Ramsey interaction with transverse decay

Xucheng Wang (王旭成)[1], Huadong Cheng (成华东)[1], and Liang Liu (刘 亮)[1,2*]

[1]*Key Laboratory of Quantum Optics, Shanghai Institute of Optics and Fine Mechanics, and Center of Cold Atom Physics, Chinese Academy of Sciences, Shanghai 201800, China*

[2]*State Key Laboratory of Precision Spectroscopy, East China Normal University, Shanghai 200062, China*

[*]*Corresponding author: liang.liu@siom.ac.cn*

Received February 16, 2012; accepted March 17, 2012; posted online August 3, 2012

The Ramsey fringe contrast of a pulsed optically pumped cold atom clock is strongly affected by the transverse decay of the atomic sample. This letter calculates the Ramsey fringe with focus on transverse decay, and analyzes the Ramsey fringe contrast with different transverse decay rates. By fitting the experimental data, we obtain the transverse decay rate in a cold atom sample at an approximate value of 30.5 s^{-1}, which is much smaller than that in a cell.

OCIS codes: 020.0020, 270.0270.

doi: 10.3788/COL201210.100201.

A typical microwave atomic clock uses the transition between two ground states as signal to lock the local oscillator. The spectral width, signal-to-noise ratio (SNR), and contrast are the key factors determining clock performance. The width is directly related to the interaction duration between the atoms and the microwave. The Ramsey method has been widely used in microwave beam clocks[1] and, recently, in cell or cold atom clocks. A typical cesium beam clock has two microwave cavities separated by L, and a cesium beam passes through the cavities with speed v[2]. The width of the Ramsey fringe is determined by $T = L/v$. Similarly, the atomic fountain[3] uses a single cavity but passes twice during the upward and downward motions of the cold atoms.

The Ramsey fringe contrast is affected by decoherence during the evolution of atoms between two Ramsey pulses. Theoretically, however, contrast is not considered in the cesium beam clock, because the coherence time is longer than the evolution time, which is similar in the atomic fountain clock[3].

Recently, compact cold atom clocks have received considerable attention due to their potential applications[4−7]. The compact cold atom clock has small volume, low weight, and high performance properties. In typical atom cooling, the interrogation and detection of a compact cold atom clock are conducted in same zone. Thus, decoherence during evolution is an important factor.

Coherence during evolution is affected by several factors, such as atom collision, external electromagnetic field, temperature and density of atomic samples[8,9], and others. Generally, decoherence time is described by transverse decay rate, which can be determined through an experiment.

In this letter, we introduce a transverse decay rate, γ, to describe decoherence in the evolution between two Ramsey pulses. We also study the Ramsey fringe contrast resulting from γ and compared it with experimental result. Finally, we estimate γ by fitting the experimental data with theory.

The up and down levels of a two-level atom system are represented by $|1\rangle$ and $|2\rangle$, respectively. The density matrix is expressed as

$$\rho = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}. \tag{1}$$

The hyperfine states that $|1\rangle$ and $|2\rangle$ have eigenvalues of E_1 and E_2, respectively, and $E_1 - E_2 = \hbar\omega_0$, ω_0 is the hyperfine transition frequency between states $|1\rangle$ and $|2\rangle$.

The interaction between a two-level atom and a microwave field is described by the optical Bloch equations[10] given by

$$\frac{\mathrm{d}(\rho_{11} - \rho_{22})}{\mathrm{d}t} = 2\mathrm{i}(V_{21}\rho_{12} - V_{12}\rho_{21}), \tag{2}$$

$$\frac{\mathrm{d}\rho_{12}}{\mathrm{d}t} = -\mathrm{i}(\omega_0 + \gamma)\rho_{12} + \mathrm{i}V_{12}(\rho_{11} - \rho_{12}), \tag{3}$$

where the perturbation term V_{12} is expressed as

$$V_{12} = \frac{1}{2}(b_1 + \mathrm{i}b_2)\mathrm{e}^{-\mathrm{i}\omega t}, \tag{4}$$

and ω is the frequency of the microwave field. Here, b_1 and b_2 have

$$b_1 = b\cos\phi, \tag{5}$$

$$b_2 = -b\sin\phi, \tag{6}$$

where b is the Rabi frequency, and ϕ is the phase of the microwave field. In Eq. (3), γ is introduced to describe atom decoherence, which is sometimes called transverse decay rate. In a previous work[10], γ is neglected. This method is also applied to cesium beam or fountain clocks[11], because the coherent times for these types of clock are longer than the interrogation between atoms and microwave, indicating that γ is indeed very small. However, in cell[12,13] or compact cold atom clocks[4−6], the coherent time of the atomic sample is a main limitation in determining the duration of the interrogation. Thus, γ must be considered. A pulsed optically pumped cell clock has been reported in a previous work[12]. In this letter, we employed a more general consideration and focused on cold atom clocks[4−7].

In accordance with a previous work[10], we define

$$\rho_{12} = \frac{1}{2}[a_1(t) + \mathrm{i}a_2(t)]\mathrm{e}^{-\mathrm{i}\omega t}, \tag{7}$$

$$\rho_{11} - \rho_{22} = a_3(t), \tag{8}$$

where $a_1(t)$, $a_2(t)$, and $a_3(t)$ are all real. Inserting Eqs. (4), (7), and (8) into Eqs. (2) and (3) gives

$$\mathrm{d}a_1(t)/\mathrm{d}t + \gamma a_1(t) + \Delta\omega a_2(t) + b_2 a_3(t) = 0, \tag{9}$$

$$-\Delta\omega a_1(t) + \mathrm{d}a_2(t)/\mathrm{d}t + \gamma a_2(t) - b_1 a_3(t) = 0, \tag{10}$$

$$-b_2 a_1(t) + b_1 a_2(t) + \mathrm{d}a_3(t)/\mathrm{d}t = 0, \tag{11}$$

where $\Delta\omega = \omega - \omega_0$. By employing Laplace and inverse Laplace transform to Eqs. (9)–(11), we obtain

$$a(b_1, b_2, \Delta\omega, \gamma, t) = R(b_1, b_2, \Delta\omega, \gamma, t)a(0), \tag{12}$$

where

$$a(b_1, b_2, \Delta\omega, \gamma, t) = \begin{bmatrix} a_1(b_1, b_2, \Delta\omega, \gamma, t) \\ a_2(b_1, b_2, \Delta\omega, \gamma, t) \\ a_3(b_1, b_2, \Delta\omega, \gamma, t) \end{bmatrix}, \tag{13}$$

$$R(b_1, b_2, \Delta\omega, \gamma, t) = \begin{bmatrix} R_{11}, R_{12}, R_{13} \\ R_{21}, R_{22}, R_{23} \\ R_{31}, R_{32}, R_{33} \end{bmatrix}, \tag{14}$$

$$R_{11} = \sum_{i=1}^{3}\Big[\frac{Z_i^2 + Z_i\gamma + b_1^2}{\Lambda_i}\mathrm{e}^{Z_i t}\Big], \tag{15}$$

$$R_{12} = \sum_{i=1}^{3}\Big[\frac{-\Delta\omega Z_i + b_1 b_2}{\Lambda_i}\mathrm{e}^{Z_i t}\Big], \tag{16}$$

$$R_{13} = \sum_{i=1}^{3}\Big[\frac{-\Delta\omega b_1 - b_2 Z_i - b_2\gamma}{\Lambda_i}\mathrm{e}^{Z_i t}\Big], \tag{17}$$

$$R_{21} = \sum_{i=1}^{3}\Big[\frac{\Delta\omega Z_i + b_1 b_2}{\Lambda_i}\mathrm{e}^{Z_i t}\Big], \tag{18}$$

$$R_{22} = \sum_{i=1}^{3}\Big[\frac{Z_i^2 + Z_i\gamma + b_2^2}{\Lambda_i}\mathrm{e}^{Z_i t}\Big], \tag{19}$$

$$R_{23} = \sum_{i=1}^{3}\Big[\frac{Z_i b_1 + b_1\gamma - \Delta\omega b_2}{\Lambda_i}\mathrm{e}^{Z_i t}\Big], \tag{20}$$

$$R_{31} = \sum_{i=1}^{3}\Big[\frac{-\Delta\omega b_1 + b_2 Z_i + b_2\gamma}{\Lambda_i}\mathrm{e}^{Z_i t}\Big], \tag{21}$$

$$R_{32} = \sum_{i=1}^{3}\Big[\frac{-b_1 Z_i - b_i\gamma - b_2\Delta\omega}{\Lambda_i}\mathrm{e}^{Z_i t}\Big], \tag{22}$$

$$R_{33} = \sum_{i=1}^{3}\Big[\frac{(Z_i + \gamma)^2 + \Delta\omega^2}{\Lambda_i}\mathrm{e}^{Z_i t}\Big]. \tag{23}$$

$\Lambda_i = 3Z_1^2 + 4Z_i\gamma + \gamma^2 + \Omega^2$, Z_i is one of the roots of the equation given as

$$Z^3 + 2\gamma Z^2 + (\gamma^2 + \Omega^2)Z + b^2\gamma = 0. \tag{24}$$

Equation (14) is exactly the same as the one given in a previous work[10] when $\gamma = 0$. Let us suppose the width of both Ramsey pulses is τ, and the duration between them is T (Fig. 1).

The Ramsey interaction process is expressed by the following matrix form:

$$\begin{aligned} a(b_1, b_2, \Delta\omega, \gamma, t') = {} & R(b_1, b_2, \Delta\omega, \gamma, \tau)R(0, 0, \Delta\omega, \gamma, T) \\ & \cdot R(b_1, b_2, \Delta\omega, \gamma, \tau)a(0), \end{aligned} \tag{25}$$

where $t' = 2\tau + T$.

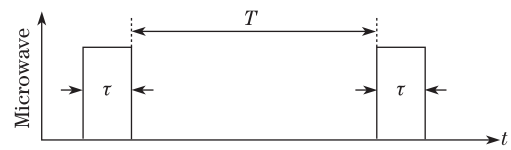

Fig. 1. Time sequence. The widths of the microwave pulses, which are separated by T, are both represented by τ.

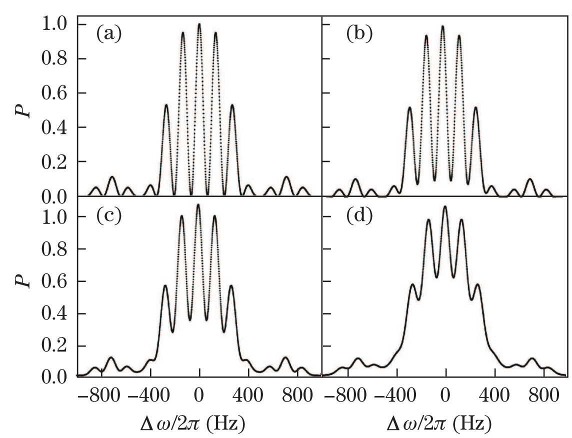

Fig. 2. Ramsey fringes for different values of γ. (a) $\gamma = 0$, (b) 30, (c) 100, and (d) 200 s^{-1}. Here, $b\tau = \pi/2$, and $T = 5$ ms.

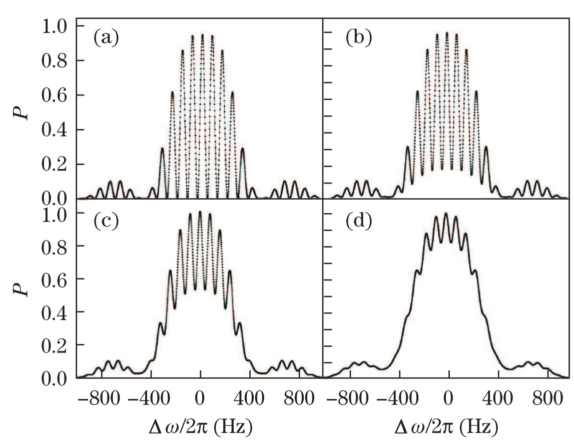

Fig. 3. Ramsey fringes for different values of γ. (a) $\gamma = 0$, (b) 30, (c) 100, and (d) 200 s^{-1}. Here, $b\tau = \pi/2$, and $T = 10$ ms.

The Ramsey signal is obtained by

$$P(b_1, b_2, \Delta\omega, \gamma, t') = [a_3(b_1, b_2, \Delta\omega, \gamma, t') + 1]/2, \quad (26)$$

with the assumption that

$$\rho_{11} + \rho_{22} = 1. \quad (27)$$

If the atoms are prepared in the single ground state $|2\rangle$ given by

$$a(0) = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}, \quad (28)$$

and $\phi = 0$, Eq. (26) becomes

$$P(b, \Delta\omega, \gamma, t') = [a_3(b, \Delta\omega, \gamma, t') + 1]/. \quad (29)$$

From Eq. (29), we can easily obtain the Ramsey fringe by varying $\Delta\omega$.

Figure 2 shows the Ramsey fringes with different transverse decay rates at evolution time $T = 5$ ms. The Ramsey fringe is discussed in detail in a previous work[10] when $\gamma = 0$; as expected, the contrast is 100%, and the contrast of the Ramsey fringe becomes small when γ increases.

A similar case happens when $T = 10$ ms (Fig. 3). Contrast decreases faster as γ increases because of longer evolution time T. In fact, γ^{-1} represents the coherent time of atomic sample. For example, when $\gamma = 100$ s^{-1}, which corresponds to 10 ms coherent time, the contrast for evolution time $T = 5$ ms is still acceptable, because T is smaller than the coherent time. Moreover, quite enough atoms remain coherent when the second Ramsey pulse is applied. However, the contrast becomes smaller because less atoms exist for the second Ramsey pulse when $T = 10$ ms. These effects are summarized in Fig. 4.

Figure 4 shows the Ramsey fringe contrast versus evolution time T for different values of γ. In the equation $\gamma = 10$ s^{-1}, which corresponds to 100 ms coherent time, the contrast drops to half at $T = 100$ ms, indicating that half of the atoms remain coherent at the second Ramsey pulse when the evolution time T is equal to the coherent time. This is also true for $\gamma = 30$ and 100 s^{-1}, which corresponds to coherent times of 33 and 10 ms, respectively.

Obtaining a Ramsey fringe with a reasonable contrast and a width smaller than $1/2T$ (100 Hz) becomes difficult when γ is 200 s^{-1} in cell clocks, corresponds to coherent time of 5 ms[12,13]. However, there is no clear γ estimation for compact cold atom clocks[4−7].

We measured the Ramsey fringe in a cold atom system[7]. The ^{87}Rb atoms were first cooled by diffusing light in an integrating sphere[14−18]. The integrating sphere was placed in a cylindrical cavity. The atoms were detected by absorption light after the application of two $\pi/2$ microwave fields. As discussed in Ref. [7], we obtained the Ramsey fringe by changing the frequency of the microwave. In our system, the temperature of the cold atoms was 75 ± 20 μK, and the vacuum was about 1×10^{-7} Pa. A constant magnetic field at 20 mGauss was applied during cooling and interrogation.

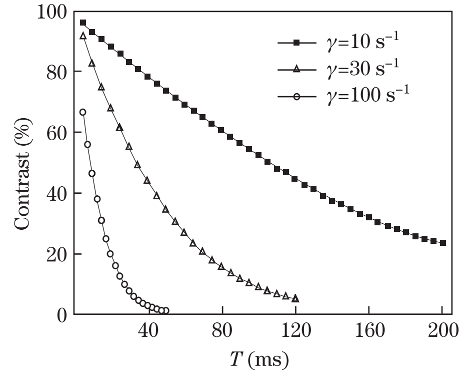

Fig. 4. Ramsey fringe contrast versus evolution time T with different values of γ. Here $b\tau = \pi/2$.

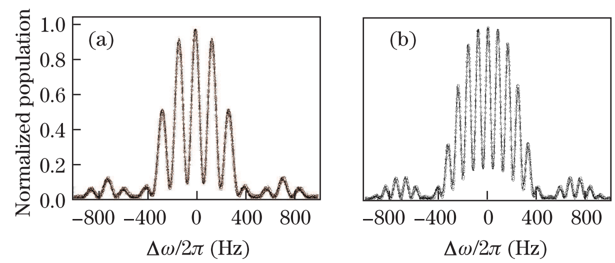

Fig. 5. Fitting of experimental data with (a) $T = 5$ and (b) 10 ms. Here, $b\tau = \pi/2$.

Figure 5 shows the fitting of experimental data by Eq. (29) with appropriate γ. Adjusting γ to fit the experimental Ramsey fringe best shows the contrast error between measurement and theory at 2.6%. Moreover, the fitting of experimental data for both $T = 5$ and 10 ms results in $\gamma = 30.5$ s^{-1}, which is the property of our cold atom system. Longer evolution time T results in more errors, because Eq. (29) is no longer applicable due to the diffusion of cold atoms and gravity. The coherent time in our system is clearly around 32.8 ms, which is around 6 times longer than that in a cell. This coherent time limits evolution time and leads to a Ramsey fringe with an even narrower width.

In conclusion, we develop a simple theory on the calculation of the Ramsey fringe of two-level atoms interacting with a microwave field. Our theory focuses on transverse decay; thus, we fit the experimental data with the theory by adjusting the transverse decay rate. Results show that the transverse decay rate in our cold atom system is 30.5 s^{-1}, which is 6 times less than a typical cell clock. Smaller transverse decay rates result in longer coherent times. This condition results in longer evolution times between two Ramsey pulses, leading to a Ramsey fringe with a narrower width. The system have longer coherent time to achieve a Ramsey fringe with an even narrower width. The main effects resulting from the decoherence are the collisions between cold-cold atoms and cold-background atoms. Therefore, high vacuum and temperature control are also important factors that must be considered.

This work was supported by the National Natural Science Foundation of China under Grant Nos. 11074262, 10874193, and 11034008.

References

1. N. F. Ramsey, Phys. Rev. **78,** 695 (1950).
2. A. Bauch, Metrologia **42,** S43 (2005).
3. R. Wynands and S. Weyers, Metrologia **42,** S64 (2005).
4. F.-X. Esnault, D. Holleville, N. Rossetto, S. Guerandel, and N. Dimarcq, Phys. Rev. A **82,** 033436 (2010).
5. F. X. Esnault, N. Rossetto, D. Holleville, J. Delporte, and N. Dimarcq, Adv. Space Res. **47,** 854 (2011).
6. S. T. Müller, D. V. Magalhães, R. F. Alves, and V. S. Bagnato, J. Opt. Soc. Am. B **28,** 2592 (2011).
7. X. Wang, H. Cheng, B. Zheng, Y. Meng, L. Xiao, L. Liu, and Y. Wang, in *Proceedings of 2011 Joint Conference of the IEEE International Frequency Control Symposium and European Frequency and Time Forum* 1 (2001).
8. W. M. Liu, B. Wu, and Q. Niu, Phys. Rev. Lett. **84,** 2294 (2000).
9. R. Qi, X. L. Yu, Z. B. Li, and W. M. Liu, Phys. Rev. Lett. **102,** 185301 (2009).
10. J. Vanier and C. Audoin, *The Quantum Physics of Atomic Frequency Standards* (Institute of Physics Publishing, Bristol, 1989) chap. 5.
11. D. Lü, B. Wang, T. Li, and L. Liu, Chin. Opt. Lett. **8,** 735 (2010).
12. A. Godone, S. Micalizio, and F. Levi, Phys. Rev. A **70,** 023409 (2004).
13. A. Godone, S. Micalizio, F. Levi, and C. Calosso, Phys. Rev. A **74,** 043401 (2006).
14. H. D. Cheng, W. Z. Zhang, H. Y. Ma, L. Liu, and Y. Z. Wang, Phys. Rev. A **79,** 023407 (2009).
15. W. Z. Zhang, H. D. Cheng, L. Xiao, L. Liu, and Y. Z. Wang, Opt. Express **17,** 2892 (2009).
16. W. Z. Zhang, H. D. Cheng, L. Liu, and Y. Z. Wang, Phys. Rev. A **79,** 053804 (2009).
17. L. Xiao, X. Wang, W. Zhang, H. Cheng, L. Liu, and Y. Wang, Chin. Opt. Lett. **8,** 253 (2010).
18. H. D. Cheng, X. C. Wang, L. Xiao, W. Z. Zhang, L. Liu, and Y. Z. Wang, Chin. Phys. B **20,** 023701 (2011).