Precision position measurement of single atom

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Atom localization in a five-level atomic system under the effect of three driving fields and one standing wave field is suggested. A spontaneously emitted photon from the proposed system is measured in a detector. Precision position measurement of an atom is controlled via phase and vacuum field detuning without considering the parity violation.

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In recent years, several schemes have been proposed for the localization of an atom, such as the use of a standing optical light field[1,2]. In these schemes, the idea of the virtual optical slit was suggested for measuring the phase-shift of the optical field in a cavity. Earlier, several experiments were conducted on the basis of resonance imaging for the precise position of the moving atoms[3]. The magnetic field gradient[4] used in these experiments was able to determine a spatial resolution of 1.7 μm. This spatial resolution was then enhanced to 200 nm by using a light shift gradient[5,6] instead of the magnetic field gradient.

The study of atom localization has potential applications, such as in laser cooling and trapping of neutral atoms[7], atom nanolithography[8], etc. Researchers have investigated a lot of localization schemes, that depend on atomic coherence and quantum interference effect. These schemes include, for example, resonance fluorescence from a two-level system[9] and the measurement of the spontaneous emission[10,11]. A study of the case of a three-level atomic system[10] revealed that a spontaneously emitted photon carries information regarding the atom; thus four equally probable positions of a single atom could be observed by decreasing the vacuum field detuning $\delta_{\mathbf{k}}$.

In the last decade, a scheme consisting of a four-level atomic system interacting with the traveling and standing wave fields that was capable of observing four equally probable positions for a single atom was suggested[12]. Furthemore, the four equally probable positions were reduced by a factor of 2 for a single frequency measurement whenever the phase of the classical standing wave field was controlled. The authors observed that control of the amplitudes of the driving field provided a strong narrowing line that yielded a better resolution in position measurement of the single atom. In the scheme[12], a parity violation was considered, and a high field was required to break this violation.

In 2009, two different systems were used for the atom localizations[13,14] to observe single position measurement. In the system[13], a four-level Raman gain process was used for subwavlength atom localization and a single peak was observed for an atom. The other system[14] was basically dependent upon two-photon measurement of the position of a quantum particle in Λ- and M-type systems, and the same behavior was observed for single atom localization.

More recently, a scheme[15] was proposed for atom localization for a single position measurement of an atom interacting with two classical standing wave fields. In the scheme[15], a quantum coherence was generated in a three-level atomic system by a classical standing wave field coupled to the upper excited two levels. In this system, the fluorescence spectrum was controlled via the phase of the driving field. The control of the fluorescence spectrum led to reduced localization peaks in the conditional position probability distibution. Initially, eight peaks per unit wavelength of the standing wave were observed in the conditional position probability distribution. The number of peaks was reduced to one via a single controllable parameter i.e., phase φ of the standing wave.

In this letter, we demonstrate the subwavelenght localization of an atom in a five-level atomic system interacting with traveling and standing wave fields, as shown in Fig. 1. We control the position and width of the localization peaks by phase and vacuum field detuning. We observe two, as well as one, localization peaks for a single frequency measurement. In our proposed system, no parity violation exits, which is why experimentally our system is more viable than that proposed in Ref. [12].

We propose a five-level atomic configuration, as shown in Fig. 2. A five-level atom with energy-levels of $|a\rangle$, $|b\rangle$,

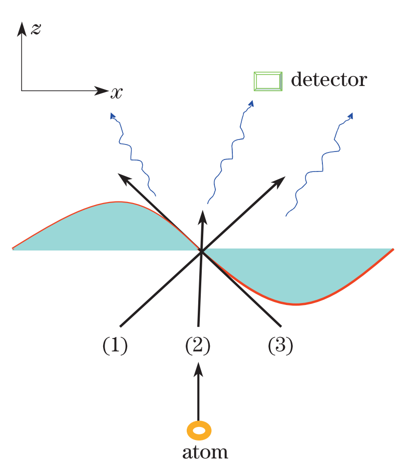

Fig. 1. Schematic of an atom interacting with four fields.

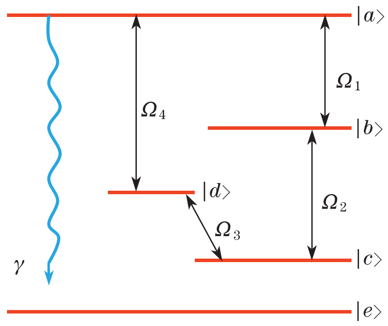

Fig. 2. Energy-level configuration.

$|c\rangle$, $|d\rangle$, and $|e\rangle$ passes through four classical driving fields as shown in Fig. 1. The atom decays from level $|a\rangle$ to level $|e\rangle$ with decay rate γ due to the fact that the atom interacts with the reservoir modes. The decaying energy-levels $|a\rangle$ couples with $|b\rangle$ and $|b\rangle$ with $|c\rangle$ through classical traveling wave fields having Rabi frequencies Ω_1 and Ω_2 respectively. At that instant, level $|a\rangle$ also couples with level $|d\rangle$ through a classical standing wave field having frequency ν and relative phase φ, the relative phase may be defined as $\varphi = \varphi_3 + \varphi_4 - \varphi_1 - \varphi_2$. The corresponding Rabi frequency between levels $|a\rangle$ and $|d\rangle$ is Ω_4. Also the energy-level $|d\rangle$ couples with $|c\rangle$ with corresponding Rabi frequency Ω_3. As the atom moves in the z-direction, during its motion, it interacts with the classical standing wave field. During the interaction of the atom with standing wave field, the corresponding Rabi frequency $\Omega_4(x)$ is position dependent i.e., $\Omega_4(x)$=$\Omega_4\sin(kx)$, where k is the wave vector of the standing wave field.

The interaction Hamiltonian for the resonant atomic system is written as

$$\begin{aligned} V = \hbar\Big[&\Omega_1 \mathrm{e}^{\mathrm{i}kx\cos\theta_1}|a\rangle\langle b| + \Omega_2 \mathrm{e}^{\mathrm{i}kx\cos\theta_2}|b\rangle\langle c| \\ &+ \Omega_3 \mathrm{e}^{\mathrm{i}kx\cos\theta_3}|d\rangle\langle c| + \Omega_4(x)\mathrm{e}^{\mathrm{i}\varphi}|a\rangle\langle d| \\ &+ \sum_{\mathbf{k}} g_{\mathbf{k}}(x)\mathrm{e}^{\mathrm{i}\delta_{\mathbf{k}}t}|a\rangle\langle \mathrm{e}|\mathrm{b}_{\mathbf{k}} + H.c\Big], \end{aligned} \tag{1}$$

where $g_{\mathbf{k}}(x)$ is the coupling constant linked to the spontaneously emitted photon, and $b_{\mathbf{k}}$ is the annihilation operator. Here, we define the vacuum field detuning as

$$\delta_{\mathbf{k}} = \omega_{ae} - \nu_{\mathbf{k}}, \tag{2}$$

where ω_{ae} is the transition frequency between levels $|a\rangle$ and $|e\rangle$ whereas $\nu_{\mathbf{k}}$ is frequency of the spontaneously emitted photon in the reservoir mode $\mathbf{k}$.

Now, we may write the complete atom-field state vector as

$$\begin{aligned} |\Psi(x;t)\rangle = \int \mathrm{d}x\Big\{&f(x)|x\rangle[A_{a,0}(x;t)|a,0\rangle \\ &+ A_{b,0}(x;t)|b,0\rangle] + A_{c,0}(x;t)|c,0\rangle \\ &+ A_{d,0}(x;t)|d,0\rangle + \sum_{\mathbf{k}} A_{e,1_{\mathbf{k}}}(x;t)|e,1_{\mathbf{k}}\rangle]\Big\}, \end{aligned}$$

where $A_{j,0}(x;t)$ represents the probability amplitudes for levels $|a\rangle$, $|b\rangle$, $|c\rangle$, and $|d\rangle$ having no photon present in the reservoir mode k; $A_{e,1_{\mathbf{k}}}(x;t)$ is the probability amplitude for level $|e\rangle$ having one photon emitted spontaneously in the reservoir mode k when the atom is in level $|e\rangle$; $f(x)$ is the centre of the mass wave function of the atom.

Because our system is associated with the position dependent atom-field interaction, the spontaneously emitted photon carries the information about the centre-of-mass motion of the atom. Therefore, the position measurement of the atom is conditioned by finding the spontaneously emitted photon. Thus, the conditional position probability distribution $W(x;t/e;1_{\mathbf{k}})$[10−12] may be defined as the probability of finding the atom at position x in the standing wave field given that a spontaneously emitted photon is detected at time t in the reservoir mode of wave vector k. This $W(x;t/e;1_{\mathbf{k}})$ can be obtained by taking the appropriate projection over the atom field state vector, which can be written as

$$W(x;t/e;1_{\mathbf{k}}) = F(x;t/e;1_{\mathbf{k}})|f(x)|^2, \tag{3}$$

where $F(x;t/e;1_{\mathbf{k}})$ is the filter function and may be defined as

$$F(x;t/e;1_{\mathbf{k}}) = |N|^2|A_{e,1_{\mathbf{k}}}(x;t)|^2, \tag{4}$$

where N is the normalization factor.

Equations (3) and (4) show that $W(x;t/e;1_{\mathbf{k}})$ depends on the probability amplitude $A_{e,1_{\mathbf{k}}}(x;t)$; thus, we should require this probability amplitude. To find the required probability amplitude $A_{e,1_{\mathbf{k}}}(x;t)$, we solve the Schrodinger wave equation by using Eq. (1). We calculate the following equation of motion using the probability amplitude method as

$$\begin{aligned} \dot{A}_{a,0} = -\,\mathrm{i}\Big[&\Omega_1 \mathrm{e}^{\mathrm{i}kx\cos\theta_1}A_{b,0} + \Omega_4 \mathrm{e}^{\mathrm{i}\varphi}\sin(kx)A_{d,0} \\ &+ g_{\mathbf{k}}(x)\sum_{\mathbf{k}} \mathrm{e}^{\mathrm{i}\delta_{\mathbf{k}}t}A_{e,1_{\mathbf{k}}}\Big], \end{aligned} \tag{5}$$

$$\dot{A}_{b,0} = -\,\mathrm{i}[\Omega_1 \mathrm{e}^{-\mathrm{i}kx\cos\theta_1}A_{a,0} + \Omega_2 \mathrm{e}^{\mathrm{i}kx\cos\theta_2}A_{c,0}], \tag{6}$$

$$\dot{A}_{c,0} = -\,\mathrm{i}[\Omega_2 \mathrm{e}^{-\mathrm{i}kx\cos\theta_2}A_{b,0} + \Omega_3 \mathrm{e}^{-\mathrm{i}kx\cos\theta_3}A_{d,0}], \tag{7}$$

$$\dot{A}_{d,0} = -\,\mathrm{i}[\Omega_3 \mathrm{e}^{\mathrm{i}kx\cos\theta_3}A_{c,0} + \Omega_4 \mathrm{e}^{-\mathrm{i}\varphi}\sin(kx)A_{a,0}], \tag{8}$$

$$\dot{A}_{e,1_{\mathbf{k}}} = -\,\mathrm{i}[g^*_{\mathbf{k}}(x)\mathrm{e}^{-\mathrm{i}\delta_{\mathbf{k}}t}A_{a,0}]. \tag{9}$$

Following the Laplace transform method and Weisskopf-wigner approximation, we calculate $A_{e,1_{\mathbf{k}}}(x;t)$ from Eqs. (5)–(9), considering that the atom is prepared initially in the level $|c\rangle$, such that only the probability amplitude is $A_{c,1_{\mathbf{k}}}(0)$=1. We follow the same method in Ref. [12] to calculate $A_{e,1_{\mathbf{k}}}(x;t)$ for a very large time, which means that the interaction time is significantly larger than the atomic decay rate γ as

$$A_{e,1_{\mathbf{k}}}(t \longrightarrow \infty) = -\mathrm{i}g^*_{\mathbf{k}}(x)(C/D), \tag{10}$$

where

$$C = -\,\mathrm{i}\delta_{\mathbf{k}}[\Omega_1\Omega_2 \mathrm{e}^{\mathrm{i}kx(\cos\theta_1+\cos\theta_2)} + \Omega_3\Omega_4 \mathrm{e}^{\mathrm{i}kx\cos\theta_3}\sin kx],$$

$$\begin{aligned} D = \delta^4_{\mathbf{k}} &- \delta^2_{\mathbf{k}}(\Omega_2^2 + \Omega_3^2) - \mathrm{i}\gamma\delta^3_{\mathbf{k}}/2 + \mathrm{i}\gamma\delta_{\mathbf{k}}(\Omega_2^2 + \Omega_3^2)/2 \\ &+ \Omega_1^2(\Omega_3^2 - \delta^2_{\mathbf{k}}) - \Omega_1\Omega_2\Omega_3\Omega_4\sin kx \\ &\cdot[\mathrm{e}^{\mathrm{i}kx(\cos\theta_1+\cos\theta_2-\cos\theta_3)-\mathrm{i}\varphi} + \mathrm{e}^{-\mathrm{i}kx(\cos\theta_1+\cos\theta_2-\cos\theta_3)+\mathrm{i}\varphi}] \\ &+ \Omega_4^2\sin^2 kx(\Omega_2^2 - \delta^2_{\mathbf{k}}), \end{aligned}$$

and $\theta_1 = \pi/4$, $\theta_2 = \pi/2 + \theta_1$, and $\theta_3 = \pi/2$. Now, the probability of finding the atom in the level $|e\rangle$ along with the spontaneous emission is given by

$$|A_{e,1_{\mathbf{k}}}(t \longrightarrow \infty)|^2 = |G_k|^2|C/D|^2. \tag{11}$$

We place Eq. (11) into Eq. (4), and find the filter function proportional to $W(x)$ to be

$$F(x;t/e;1_{\mathbf{k}}) = |N|^2|G_{\mathbf{k}}|^2 \frac{\delta_{\mathbf{k}}^2 \Omega_3^2 \Omega_4^2 (\sin kx + S_1)(\sin kx + S_2)}{\gamma^2/4\delta_{\mathbf{k}}^2(\Omega_2^2+\Omega_3^2-\delta_{\mathbf{k}}^2)^2 + \Omega_4^4(\Omega_2^2-\delta_{\mathbf{k}}^2)^2(\sin kx - X_1)^2(\sin kx - X_2)^2}, \tag{12}$$

with $S_1 = \frac{\Omega_1\Omega_2}{\Omega_3\Omega_4}\mathrm{e}^{-\mathrm{i}\varphi}$, $S_2 = \frac{\Omega_1\Omega_2}{\Omega_3\Omega_4}\mathrm{e}^{\mathrm{i}\varphi}$, and

$$X_1 = \frac{\Omega_1\Omega_2\Omega_3\cos\varphi + \sqrt{\Omega_1^2\Omega_2^2\Omega_3^2\cos^2\varphi - (\Omega_2^2-\delta_{\mathbf{k}}^2)(\delta_{\mathbf{k}}^2-\Omega_1^2-\Omega_2^2-\Omega_3^2+\Omega_1^2\Omega_3^2)}}{\Omega_4(\Omega_2^2-\delta_{\mathbf{k}}^2)}, \tag{13}$$

$$X_2 = \frac{\Omega_1\Omega_2\Omega_3\cos\varphi - \sqrt{\Omega_1^2\Omega_2^2\Omega_3^2\cos^2\varphi - (\Omega_2^2-\delta_{\mathbf{k}}^2)(\delta_{\mathbf{k}}^2-\Omega_1^2-\Omega_2^2-\Omega_3^2+\Omega_1^2\Omega_3^2)}}{\Omega_4(\Omega_2^2-\delta_{\mathbf{k}}^2)}. \tag{14}$$

We begin our discussion by considering a five-level atomic system interacting with four classical fields. The three classical fields are considered as traveling waves and are mutually perpendicular to one another, as labelled by (1), (2) and (3) in Fig. 1. The fourth field is the classical standing wave field aligned in the x-axis, as shown in Fig. 1. Here we consider the initial position distribution of the atom, for example $|f(x)|^2$, is constant in many wavelengths of the standing wave field. Equation (12) shows that the filter function $F(x;t/e;1_{\mathbf{k}})$ depends upon the spontaneously emitted photon $\nu_{\mathbf{k}}$, which is proportional to $\delta_{\mathbf{k}}$, as shown in Eq. (2). The filter function also depends upon the amplitudes and phase of the driving fields. From the analysis of the filter function, clearly the peak maxima occurs when $\sin kx = X_1$ or $\sin kx = X_2$. This phenomenon means that, when $\Omega_1=\Omega_2=\Omega_3=\Omega$ and $\varphi = \pi/2$, we obtain peak maxima in the filter function when $\delta_{\mathbf{k}}$ satisfies the following condition

$$\delta_{\mathbf{k}} = \pm\sqrt{\frac{\Omega^2\Omega_4^2\sin^2 kx - 3\Omega^2 + \Omega^4}{\Omega_4^2\sin^2 kx - 1}}. \tag{15}$$

Now, the frequency of the spontaneously emitted photon $\nu_{\mathbf{k}}$ from level $|a\rangle$ to level $|e\rangle$ is

$$\nu_{\mathbf{k}} = \omega_{ae} \mp \sqrt{\frac{\Omega^2\Omega_4^2\sin^2 kx - 3\Omega^2 + \Omega^4}{\Omega_4^2\sin^2 kx - 1}}. \tag{16}$$

Clearly, if we measure the spontaneously emitted photon, we find the conditional position probability distribution $W(x)$ of the atom inside the standing wave field. Then, the peaks are located at the following normalized position in the conditional probability distribution as

$$kx = \pm\sin^{-1}\sqrt{\frac{\delta_{\mathbf{k}}^2 - 3\Omega^2 + \Omega^4}{\Omega_4^2}} + n\pi, \tag{17}$$

where n is an integer having the values 0, 1, 2, $\cdots$.

We plot the conditional position probability distribution $W(x)$ versus the normalized position kx for different phase values of the standing wave field φ, see Fig. 3. We observe that how the phase of the standing wave field affects the precise position measurement of the spontaneous emission. when we set the phase of the standing wave fields at π and 0, the localization peaks reduce in the conditional position probability distribution $W(x)$, as shown in Fig. 3. For phase $\varphi = \pi$ of the standing wave field, we observe a single localization peak in the conditional position probability distribution $W(x)$ at left side, as shown in Fig. 3(a). It is due to the fact that

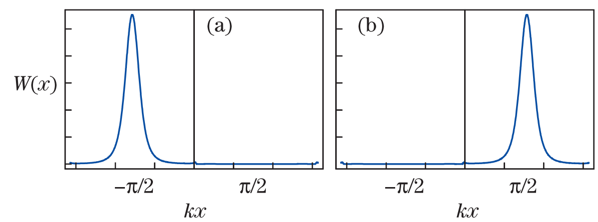

Fig. 3. $W(x)$ verses normalized position kx ($\Omega_1=\Omega_2=\Omega_3=1\gamma$, $\delta_{\mathbf{k}}=0.9\gamma$, and $\Omega_4=10.5\gamma$) with (a) $\varphi=\pi$ and (b) $\varphi=0$.

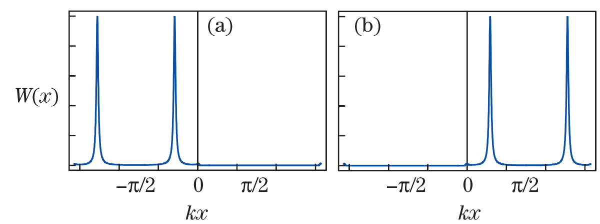

Fig. 4. $W(x)$ verses normalized position kx ($\Omega_1=\Omega_2=\Omega_3=1\gamma$, $\delta_{\mathbf{k}}=0.8\gamma$, and $\Omega_4=10.5\gamma$) with (a) $\varphi=\pi$ and (b) $\varphi=0$.

a quenching occurs in the spontaneous emission in the normalized position kx ranging for $0 \longrightarrow \pi$. Similarily to the control of the localization peak, we observe when we set the phase of the standing wave field $\varphi = 0$, a single localization peak in the conditional position probability distribution $W(x)$ at right side and a quenching occurs in the left side, see Fig. 3(b).

In Fig. 3, we see that the quenching in the spontaneous emission contributes to enhancing the precision position measurement of a single atom by controlling the phase of the standing wave field. We obtain a single peak in the conditional position probability distribution $W(x)$. A similar single peak was observed in three-level system by Ghafoor et al.[15], however, the position of an atom was imprecise. Furthermore, to enhance the precision in the position of an atom, we decrease $\delta_{\mathbf{k}}$ from 0.9γ to 0.8γ. The observed single peak splits into two peaks and we obtain two precise positions of a single atom at phases π and 0, see Fig. 4. We note that the position of the maxima in the conditional position probability distribution strongly depends on the value of the detuning $\delta_{\mathbf{k}}$.

In conclusion, we suggest a scheme for atom localization. Our scheme is based on the spontaneously emitted photon that carries the information of an atom. The spontaneously emitted photon localizes the atom in real time by measuring the frequency. We control the localization peaks by phase of the standing wave field and vacuum field detuning thereby achieving precise position of an atom without parity violation.

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