Theoretical investigations of impulsive synchronization on semiconductor laser chaotic systems

Mengfan Cheng (程孟凡) and Hanping Hu (胡汉平)*

Institute for Pattern Recognition and Artificial Intelligence, Huazhong University of Science and Technology, Wuhan 430074, China

**Corresponding author: hphuhust@qq.com*

Received February 27, 2012; accepted April 17, 2012; posted online August 3, 2012

The impulsive synchronization problem for semiconductor laser (SL) chaotic systems is studied. SL systems can be described by the rate equations governing photon density and carrier density. Because the carrier density is not easy to observe or measure, only photon density is used to design the impulsive controller. Several sufficient conditions for the synchronization of SL chaotic systems via impulsive control are derived. Numerical simulations are presented to show the effectiveness of the proposed method.

OCIS codes: 190.3100, 140.1540, 140.5960.

doi: 10.3788/COL201210.101901.

Since the first experimental demonstration was reported in 1998[1−3], optical chaotic communication systems had attracted increasing interest. In recent years, chaos synchronization and secure communication of laser systems have also attracted considerable attention[4−11] due to their unique advantages, such as higher security, broader signal bandwidth, and greatly enhanced signal transmission capability.

Extensive research efforts have been concentrated on how to generate and control chaotic signal from laser systems[12−15]. For semiconductor lasers (SLs), various routes to chaos under different perturbation, such as current modulation[12], optical feedback[13], optical injection[14], and optoelectronic feedback[15], have been observed and investigated. In addition, chaos synchronization of SLs has become more important due to its applications to communication. Thus far, various synchronization and communication schemes for laser chaotic systems with different coupled modes have been studied by many researchers. Kouomou *et al.* investigated cluster synchronization of coupled current-modulated SLs[6]. Deng *et al.* investigated a bidirectional chaos-synchronization scheme of SLs with optoelectronic feedback[7]. Xia *et al.* investigated cascaded chaotic synchronization and communication based on optoelectronic negative feedback SLs[8]. Saha *et al.* studied the synchronization of two coupled single-mode Nd:YAG lasers[9]. Banerjee *et al.* explored the synchronization phenomena in spatiotemporal SL systems and proposed a colored-image-encryption scheme[10]. These methods all adopt the continuous chaotic synchronization scheme. To increase the efficiency of bandwidth usage, impulsive chaotic synchronization[16] has been proposed. In the synchronization process, the control signals are transmitted from the driving system to the driven system only at discrete time instants, thereby reducing the amount of information transmitted between the two systems. Various theoretical and experimental results of impulsive chaotic synchronization can be found in Refs. [16-18]. To the best of our knowledge, very few results on impulsive synchronization of SL chaotic systems have been published. For SL chaotic systems, the carrier density is not easy to observe or measure in the real world, which means not all states of the system equation are available. Therefore, the mentioned impulsive synchronization method cannot be used to synchronize SL chaotic systems.

In our work, the general case of SL chaotic system is considered, and an impulsive control scheme is investigated. The proposed synchronization method can be used to share an identical chaotic waveform as the carrier in both the transmitter and the receiver for analog signal concealment. It can also be used as a pseudo-random sequence generator for digital signal encoding and decoding by sampling and quantizing the synchronized chaotic signal in both sides.

Consider a drive-response synchronization scheme with the dynamics of the SL drive system described by the following rate equations governing the photon density (P) and carrier density (N):

$$\dot{x} = Ax + f(x(t)), \tag{1}$$

where $x = [N_1, P_1]^{\mathrm{T}}$, $A \in R^{2\times 2}$, and $f : R^2 \rightarrow R^2$ are nonlinear continuous functions with respect to its arguments. The response system is characterized by

$$\begin{cases} \dot{y} = Ay + f(y(t)), \quad t \neq t_i, \\ \Delta P_2 = P_2(t_i^+) - P_2(t_i^-) = P_2(t_i^+) - P_2(t_i) \\ \qquad = \mu(P_2 - P_1), \quad t = t_i \\ P_2(t_0^+) = P_2(t_0), \end{cases} \tag{2}$$

where $y = [N_2, P_2]^{\mathrm{T}}$, P_2 is left continuous at $t = t_i$, μ is a constant, and f is the same function as defined before. The impulse instant sequence $\{t_i\}$ satisfies $0< t_1 < t_2 < \cdots < t_i < \cdots$, $t_i \rightarrow \infty$ as $i \rightarrow \infty$. Note that the carrier density is not easy to observe or measure, and that only photon density is used to design the impulsive controller.

The nonlinear function f is assumed to be a Lipschitz function with respect to its argument

$$\parallel f(y(t)) - f(x(t)) \parallel \leqslant k \parallel y(t) - x(t) \parallel . \tag{3}$$

Thus, we have

$$f(y(t)) - f(x(t)) = K(x(t), y(t))(y(t) - x(t)), \tag{4}$$

where $K \in R^{2\times 2}$ is a bounded matrix ($||K|| \leqslant k$) with its elements depending on $x(t)$ and $y(t)$.

Defining the synchronization error as $e(t) = [e_N, e_P]^{\mathrm{T}} = y(t) - x(t)$, we can obtain the following error system:

$$\begin{cases} \dot{e} = Ae + f(y(t)) - f(x(t)), & t \neq t_{\mathrm{i}} \\ \Delta e = [0 \quad \mu e_P]^{\mathrm{T}}, & t = t_i \end{cases}. \tag{5}$$

From Eqs. (3)–(5), we can obtain

$$\begin{cases} \dot{e} = (A + K(x,y))e, & t \neq t_i \\ \Delta e = [0 \quad \mu e_P]^{\mathrm{T}}, & t = t_i \end{cases}. \tag{6}$$

In this letter, our goal is to find several conditions on the control gains μ, the impulse intervals $\tau_{i+1} = t_{i+1} - t_i < \infty$ $(i = 1, 2, \cdots)$ such that the response system is synchronized.

In what follows, we shall develop some stabilizable sufficient conditions for the error system of the SL chaotic systems.

Lemma 1[19]: If the following impulsive differential inequality is satisfied as

$$\begin{cases} \dot{z}(t) \leqslant pz(t) + q(t), & t \neq t_i, \quad t \geqslant t_0 \\ z(t_i^+) \leqslant d_i z(t_i), & t = t_i, \quad i = 1, 2, \cdots \end{cases},$$

where $z(t)$ belong to the set PC(l), PC(l)=$\{\varphi : [0, \infty) \to R^{\mathrm{l}}$, $\varphi(t)$ is continuous everywhere except for the finite number of points t_i at which $\varphi(t_i) = \varphi(t_i^-)$ and $\varphi(t_i^+)$ exist$\}$. Here p, $d_i \in R$ and $q \in C(R_+, R)$. Then

$$z(t) \leqslant z(t_0) \prod_{t_0<t_i<t} d_i e^{p(t-t_0)} + \int_{t_0}^{t} \prod_{t_0<t_i<t} d_i e^{p(t-t_0)} q(s)\mathrm{d}s, \quad t \geqslant t_0.$$

Lemma 2[19]: If the following integral inequality is satisfied as

$$z(t) \leqslant h(t) + \int_{t_0}^{t} p(s)z(s)\mathrm{d}s, \quad t \geqslant t_0,$$

where h, $z \in$PC(R_+, R) and $d_i \in R$, $p \in$C(R_+, R_+), then,

$$z(t) \leqslant h(t) + \int_{t_0}^{t} \exp \int_{s}^{t} p(\sigma)\mathrm{d}\sigma p(s)h(s)\mathrm{d}s, \quad t \geqslant t_0.$$

Theorem 1: If positive scalars λ_1, λ_2, δ_1, and δ_2 exists, such that the following inequalities hold:

$$\begin{bmatrix} A_{11} + K_{11} + \lambda_1 & 0.5(A_{12} + K_{12}) \\ * & -\delta_1 \end{bmatrix} < 0, \tag{7}$$

$$\begin{bmatrix} -\lambda_2 & 0.5(A_{21} + K_{21}) \\ * & A_{22} + K_{22} - \delta_2 \end{bmatrix} < 0, \tag{8}$$

$$\frac{4\lambda_2\delta_1}{\omega\left(\frac{\ln(1/\omega)}{\beta} - 2\delta_2 - 2\lambda_1\right)} - 2\lambda_1 < 0, \tag{9}$$

where $\omega = (1 + \mu)^2$ and $\tau_i \leqslant \beta$, then the origin of the error system will be stable. Thus, SL chaotic system and SL chaotic system are synchronized.

Proof: Choose a candidate Lyapunov function $V(t) = V(e(t))$=$0.5e^{\mathrm{T}}e$.

Set $V_1 = 0.5e_N^2$ and $V_2 = 0.5e_P^2$.

The time derivative of V_1 is

$$\begin{aligned} \dot{V}_1 &= \dot{e}_N e_N = (A_{11}e_N + A_{12}e_P + \phi_1)e_N \\ &= (A_{11}e_N + A_{12}e_P + K_{11}e_N + K_{12}e_P)e_N \\ &= (A_{11} + K_{11})e_N^2 + (A_{12} + K_{12})e_N e_P \\ &= \begin{bmatrix} e_N & e_P \end{bmatrix} \begin{bmatrix} A_{11} + K_{11} + \lambda_1 & 0.5(A_{12} + K_{12}) \\ * & -\delta_1 \end{bmatrix} \\ &\quad \cdot \begin{bmatrix} e_N \\ e_P \end{bmatrix} - 2\lambda_1 V_1 + 2\delta_1 V_2. \end{aligned}$$

From Eq. (7), we have

$$\dot{V}_1 \leqslant -2\lambda_1 V_1 + 2\delta_1 V_2. \tag{10}$$

For $t \in (t_i,\ t_{i+1}]$, $i = 1, 2, \cdots$, taking the derivative of $V_2(t)$ with respect to t along the trajectory of Eq. (6) yields

$$\begin{aligned} \dot{V}_2 &= \dot{e}_P e_P = (A_{21}e_N + A_{22}e_P + \phi_2)e_P \\ &= (A_{21}e_N + A_{22}e_P + K_{21}e_N + K_{22}e_P)e_P \\ &= (A_{22} + K_{22})e_P^2 + (A_{21} + K_{21})e_N e_P \\ &= \begin{bmatrix} e_N & e_P \end{bmatrix} \begin{bmatrix} -\lambda_2 & 0.5(A_{21} + K_{21}) \\ * & A_{22} + K_{22} - \delta_2 \end{bmatrix} \\ &\quad \cdot \begin{bmatrix} e_N \\ e_P \end{bmatrix} + 2\lambda_2 V_1 + 2\delta_2 V_2. \end{aligned}$$

From Eq. (8), we have

$$\dot{V}_2 \leqslant 2\lambda_2 V_1 + 2\delta_2 V_2. \tag{11}$$

From Eq. (6), we have

$$\begin{aligned} V_2(t_i^+) &= 0.5e_P^2(t_i^+) = 0.5(1 + \mu)^2 e_P^2(t_i^-) \\ &= (1 + \mu)^2 V_2(t_i^-) = \omega V_2(t_i). \end{aligned}$$

From Eq. (10), we can obtain

$$V_1 \leqslant e^{-2\lambda_1 t} V_1(0) + 2\delta_1 \int_0^t e^{-2\lambda_1(t-s)} V_2(s)\mathrm{d}s. \tag{12}$$

Substituting Eq. (12) into Eq. (11) yields

$$\begin{cases} \dot{V}_2 \leqslant 2\delta_2 V_2 + 2\lambda_2(e^{-2\lambda_1 t}V_1(0) + 2\delta_1 \displaystyle\int_0^t e^{-2\lambda_1(t-s)} \\ \quad \cdot V_2(s)\mathrm{d}s), \quad t \neq t_i \\ V_2(t_k) = \omega V_2(t_k^-), \quad t = t_i \end{cases}$$

According to Lemma 1, and setting $\gamma = \frac{1}{\beta}\ln(1/\omega) - 2\delta_2$, we can obtain

$$V_2 \leqslant V_2(0)\prod_{0<t_k<t}\omega e^{2\delta_2 t} + \int_0^t \prod_{s<t_k<t}\omega e^{2\delta_2(t-s)}2\lambda_2 \cdot (e^{-2\lambda_1 s}V_1(0) + 2\delta_1\int_0^s e^{-2\lambda_1(s-\sigma)}V_2(\sigma)d\sigma)\mathrm{d}s$$

$$\leqslant V_2(0)\frac{1}{\omega}e^{-\gamma t} + \int_0^t \frac{1}{\omega}e^{-\gamma(t-s)}2\lambda_2\Big[e^{-2\lambda_1 s}V_1(0) + 2\delta_1\int_0^s e^{-2\lambda_1(s-\sigma)}V_2(\sigma)\mathrm{d}\sigma\Big]\mathrm{d}s$$

$$= \frac{V_2(0)}{\omega}e^{-\gamma t} + \frac{2\lambda_2 V_1(0)}{\omega(\gamma-2\lambda_1)}(e^{-2\lambda_1 t} - e^{-\gamma t}) + \frac{4\lambda_2\delta_1}{\omega}\int_0^t\int_\sigma^t e^{-\gamma(t-s)}e^{-2\lambda_1(s-\sigma)}V_2(\sigma)\mathrm{d}s\mathrm{d}\sigma$$

$$= \frac{V_2(0)}{\omega}e^{-\gamma t} + \frac{2\lambda_2 V_1(0)}{\omega(\gamma-2\lambda_1)}(e^{-2\lambda_1 t} - e^{-\gamma t}) + \frac{4\lambda_2\delta_1}{\omega(\gamma-2\lambda_1)}\int_0^t (e^{-2\lambda_1(t-\sigma)} - e^{-\gamma(t-\sigma)})V_2(\sigma)\mathrm{d}\sigma.$$

According to Eq. (9), $\gamma - 2\lambda_1 > 0$; thus, we have

$$V_2 \leqslant \frac{V_2(0)}{\omega}e^{-\gamma t} + \frac{2\lambda_2 V_1(0)}{\omega(\gamma-2\lambda_1)}e^{-2\lambda_1 t} + \frac{4\lambda_2\delta_1}{\omega(\gamma-2\lambda_1)} \cdot \int_0^t e^{-2\lambda_1(t-\sigma)}V_2(\sigma)\mathrm{d}\sigma.$$

Set $V_3 = V_2 e^{2\lambda_1 t}$, then

$$V_3 \leqslant \frac{V_2(0)}{\omega}e^{(2\lambda_1-\gamma)t} + \frac{2\lambda_2 V_1(0)}{\omega(\gamma-2\lambda_1)} + \frac{4\lambda_2\delta_1}{\omega(\gamma-2\lambda_1)} \cdot \int_0^t V_3(\sigma)\mathrm{d}\sigma.$$

According to Lemma 2, we can obtain

$$V_3 \leqslant \frac{V_2(0)}{\omega}e^{(2\lambda_1-\gamma)t} + \frac{2\lambda_2 V_1(0)}{\omega(\gamma-2\lambda_1)} + \int_0^t e^{\frac{4\lambda_2\delta_1}{\omega(\gamma-2\lambda_1)}(t-s)} \cdot \left(\frac{V_2(0)}{\omega}e^{(2\lambda_1-\gamma)s} + \frac{2\lambda_2 V_1(0)}{\omega(\gamma-2\lambda_1)}\right)\mathrm{d}s$$

$$= \frac{V_2(0)}{\omega}e^{(2\lambda_1-\gamma)t} + \frac{2\lambda_2 V_1(0)}{\omega(\gamma-2\lambda_1)} + \frac{V_2(0)(\gamma-2\lambda_1)}{\omega(\gamma-2\lambda_1)^2+4\lambda_2\delta_1} \cdot e^{\frac{4\lambda_2\delta_1}{\omega(\gamma-2\lambda_1)}t}(1 - e^{(2\lambda_1-\gamma-\frac{4\lambda_2\delta_1}{\omega(\gamma-2\lambda_1)})t}) + \frac{2\lambda_2 V_1(0)}{4\lambda_2\delta_1}e^{\frac{4\lambda_2\delta_1}{\omega(\gamma-2\lambda_1)}t}(1 - e^{\frac{-4\lambda_2\delta_1}{\omega(\gamma-2\lambda_1)}t})$$

$$\leqslant \frac{V_2(0)}{\omega}e^{(2\lambda_1-\gamma)t} + \frac{2\lambda_2 V_1(0)}{\omega(\gamma-2\lambda_1)} + \frac{V_2(0)(\gamma-2\lambda_1)}{\omega(\gamma-2\lambda_1)^2+4\lambda_2\delta_1} \cdot e^{\frac{4\lambda_2\delta_1}{\omega(\gamma-2\lambda_1)}t} + \frac{2\lambda_2 V_1(0)}{4\lambda_2\delta_1}e^{\frac{4\lambda_2\delta_1}{\omega(\gamma-2\lambda_1)}t}.$$

Then, we can obtain

$$V_2 \leqslant \frac{V_2(0)}{\omega}e^{-\gamma t} + \frac{2\lambda_2 V_1(0)}{\omega(\gamma-2\lambda_1)}e^{-2\lambda_1 t} + \left(\frac{V_2(0)(\gamma-2\lambda_1)}{\omega(\gamma-2\lambda_1)^2+4\lambda_2\delta_1} + \frac{2\lambda_2 V_1(0)}{4\lambda_2\delta_1}\right)e^{\left[\frac{4\lambda_2\delta_1}{\omega(\gamma-2\lambda_1)}-2\lambda_1\right]t}$$

$$\leqslant \frac{V_2(0)}{\omega}e^{-\gamma t} + \alpha e^{(\frac{4\lambda_2\delta_1}{\omega(\gamma-2\lambda_1)}-2\lambda_1)t}, \quad (13)$$

where $\alpha = \frac{2\lambda_2 V_1(0)}{\omega(\gamma-2\lambda_1)} + \frac{V_2(0)(\gamma-2\lambda_1)}{\omega(\gamma-2\lambda_1)^2+4\lambda_2\delta_1} + \frac{2\lambda_2 V_1(0)}{4\lambda_2\delta_1}$.

Substituting Eq. (13) into Eq. (12) yields

$$V_1 \leqslant V_1(0)e^{-2\lambda_1 t} + 2\delta_1\int_0^t e^{-2\lambda_1(t-s)}V_2(s)\mathrm{d}s \leqslant V_1(0)e^{-2\lambda_1 t} + \frac{2\delta_1 V_2(0)}{\omega(\gamma-2\lambda_1)}e^{-2\lambda_1 t} + \frac{\alpha\delta_1\omega(\gamma-2\lambda_1)}{2\lambda_2\delta_1}e^{(\frac{4\lambda_2\delta_1}{\omega(\gamma-2\lambda_1)}-2\lambda_1)t}.$$

From Eq. (9), we have

$$\frac{4\lambda_2\delta_1}{\omega(\gamma-2\lambda_1)} - 2\lambda_1 < 0.$$

Therefore, we can conclude that $V(t)$ decrease as t goes to infinity, which means that the synchronization error $|e|$ decreases. Clearly, the order of the synchronization errors is 1. This completes the proof.

To verify the effectiveness of proposed method, we conduct simulation studies on two directly modulated SL chaotic systems. This method can also be used to synchronize other types of SL chaotic systems. The corresponding rate equations can be represented as

$$\begin{cases} \frac{\mathrm{d}N_1}{\mathrm{d}t} = \frac{1}{\tau_e}\left(\frac{I}{I_h} - N_1 - \frac{N_1-\delta}{1-\delta}P_1\right) \\ \frac{\mathrm{d}P_1}{\mathrm{d}t} = \frac{1}{\tau_p}\left(\frac{N_1-\delta}{1-\delta}(1-\varepsilon P_1)P_1 - P_1 + \eta N_1\right) \end{cases}, \quad (14)$$

$$\begin{cases} \frac{\mathrm{d}N_2}{\mathrm{d}t} = \frac{1}{\tau_e}\left(\frac{I}{I_h} - N_2 - \frac{N_2-\delta}{1-\delta}P_2\right) \\ \frac{\mathrm{d}P_2}{\mathrm{d}t} = \frac{1}{\tau_p}\left(\frac{N_2-\delta}{1-\delta}(1-\varepsilon P_2)P_2 - P_2 + \eta N_2\right) + \mu(P_2(t_i^-) - P_1(t_i^-))\sum_{i=0}^{t}\delta(t-t_i) \\ I = I_\mathrm{b} + I_\mathrm{m}\sin(2\pi f_\mathrm{m} t) \end{cases}, \quad (15)$$

where τ_e and τ_p are the electron and photon lifetimes; I is the driving current; $\delta = n_0/n_\mathrm{th}$ and $\varepsilon = \varepsilon_\mathrm{NL}S_0$ are dimensionless parameters, where n_0 is the carrier density required for transparency, $n_\mathrm{th} = (\tau_\mathrm{e}I_\mathrm{h}/\mathrm{eV})$ is the threshold carrier density, ε_NL is the factor governing the nonlinear gain reduction occurring with an increase in S, and $S_0 = \Gamma(\tau_p/\tau_e)n_{th}$; I_h is the threshold current; e is the electron charge; V is the active volume; Γ is the confinement factor. Here, I_b is the bias current, I_m is the amplitude of the modulation current, f_m is the modulation frequency, and η is the spontaneous emission factor. Initially, the two lasers are set to operate at different regions of the phase space.

Equations (14) and (15) can be rewritten in the form of Eqs. (1) and (2), and the error system of SL in Eqs. (14) and (15) can be written in the form of Eq. (6); thus, we can obtain

$$A = \begin{bmatrix} -\dfrac{1}{\tau_e} & \frac{1}{\tau_e}\dfrac{\delta}{1-\delta} \\ \dfrac{\eta}{\tau_p} & -\frac{1}{\tau_p}\left(\dfrac{1}{1-\delta}\right) \end{bmatrix},$$

$$K = \begin{bmatrix} -\dfrac{1}{\tau_e(1-\delta)}P_1 & -\dfrac{1}{\tau_e(1-\delta)}N_2 \\ \dfrac{1}{\tau_p(1-\delta)}(P_2+\varepsilon P_2^2) & \dfrac{1}{\tau_p(1-\delta)}(N_1+\varepsilon N_1 P_1+\varepsilon N_1 P_2+\varepsilon\delta P_1+\varepsilon\delta P_2) \end{bmatrix}.$$

The numerical integrations were accomplished using the Runge-Kutta fourth-order method. The values of the parameters used for numerical calculations were chosen as $\tau_p = 6\times10^{-12}$ s, $\tau_e = 3\times10^{-9}$ s, $I_h = 1.7\times10^{-2}$ A, δ=0.692, I_m=0.3I_h, I_b=1.5I_h, f_m=8×10^8 Hz, η=5×10^{-5}, and ε=10^{-4}. From the simulation results, we can obtain $-6.4\times10^9 < K_{11} < 0$, $-1.17\times10^9 < K_{12} < -1.02\times10^9$, $0 < K_{21} < 3.25\times10^{12}$, and $0.51\times10^{12} < K_{22} < 0.59\times10^{12}$. Therefore, according to Eqs. (7) and (8), the positive scalars $\lambda_1=0.2\times10^9$, $\lambda_2=0.1\times10^{12}$, $\delta_1=0.34\times10^9$, and $\delta_2=2.67\times10^{13}$ exist. We consider $\mu = -0.9$. Thus, according to Eq. (9), the impulsive intervals are chosen as $\tau_i \leqslant \beta \leqslant 0.5\times10^{-13}$. The simulation results are shown in Figs. 1 and 2, with the synchronization errors plotted in the log scale.

In conclusion, we investigate the impulsive synchronization problem for SL chaotic systems. The conditions for impulsive synchronization of SL chaotic systems are derived. A numerical example is given to illustrate the effectiveness of the proposed control scheme.

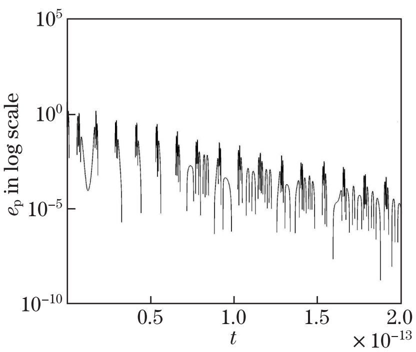

Fig. 1. Synchronization error of photon density.

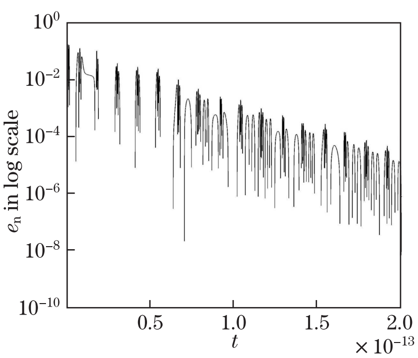

Fig. 2. Synchronization error of carrier density.

References

1. G. D. Van Wiggeren and R. Roy, Science **279,** 1198 (1998).
2. J. P. Goedgebuer, L. Larger, and H. Porte, Phys. Rev. Lett. **80,** 2249 (1998).
3. L. Larger, J. P. Goedgebuer, and F. Delorme, Phys. Rev. E **57,** 6618 (1998).
4. J. Hu, K. Jia, and J. Ma, Int. J. Light Electron. Opt. **122,** 2071 (2011).
5. S. Banerjee, L. Rondoni, and S. Mukhopadhyay, Opt. Commun. **284,** 4623 (2011).
6. Y. C. Kouomou and P. Woafo, Opt. Commun. **223,** 283 (2003).
7. T. Deng, G. Xia, L. Cao, J. Chen, X. Lin, and Z. Wu, Opt. Commun. **282,** 2243 (2009).
8. G. Xia, Z. Wu, and J. Liao, Opt. Commun. **282,** 1009 (2009).
9. P. Saha, S. Banerjee, and A. R. Chowdhury, Chaos Solitons Fractals **14,** 1083 (2002).
10. S. Banerjee, L. Rondoni, S. Mukhopadhyay, and A. P. Misra, Opt. Commun. **284,** 2278 (2011).
11. Y. Zhang, J. Zhang, M. Zhang, and Y. Wang, Chin. Opt. Lett. **9,** 031404 (2011).
12. T. Kuruvilla and V. M. Nandakumaran, Phys. Lett. A **254,** 59 (1999).
13. J. Mork, B. Tromborg, and J. Mark, IEEE J. Quantum Electron. **28,** 93 (1992).
14. S. K. Hwang and J. M. Liu, Opt. Commun. **183,** 195 (2000).
15. F. Y. Lin and J. M. Liu, Appl. Phys. Lett. **81,** 3128 (2002).
16. E. Gilbert and G. Harasty, IEEE Trans. Autom. Control **AC-16,** 1 (1971) .
17. T. Yang, L. B. Yang, and C. M. Yang, Phys. D **110,** 18 (1997).
18. X. Yang, J. Cao, and J. Lu, Nonlinear Anal. Real World Appl. **12,** 2252 (2011).
19. X. Song, H. Guo, and X. Shi, *Impulsive Differential Equation Theory and Its Applications* (in Chinese) (Science Press, Beijing, 2011).