Different entanglement dynamics and transfer behaviors due to dipole-dipole interaction

Likai Cui (崔利凯), Yingjie Zhang (张英杰), Zhongxiao Man (满忠晓), and Yunjie Xia (夏云杰)*

Shandong Provincial Key Laboratory of Laser Polarization and Information Technology,

Department of Physics, Qufu Normal University, Qufu 273165, China

**Corresponding author: yjxia@mail.qfnu.edu.cn*

Received April 4, 2012; accepted April 25, 2012; posted online , 2012

We analyze entanglement dynamics and transfer in a system composed of two initially correlated two-level atoms, in which each atom is coupled with another atom interacting with its own reservoir. Considering atomic dipole-dipole interactions, the results show that dipole-dipole interactions restrain the entanglement birth of the reservoirs, and a parametric region of dipole-dipole interaction strength exists in which the maximal entanglement of two initially uncorrelated atoms is reduced. The transfer of entanglement shows obvious different behaviors in two initial Bell-like states.

OCIS codes: 020.5580, 060.5565.

doi: 10.3788/COL201210.100202.

Entanglement, a unique feature of quantum mechanical systems with no classical analog, is a crucial resource for various aspects of quantum information processing[1]. Entanglement dynamics and entanglement control have recently attracted extensive studies, and various aspects of entanglement, especially multipartite entanglement and its evolution, require further exploration[2]. A peculiar aspect of entanglement dynamics is the well known "entanglement sudden death (ESD)" phenomenon. In the process of entanglement distribution and qubit manipulation, each qubit is unavoidably exposed to its own uncontrollable environment. This phenomenon leads to local decoherences that spoil the necessary entanglement. In previous studies, various types of environments were studied, such as fermionic symmetry-broken[3], quantum critical[4], dephasing[5], multimode electromagnetic field[6,7], and quantum spin environments[8], among others.

The evolution of open quantum systems is divided into the Markovian and non-Markovian regimes. For the Markovian regime, the correlation time between the system and environment is infinitesimally small such that the dynamical map has no memory effects and results in a monotonic flow of information from the system to the environment. In contrast, a non-Markovian regime with memory has dynamical traits that give rise to the backflow of information from the environment to the system and can lead to distinctly different effects on the decoherence and disentanglement of open systems compared with the Markovian regime[9,10]. Several studies are currently focused on the non-Markovian regime for its dynamical behaviors that differ significantly from those of the Markovian regime, including those involving non-Markovianity[9,10], positivity[11,12], and several other dynamical properties and approaches.

The dynamics of entanglement transfer in interacting and non-interacting systems has been extensively studied[13−15]. Conservation for entanglement depends on how qubits are initially correlated[16]. Zhang *et al.* focused on the case in which two atoms off-resonantly interact with their loose cavities and examined the complete entanglement transfer from the atoms to their independent reservoirs[14]. In this letter, we study a system consisting of two initially correlated two-level atoms A and B, each coupled with another atom $C(D)$ interacting with its own reservoir $a(b)$. The atoms A and B are initially entangled, whereas C and D are in the ground states. Dipole-dipole interactions exist between the atoms in each subsystem. We consider the Markovian and non-Markovian effects of reservoirs on the entanglement evolution and transfer for the remote parties AB, CD, and ab.

We consider two independent subsystems, each formed by two two-level atoms coupled with a thermal reservoir. In each subsystem, the atoms interact with each other via dipole-dipole interactions. Considering a subsystem ACa as an example, the Hamiltonian H is

$$H = H_0 + H_I, \tag{1}$$

with H_0 as the free Hamiltonian, H_I describing the interaction part.

$$H_0 = \Omega(\sigma_+^A\sigma_-^A + \sigma_+^C\sigma_-^C) + \sum_k \omega_k a_k^\dagger a_k, \tag{2}$$

$$H_I = \sum_k [g_k(\alpha_A\sigma_A^- + \alpha_C\sigma_C^-)a_k^\dagger + g_k^*(\alpha_A\sigma_A^+ + \alpha_C\sigma_C^+)a_k] + D(\sigma_A^-\sigma_C^+ + \sigma_C^-\sigma_A^+), \tag{3}$$

where Ω and $\sigma_\pm^i(i = A, C)$ are the atomic transition frequency and the inversion operators of the ith atom, respectively; $a_k^\dagger(a_k)$ is the creation (annihilation) operator of the photon of the reservoir; ω_k and g_k are the frequency of the mode k of the reservoir and its coupling strength with the atoms, respectively; D is the strength of the dipole-dipole interaction between atoms. The actual coupling strength between the ith atom and the kth mode photon is measured by $|g_k|\,\alpha_i$.

Suppose that the atoms A and C are initially in the state $|eg\rangle_{AC}$ and reservoir a is in the vacuum state $|\bar{0}\rangle_r = \prod_k |0_k\rangle_r$. The quantum state of subsystem ACa

at time t can be written as

$$|\Phi(t)\rangle = [c_1(t)\,|eg\rangle + c_2(t)\,|ge\rangle]\otimes|\bar{0}\rangle + c_3(t)\,|gg\rangle\otimes|\bar{1}\rangle\,, \quad (4)$$

where $|\bar{1}\rangle = \frac{1}{c_3(t)}\sum_k \gamma_k(t)\,|\bar{1}_k\rangle\,, |\bar{1}_k\rangle$ means that there is one photon in the kth mode of the reservoir. The coefficients satisfy $c_1(t)^2 + c_2(t)^2 + c_3(t)^2 = 1$. We can obtain the coefficients $c_1(t)$, $c_2(t)$, and $c_k(t)$ by solving the equation of motion $\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\,|\Phi(t)\rangle = H_I\,|\Phi(t)\rangle$. From Eqs. (3) and (4), we have

$$\mathrm{i}\frac{d}{\mathrm{d}t}c_1(t) = Dc_2(t) + \alpha_A \sum_k g_k^* \mathrm{e}^{\mathrm{i}(\Omega-\omega_k)t} c_k(t), \quad (5)$$

$$\mathrm{i}\frac{d}{\mathrm{d}t}c_2(t) = Dc_1(t) + \alpha_C \sum_k g_k^* \mathrm{e}^{\mathrm{i}(\Omega-\omega_k)t} c_k(t), \quad (6)$$

$$\mathrm{i}\frac{d}{\mathrm{d}t}\gamma_k(t) = g_k \mathrm{e}^{-\mathrm{i}(\Omega-\omega_k)t}(\alpha_A c_1(t) + \alpha_C c_2(t)). \quad (7)$$

Integrating Eq. (7) with the initial condition $\gamma_k(0) = 0$, we can obtain

$$\gamma_k(t) = -\mathrm{i}\int_0^t \mathrm{d}t' g_k e^{-i(\Omega-\omega_k)t'}(\alpha_A c_1(t') + \alpha_C c_2(t')). \quad (8)$$

Then, substituting Eq. (8) into Eqs. (5) and (6), we can obtain

$$\frac{d}{\mathrm{d}t}c_1(t) = -\mathrm{i}Dc_2(t) - \int_0^t \mathrm{d}t' \sum_k |g_k|^2 \mathrm{e}^{-\mathrm{i}(\omega_k-\Omega)(t-t')} \cdot (\alpha_A^2 c_1(t') + \alpha_A\alpha_C c_2(t')), \quad (9)$$

$$\frac{d}{\mathrm{d}t}c_2(t) = -\mathrm{i}Dc_1(t) - \int_0^t \mathrm{d}t' \sum_k |g_k|^2 \mathrm{e}^{-\mathrm{i}(\omega_k-\Omega)(t-t')} \cdot (\alpha_A\alpha_C c_1(t') + \alpha_C^2 c_2(t')). \quad (10)$$

In the continuum limit for the reservoir spectrum, the sum over the modes is replaced by the integral $\sum_k |g_k|^2 \to \int \mathrm{d}\omega J(\omega)$, where $J(\omega)$ is the reservoir spectral density. We consider a spectrum of the field displaying a Lorentzian broadening with

$$J(\omega) = \frac{R^2}{\pi}\frac{\lambda}{(\omega-\omega_\mathrm{c})^2 + \lambda^2}, \quad (11)$$

where ω_c is the fundamental frequency of the field, R specifies the atom-reservoir coupling, and λ is the half-width at half-height of the field spectrum profile. According to Refs. [17,18], weak-coupling is represented by $\lambda > 2R$, where the behavior of the qubit-reservoir system is Markovian and irreversible decay occurs. In contrast, a strong-coupling regime is represented by $\lambda < 2R$, where non-Markovian dynamics occurs accompanied by oscillatory reversible decay, and a structured, rather than flat, reservoir situation applies.

Here, we set $f(t-t') = \int_{-\infty}^{\infty} \mathrm{d}\omega J(\omega)\mathrm{e}^{-(\omega-\Omega)(t-t')}$, and the equations of $c_1(t)$ and $c_2(t)$ are

$$\frac{\mathrm{d}}{\mathrm{d}t}c_1(t) = -\mathrm{i}Dc_2(t) - \int_0^t \mathrm{d}t'(\alpha_A^2 c_1(t') + \alpha_A\alpha_C c_2(t'))f(t-t'), \quad (12)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}c_2(t) = -\mathrm{i}Dc_1(t) - \int_0^t \mathrm{d}t'(\alpha_A\alpha_C c_1(t') + \alpha_C^2 c_2(t'))f(t-t'). \quad (13)$$

Using Laplace transformation, we can obtain the exact solutions of $c_1(t)$ and $c_2(t)$.

Results from previous calculations can be used directly to obtain the solution for our model. We consider the subsystems ACa and BDb with no interaction between them; the only correlation between ACa and BDb is the initial entanglement of atoms AB. Here, atoms A and B are considered to be initial in the Bell-like pure states $(\alpha\,|gg\rangle + \beta\,|ee\rangle)_{AB}$ and $(\alpha\,|eg\rangle + \beta\,|ge\rangle)_{AB}$. The atoms C and D are in the ground state, whereas the reservoirs a and b are in the vacuum state. The initial states of the system are

$$|\Psi(0)\rangle = (\alpha\,|gg\rangle_{AB} + \beta\,|ee\rangle_{AB})\otimes|gg\rangle_{CD}\otimes|\bar{0}\bar{0}\rangle_{ab}\,, \quad (14)$$

$$|\Phi(0)\rangle = (\alpha\,|eg\rangle_{AB} + \beta\,|ge\rangle_{AB})\otimes|gg\rangle_{CD}\otimes|\bar{0}\bar{0}\rangle_{ab}\,. \quad (15)$$

We can obtain the evolved states at $t > 0$ as

$$\begin{aligned}|\Psi(t)\rangle =& \alpha|gg\rangle_{AB}|gg\rangle_{CD}|\bar{0}\bar{0}\rangle_{ab} + \beta[c_1^2(t)|ee\rangle_{AB}|gg\rangle_{CD}|\bar{0}\bar{0}\rangle_{ab} \\ &+ c_1(t)c_2(t)\,|eg\rangle_{AB}\,|ge\rangle_{CD}\,|\bar{0}\bar{0}\rangle_{ab} \\ &+ c_1(t)c_3(t)\,|eg\rangle_{AB}\,|gg\rangle_{CD}\,|\bar{0}\bar{1}\rangle_{ab} \\ &+ c_1(t)c_2(t)\,|ge\rangle_{AB}\,|eg\rangle_{CD}\,|\bar{0}\bar{0}\rangle_{ab} \\ &+ c_2^2(t)\,|gg\rangle_{AB}\,|ee\rangle_{CD}\,|\bar{0}\bar{0}\rangle_{ab} \\ &+ c_2(t)c_3(t)\,|gg\rangle_{AB}\,|eg\rangle_{CD}\,|\bar{0}\bar{1}\rangle_{ab} \\ &+ c_1(t)c_3(t)\,|ge\rangle_{AB}\,|gg\rangle_{CD}\,|\bar{1}\bar{0}\rangle_{ab} \\ &+ c_2(t)c_3(t)\,|gg\rangle_{AB}\,|ge\rangle_{CD}\,|\bar{1}\bar{0}\rangle_{ab} \\ &+ c_3^2(t)\,|gg\rangle_{AB}\,|gg\rangle_{CD}\,|\bar{1}\bar{1}\rangle_{ab}], \end{aligned} \quad (16)$$

$$\begin{aligned}|\Phi(t)\rangle =& \alpha(c_1(t)\,|eg\rangle_{AB}\,|gg\rangle_{CD}\,|\bar{0}\bar{0}\rangle_{ab} \\ &+ c_2(t)\,|gg\rangle_{AB}\,|eg\rangle_{CD}\,|\bar{0}\bar{0}\rangle_{ab} \\ &+ c_3(t)\,|gg\rangle_{AB}\,|gg\rangle_{CD}\,|\bar{1}\bar{0}\rangle_{ab}) \\ &+ \beta_1(t)\,|\mathrm{ge}\rangle_{AB}\,|gg\rangle_{CD}\,|\bar{0}\bar{0}\rangle_{ab} \\ &+ c_2(t)\,|gg\rangle_{AB}\,|ge\rangle_{CD}\,|\bar{0}\bar{0}\rangle_{ab} \\ &+ c_3(t)\,|gg\rangle_{AB}\,|gg\rangle_{CD}\,|\bar{0}\bar{1}\rangle_{ab}). \end{aligned} \quad (17)$$

We analyze the evolution of entanglement in the models above. To quantify two qubit entanglement, we use the Wootters concurrence[19], defined as $C(t) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}$, where $\lambda_1 \geqslant \lambda_2 \geqslant \lambda_3 \geqslant \lambda_4 \geqslant 0$ are the eigenvalues of the matrix $\xi = \rho_{12}(\sigma_y^1\otimes\sigma_y^2)\rho_{12}^*(\sigma_y^1\otimes\sigma_y^2)$ and ρ_{12}^* is the complex conjugate of ρ_{12}. $C(t) = 1$ indicates the maximally entangled state, whereas $C(t) = 0$ indicates a separable state. For Bell-like states, the density matrix of the atomic system has an X form based on $\{|00\rangle\,, |01\rangle\,, |10\rangle\,, |11\rangle\}$

$$\rho_X = \begin{pmatrix} a & 0 & 0 & w \\ 0 & b & z & 0 \\ 0 & z^* & c & 0 \\ w^* & 0 & 0 & d \end{pmatrix}. \quad (18)$$

For the X-state Eq. (18), the concurrence can be derived as

$$C(\rho) = 2\max\{0, |w| - \sqrt{bc}, |z| - \sqrt{ad}\}. \quad (19)$$

We set $\alpha_A = \alpha_C = 1$, $c_1(0) = 1, c_2(0) = 0$ for convenience and ignore the detuning between atoms and reservoir. The coefficients $c_1(t)$ and $c_2(t)$ have the forms

$$c_1(t) = \frac{1}{2}e^{iDt} + \frac{1}{2}e^{-\frac{1}{2}t(\lambda+iD)} \cdot \left[\cosh\left(\frac{\Theta t}{2}\right) + \frac{\lambda - iD}{\Theta}\sinh\left(\frac{\Theta t}{2}\right)\right], \quad (20)$$

$$c_2(t) = -\frac{1}{2}e^{iDt} + \frac{1}{2}e^{-\frac{1}{2}t(\lambda+iD)} \cdot \left[\cosh\left(\frac{\Theta t}{2}\right) + \frac{\lambda - iD}{\Theta}\sinh\left(\frac{\Theta t}{2}\right)\right], \quad (21)$$

$$c_3(t) = \sqrt{1 - c_1(t)^2 - c_2(t)^2}, \quad (22)$$

with

$$\Theta = \sqrt{(\lambda - iD)^2 - 8R^2}. \quad (23)$$

We attempt to analyze the evolution of entanglement in systems with different parameters. In the model we discussed above, the dynamics of the system varies with the parameters α, β, R/λ, and D.

First, we investigate the entanglement of state $|\Psi(t)\rangle$ in Eq. (16). Using the method above, the concurrences of AB, CD, and ab will be

$$\begin{cases} C_{AB}=2\max\left\{0, \left|\alpha\beta c_1^2(t)\right|-\left|\beta^2[c_1^2(t)(1-c_1^2(t))]\right|\right\} \\ C_{CD}=2\max\left\{0, \left|\alpha\beta c_2^2(t)\right|-\left|\beta^2[c_2^2(t)(1-c_2^2(t))]\right|\right\} \\ C_{ab}=2\max\left\{0, \left|\alpha\beta c_3^2(t)\right|-\left|\beta^2[c_3^2(t)(1-c_3^2(t))]\right|\right\} \end{cases}. \quad (24)$$

For the initial state $|\Psi(0)\rangle$, an ESD appears for atoms AB even with maximal initial entanglement $C_{AB} = 1$. Entanglement sudden birth simultaneously occurs for atoms CD and reservoirs ab. Figure 1 shows the evolution of entanglement of atoms AB and CD and reservoirs ab in a non-Markovian regime where $\lambda = 0.1R$. We set $\alpha/\beta = \sqrt{3/4}$. Here, we provide three cases of systems with weak ($D = 0.1R$), intermediate ($D = R$), and strong ($D = 5R$) dipole-dipole interactions. The entanglement of AB decays asymptotically with oscillation and ESD, whereas the entanglements of CD and ab approach a peak value and then decay asymptotically with oscillation (Fig. 1(a)). Figure 1 shows that the transfer of entanglement between atoms AB and CD is strengthened with increasing dipole-dipole interactions. Dipole-dipole interactions delay the decay of entanglement of AB and restrain the birth of entanglement of reservoirs ab. However, for the atoms CD, a parametric region of dipole-dipole interaction causes the entanglement to weaken compared with the case with no dipole-dipole interaction. Figure 2 shows the maximum entanglement values of atoms CD with dipole-dipole interactions increasing from 0 to $2.5R$ in the non-Markovian regime where $\lambda = 0.1R$. When no dipole-dipole interaction exists between atoms, reservoirs ab act as a medium in the transfer of entanglement from atoms AB to CD. When we consider dipole-dipole interactions, two pathways for the transfer of entanglement appear. On the one hand, dipole-dipole interactions between atoms weaken the interaction between atoms and the reservoir. On the other hand, weaker dipole-dipole interactions do not play an obvious role during entanglement transfer that reduces the entanglement transferred to CD. The entanglement of atoms CD remains at the lowest level with time evolution, particularly when the dipole-dipole interaction strength is equivalent to the strength of the coupling between atoms and reservoirs.

Figure 3 shows the evolution of entanglement of atoms AB and CD and reservoirs ab in the Markovian regime where $\lambda = 8R$ with $\alpha/\beta = \sqrt{3/4}$. Here, we demonstrate weak ($D = 0.1R$), intermediate ($D = R$), and strong ($D = 5R$) dipole-dipole interactions. With weak dipole-dipole interactions, $D = 0.1R$, as in Fig. 3(a), the entanglement of atoms AB decays asymptotically,

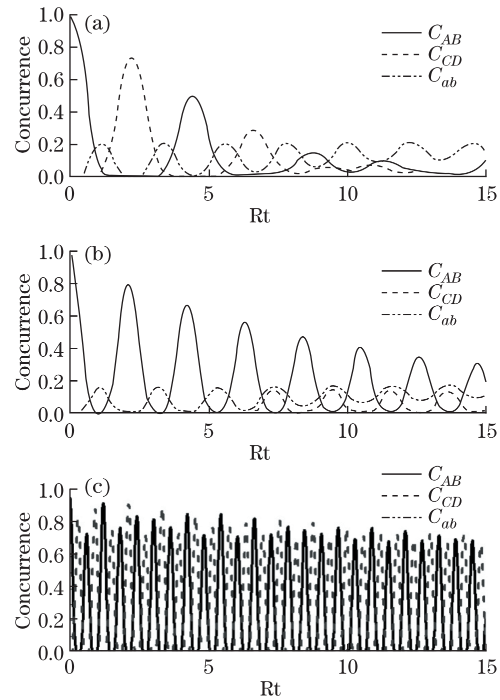

Fig. 1. Time evolution of concurrences of AB, CD, and ab in the non-Markovian regime where $\lambda = 0.1R$ for the state $|\Psi(t)\rangle$ with $\alpha/\beta = \sqrt{3/4}$ in the case of (a) weak ($D = 0.1R$), (b) intermediate ($D = R$), and (c) strong ($D = 5R$) dipole-dipole interactions.

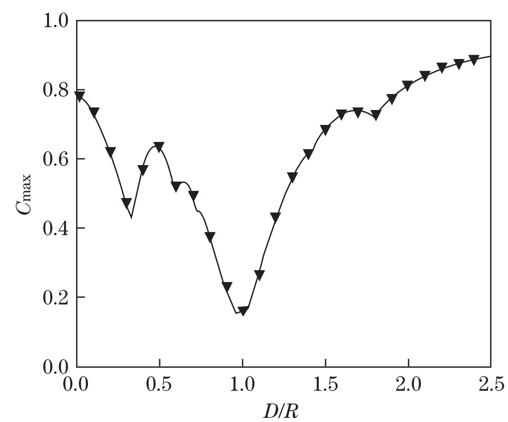

Fig. 2. Maximum values of concurrence of CD with dipole-dipole interaction strengths changing from 0 to $2.5R$ in the non-Markovian regime where $\lambda = 0.1R$ for the state $|\Psi(t)\rangle$ with $\alpha/\beta = \sqrt{3/4}$.

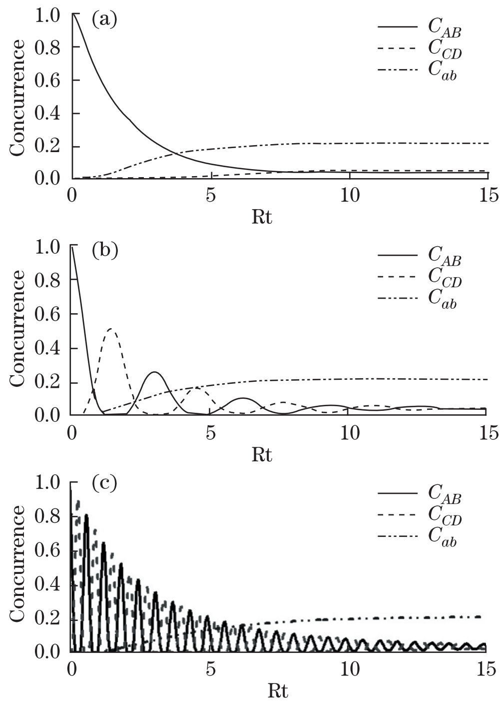

Fig. 3. Time evolution of concurrences of AB, CD, and ab in the Markovian regime where $\lambda = 8R$ for the state $|\Psi(t)\rangle$ with $\alpha/\beta = \sqrt{3/4}$ in the case of (a) weak ($D = 0.1R$), (b) intermediate ($D = R$), and (c) strong ($D = 5R$) dipole-dipole interactions.

and the entanglement of atoms CD or reservoirs ab gradually increases without oscillation. When the dipole-dipole interaction is equivalent to the atom-reservoir coupling strength such that $D = R$, the entanglement of AB evolves as in the non-Markovian regime with ESD and oscillation occurs. The entanglement of CD reaches a peak value and subsequently oscillates. The entanglement evolutions of atoms AB and CD reflect non-Markovianity. With the strong dipole-dipole interactions, $D = 5R$, the non-Markovianity of the system becomes more obvious, entanglement transfer between AB and CD is strengthened, and the time windows of sudden death are shortened. The entanglement of ab reflects no oscillation but a much stronger dipole-dipole interaction restrains its birth time more obviously.

We observed entanglement transfer from atoms AB to CD and reservoirs ab. However, with the coupling of atoms and reservoirs, the transfer was incomplete in these three parts. Figure 4 shows the entanglements of atoms AB and CD and reservoirs ab and the sum of these three bipartite entanglements in the non-Markovian regime where $\lambda = 0.1R$ with $D = R$ and $\alpha/\beta = \sqrt{3/4}$. The initial entanglement is only partly distributed in atoms AB and CD and reservoirs ab with time evolution, and part of it is transferred to multi-qubit form. Figure 5 shows a plot of the sum of the entanglements AB, CD, and ab with different ratios of α and β. When $\alpha/\beta < 1/2$, no entanglement is distributed in these three parts. When $\alpha/\beta = 1/3$, the entanglement quickly decays to zero. When $\alpha/\beta = 1$, indicating an initial maximum entanglement state, the sum entanglement remains at a fixed value.

For the state $|\Phi(t)\rangle$ in Eq. (17), the behavior of entanglement dynamics differs from the state $|\Psi(t)\rangle$. For this state, the concurrences of atoms AB and CD and reservoirs ab have the form

$$\begin{cases} C_{AB} = 2\max\left\{0, \left|\alpha\beta c_1^2(t)\right|\right\} \\ C_{CD} = 2\max\left\{0, \left|\alpha\beta c_2^2(t)\right|\right\} \\ C_{ab} = 2\max\left\{0, \left|\alpha\beta c_3^2(t)\right|\right\} \end{cases}. \tag{25}$$

From Eq. (25), the entanglements of AB, CD, and ab satisfy the equation $C_{AB}+C_{CD}+C_{ab}=2\alpha\beta$, indicating that the initial entanglement of AB thoroughly distributes

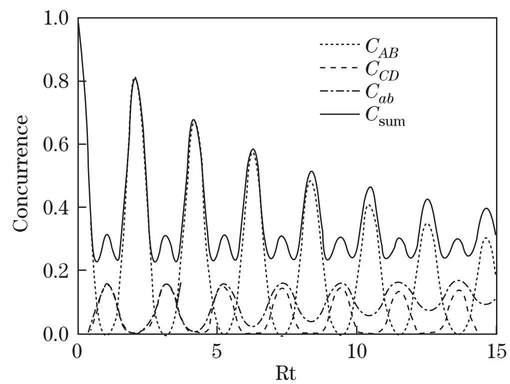

Fig. 4. Time evolution of concurrences of AB, CD, and ab and the sum of these values in the non-Markovian regime where $\lambda = 0.1R$ for the state $|\Psi(t)\rangle$ with $\alpha/\beta = \sqrt{3/4}$ and $D = R$.

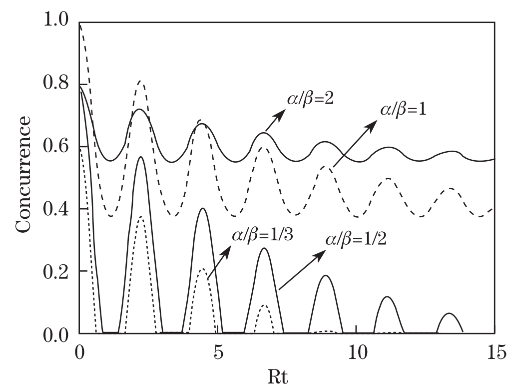

Fig. 5. Sum of entanglements of AB, CD, and ab in the non-Markovian regime $\lambda = 0.1R$ for the state $|\Psi(t)\rangle$ with $D = R$.

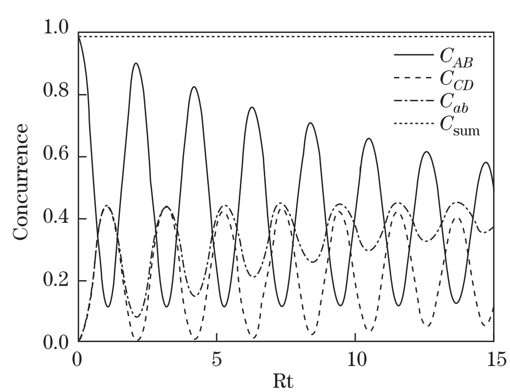

Fig. 6. Time evolution of concurrences of AB, CD, and ab and the sum of these values in the non-Markovian regime where $\lambda = 0.1R$ for the state $|\Phi(t)\rangle$ with $\alpha/\beta = \sqrt{3/4}$ and $D = R$.

in these three parts with time evolution. Figure 6 shows the entanglements of AB, CD, and ab, and the sum of these values in the non-Markovian regime where $\lambda = 0.1R$ with $D = R$. In this state, no sudden death or sudden birth phenomenon with or without dipole-dipole interactions occurs. The initial entanglement completely transfers in these three bipartite entanglements.

In conclusion, we discuss a system consisting of two initially correlated two-level atoms, each coupled with another atom interacting with its own reservoir. We study the entanglement dynamics and transfer of two subsystems with different initial entanglements, and atomic dipole-dipole interactions are considered. For the condition with initial atomic entanglement$(\alpha\,|eg\rangle + \beta\,|ge\rangle)_{AB}$, no ESD or sudden birth phenomenon occurs with time evolution. The total entanglement of atoms AB and CD and reservoirs ab is conservative and equal to the initial entanglement of atoms AB. For the condition with initial atomic entanglement $(\alpha\,|gg\rangle + \beta\,|ee\rangle)_{AB}$, the total entanglement of atoms AB and CD and reservoirs ab is not conserved and cannot reach a maximum value similar to that of the initial entanglement of atoms AB. If parameters α, β satisfy $\alpha/\beta < 1/2$, the initial entanglement will completely transfer to the multi-qubit form. Atomic dipole-dipole interactions can accelerate the entanglement transfer between AB and CD and simultaneously restrain the entanglement birth of reservoirs ab. However, for the entanglement evolution of atoms CD, a parametric region of dipole-dipole interaction exists such that the dipole-dipole interaction results in weaker entanglement compared with the case with no dipole-dipole interaction.

This work was supported by the National Natural Science Foundation of China (Nos. 61178012 and 10947006) and the Specialized Research Fund for the Doctoral Program of Higher Education (No. 20093705110001).

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