly look the same, making it difficult to distinguish these vortices, nor is it possible to detect a vortex with large TC.

In this letter, a method to determine TC using a non-uniformly-distributed multi-pinhole plate (MHP) in a circle was proposed. The phase difference change between any two pinholes is different from the TC increment when the pinholes are non-uniformly distributed. As a result, the interference pattern of the vortex with non-uniformly-distributed multi-pinhole is different from that with uniformly-distributed multi-pinhole. The TC can be determined by distinguishing the interferograms. This method overcomes the limit of large TC detection from the number of pinholes.

Similar with Ref. [19], the Fraunhofer limit was considered in this letter. The far-field amplitude of the nth pinhole is related to $\exp(-\mathrm{i}l\alpha_n)\exp\left[\mathrm{i}\frac{ka}{z}(x\cos\alpha_n+y\sin\alpha_n)\right]$. The intensity pattern I_l^N behind a MHP, which is illuminated by an on-axis, and the Laguerre-Guass beam with TC l, is given by the Fourier transform of the field distribution in the aperture plane as

$$I_l^N \propto \left|\sum_{n=0}^{N-1}\exp(-\mathrm{i}l\alpha_n)\exp\left[\mathrm{i}\frac{ka}{z}(x\cos\alpha_n+y\sin\alpha_n)\right]\right|^2, \quad (1)$$

where $k = 2\pi/\lambda$ is the wave number and α_n is the azimuthal angle of the nth pinhole. The azimuth of the pinholes is represented as $\alpha_n = 2\pi n/N$, where N is the total number of pinholes, when the pinholes are uniformly distributed in a circle. Berkhout *et al.*[19] used this kind of MHP to determine the TC of the vortex beam by distinguishing the interferograms. However, the interferograms will repeat when the TC is larger than the number of pinholes. Far-field intensity patterns behind a uniformly-distributed MHP with six and seven pinholes and illuminated by optical vortices with different TCs are shown in Fig. 1. These patterns are consistent with the result in Ref. [19]. The intensity patterns are the same when MHP with six pinholes are illuminated by a vortex beam with $l = 0$ and 6, which is the same for $N = 6$, $l = 1$, 7; $N = 7$, $l = 0$, 7; $N = 7$, $l = 1$, 8. The reason is that l can be expressed as $l = aN + b$, where $a = \mathrm{fix}(l/N)$ and $b = \mathrm{mod}(l, N)$, when the TC value l is larger than N. The far-field amplitude of the nth pinhole was $\exp(-\mathrm{i}2\pi a)\exp(-\mathrm{i}b\alpha_n)\exp\left[\mathrm{i}\frac{ka}{z}(x\cos\alpha_n + y\sin\alpha_n)\right]$, which was numerically the same with $\exp(-\mathrm{i}b\alpha_n)\exp\left[\mathrm{i}\frac{ka}{z}(x\cos\alpha_n + y\sin\alpha_n)\right]$. The intensity patterns I_l^N and I_b^N are therefore the same, making the pattern impossible to be distinguished using this method.

Our previous analysis shows that the interference patterns will repeat when the optical vortex TC is larger than the number of pinholes, making it difficult to distinguish the vortices with different TCs. However, this limitation can be overcome when the pinholes are non-uniformly-distributed in a circle. Figure 2 shows that the phase difference between any two pinholes is different for any vortex beam, making the factor $\exp(-\mathrm{i}l\alpha_n)$ aperiodic for different TCs and different pinhole numbers. The interference pattern is then unique for different TCs. As an example, for six pinholes, $\theta = \pi/11$ was chosen and

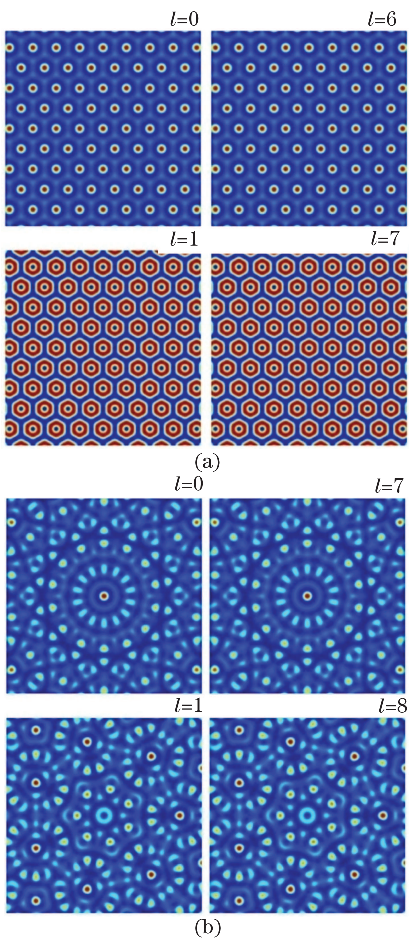

Fig. 1. (Color online) Far-field intensity patterns behind a uniformly-distributed MHP with six and seven pinholes. The MHP is illuminated by the optical vortex with (a) $l = 0$, 1, 6, 7 for six pinholes and (b) $l = 0$, 1, 7, 8 for seven pinholes.

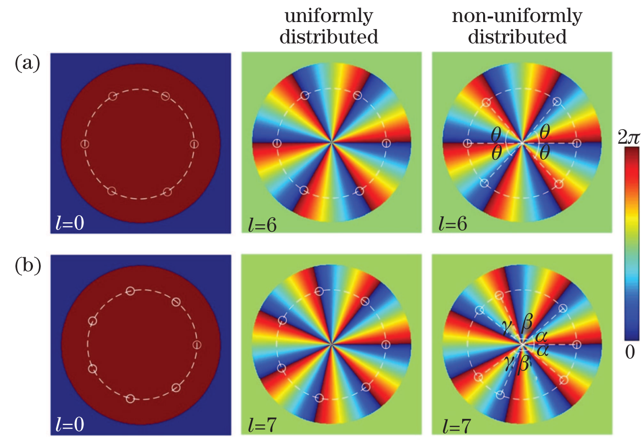

Fig. 2. (Color online) Light phase at each pinhole for MHP with (a) six and (b) seven pinholes uniformly and non-uniformlydistributed.

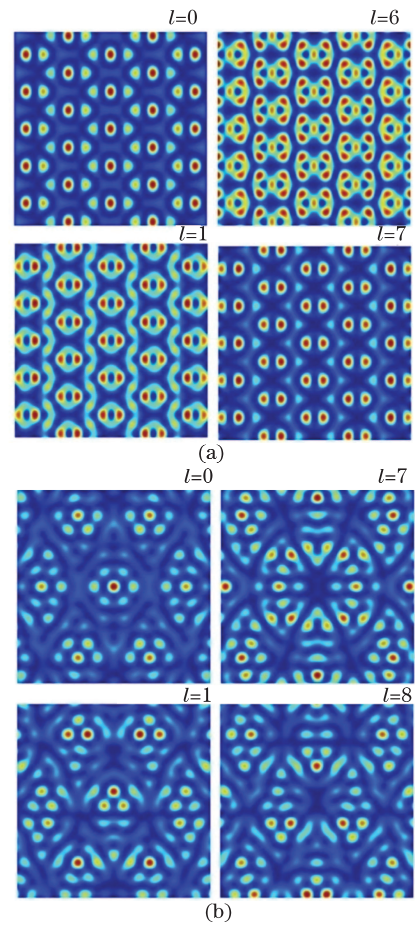

Fig. 3. (Color online) Far-field intensity patterns behind a non-uniformly distributed MHP with (a) six and (b) seven pinholes and illuminated by an optical vortex with different TCs.

$\alpha = \gamma = \pi/7$ and $\beta = 3\pi/7$ were chosen for the seven pinholes. The far-field intensity pattern changes with TC and the repetition of the interference patterns of light beams with topological differences is eliminated, as shown in Fig. 3. Larger differences exist among the interference patterns of different vertices when the pinholes are more non-uniformly-distributed. Therefore, the intensity patterns of optical vortices with different TCs are different, making it easy to characterize them from the interferograms.

The method used makes good use of the azimuthal property of the phase distribution of an optical vortex. It does not require an additional plane wave front to interfere nor does it require a large detection area. The method relies simply on the measurements of a finite number of points and can therefore be scaled to arbitrary size, making it applicable in astronomy, where the pinholes can be replaced by telescopes. Interest in light with OAM in astrophysics has grown in the last decade[12]. Several possible sources of OAM have been suggested, from bright point source behind a turbulent interstellar medium to the cosmic microwave background (CMB). Recently, Tamburiniet *et al.* predicted that light would be imprinted with OAM[24] when it passed Kerr black pinholes, which were massive rotating astrophysical objects predicted from general relativity theory of Einstein. These pinholes drag and intermix their surrounding space and time. In the diffraction-limited regime[25], our method would be an efficient way for the detection and measurement of the twisted light by using a few telescopes; thus, allowing a direct observational demonstration of the existence of rotating black holes.

In conclusion, a method for the determination of an optical vortex TC by distinguishing the far-field intensity patterns of a non-uniformly-distributed MHP is proposed. This method overcomes the interferogram repetitions and can be used to detect vortex beams with arbitrary size and arbitrary TC. Therefore, this method has potential applications in astrophysics for the OAM detection of light from the cosmos.

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