Propagation dynamics of nonlinear chirped optical laser pulses in a two-level medium

Haifeng Yao (姚海凤)[1,2], Yueping Niu (钮月萍)[1,3], Yandong Peng (彭延东)[1,2], and Shangqing Gong (龚尚庆)[3,1*]

[1]*State Key Laboratory of High Field Laser Physics, Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai 201800, China*

[2]*Graduate University of Chinese Academy of Sciences, Beijing 100049, China*

[3]*Department of Physics, East China University of Science and Technology, Shanghai 200237, China*

[*]*Corresponding author: sqgong@ecust.edu.cn*

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We investigate the propagation dynamics of nonlinear chirped optical laser pulses in a two-level medium. For certain chirp strength and chirp width, an incident 2π nonlinear chirped pulse will split into optical precursors and a stable self-induced transparency soliton. This is caused by the particular Fourier spectrum that includes not only central resonant frequency components but also high-frequency and low-frequency sidebands. Moreover, the effects of chirp parameters on the evolution of nonlinear chirped pulses are also discussed.

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The coherent interaction between a laser pulse and a two-level system has received much experimental and theoretical attention[1]. For example, the famous area theorem[2,3] could explain many interesting features, such as self-induced transparency (SIT). It predicts that an incident pulse with area (the integral of the absolute magnitude of the field over time) greater than π is reshaped into a number of 2π sech-shaped pulses by propagating. In the last decades, SIT effect has been an area of active research in atomic gases and semiconductors[4−7]. With the development of ultrafast laser technology, much work has been done to extend this analysis to few-cycle ultrashort pulses[7−10]. Xiao *et al*. investigated the propagation of 5-fs pulse in a two-level atom medium[7]. They found that the variation of the few-cycle pulse area was caused by the splitting of the pulse, but not by the broadening or compression of the pulse as in the case of long pulse[2]. For large area pulse, Hughes proposed that the area under individual carriers might cause themselves Rabi flopping, which could lead to carrier-wave reshaping and significantly higher spectral components[8] and even the generation of soft X-ray[9].

On the other hand, extensive studies have manifested that chirping can have much impact on laser pulses when it propagates through different media. Eberly has rederived the area theorem, modifying it to include pulse chirping and homogeneous damping, and obtained a new equation for the evolution of the pulse phase[4]. Hmurcik *et al*. found that when the incident pulse was chirped, the McCall-Hahn area theorem was no longer valid and that higher area was required for the SIT to occur in coherent pulse propagation[11,12]. The results demonstrated that the amplitude of the asymptotically emerging solitons decreased monotonously with increasing chirp strength and that when the linear chirp rate was sufficiently large, the soliton character would be lost completely and only linear dispersive radiation existed. Moreover, in contrast to the smooth degradation of the soliton content with increasing chirp strength, Desaix *et al*. found a different behavior wherein the initial pulse split into two or more separated soliton pairs[13]. However, they only focused on the critical parameters such as pulse area and chirp strength for the SIT to occur by means of the numerical solution of the Zakh-Shabat eigenvalue problem. Meanwhile, Song *et al*. investigated the impact of linear chirp rate on the propagation and spectral features during propagation[14]. Recently, a nonlinear chirped laser pulse with phase variation $\phi(t) = \beta \tanh(t/\sigma)$ has attracted wide attention[15,16]. The phase of the pulse is still linear for small t, but the increase, although still monotonous, saturates at a finite value as $t \to \pm\infty$. This kind of chirp form is the source of motivation of this study.

In this letter, we investigate the propagation dynamics of nonlinear chirped laser pulses in a two-level medium. An incident 2π nonlinear chirped pulse with certain chirp strength and chirp width will evolve into sub-pulses, including optical precursors and a stable SIT soliton. By increasing the chirp strength $|\beta|$ or decreasing the chirp width σ, a larger intensity SIT soliton forms. Moreover, the splitting of an initial pulse into separated optical precursors and SIT soliton pulse could also occur for other forms of chirp.

Considering the evolution of a laser pulse along the z axis in vacuum into an input interface of a resonant two-level medium at $z = 31$ μm, the propagation dynamics can be described by the full-wave Maxwell-Bloch equations:

$$\begin{aligned} \partial_t H_y &= -\frac{1}{\mu_0}\partial_z E_x, \\ \partial_t E_x &= -\frac{1}{\varepsilon_0}\partial_z H_y - \frac{1}{\varepsilon_0}\partial_t P_x, \\ \partial_t u &= -\omega_0 v - \frac{u}{T_1}, \end{aligned} \tag{1}$$

$$\partial_t v = \omega_0 u + 2\Omega w - \frac{v}{T_1},$$

$$\partial_t w = -2\Omega v - \frac{w - w_0}{T_2},$$

where E_x and H_y are the electric and magnetic fields, and ε_0 and μ_0 are the electric permittivity and the magnetic permeability in vacuum, respectively. T_1 and T_2 are the polarization and population relaxation times, respectively. ω_0 is the transition frequency of the system and $\Omega = \mathrm{d}E_x/\hbar$ is the Rabi frequency. The terms u, v, and w represent the dispersion, absorption, and population difference between the upper and lower states, respectively. The macroscopic polarization is $P_x = N\mathrm{d}u$, where N is the density of the medium and d is the dipole moment.

We consider a sech-shaped laser pulse with a phase variation, which can be written as

$$\begin{aligned} E\left(z_0, t\right) = \ & E_0 \mathrm{sech}\left[1.76\left(t - t_0\right)/\tau_p\right] \\ & \cdot \cos\left[\omega_p\left(t - t_0\right) + \phi\left(t - t_0\right)\right] \ , \end{aligned} \tag{2}$$

where ω_p is the central carrier frequency, E_0 is the maximum electric field, t_0 is the middle of the pulse envelope, and τ_p is the duration of the incident pulse intensity envelope.

The phase variation has the following time-varying hyperbolic tangent form:

$$\phi(t - t_0) = \beta \tanh\left[(t - t_0)/\sigma\right] \ , \tag{3}$$

where the parameter β denotes the chirp strength and $\sigma = nT$ with $T = 2\pi/\omega_\mathrm{p}$ is the chirp width, which determines the temporal width of the chirp variation.

In the following analysis, the medium is initialized with $u = v = 0$ and the population difference is $w_0 = -1$ at $t = 0$. The material and laser pulse parameters based on Ref. [7] are $\omega_p = \omega_0 = 2.3$ fs^{-1} ($\lambda = 830$ nm), $d = 2 \times 10^{-29}$ C · m, $N = 2 \times 10^{18}$ cm^{-3}, and $T_1 = T_2 = 1$ ns. The input envelope area for the pulse is $A = dE_0\tau_\mathrm{p}\pi/1.76\hbar$, with a 2π pulse corresponding to a peak amplitude $E_0 = 4.673 \times 10^8$ V/m or an intensity of $I = 5.76 \times 10^{10}$ W/cm^2 for $\tau_p = 40$ fs.

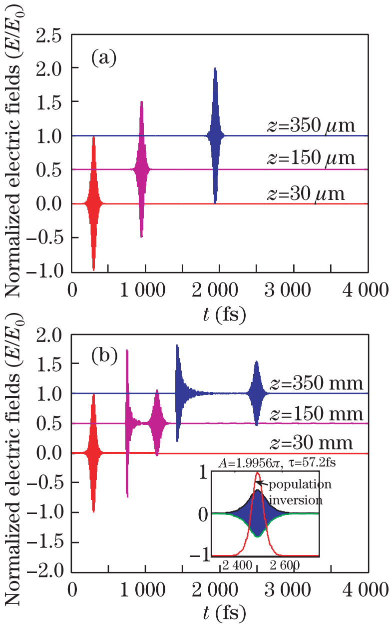

Fig. 1. (Color online) Normalized electric fields at the respective propagation distances of 30, 150, 350 μm for the incident (a) chirp-free pulse and (b) nonlinear chirped pulse with parameters of $\beta = 3$, $\sigma = 3T$; the inset shows the delayed soliton and the corresponding population inversion at $z = 350$ μm.

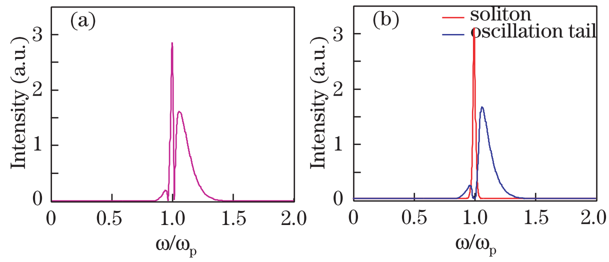

Fig. 2. (Color online) (a) Spectrum of the incident pulse with $\beta = 3$, $\sigma = 3T$; (b) spectra of the soliton (red dashed line) and the oscillatory tail before the soliton (blue solid line) at $z = 350$ μm.

Figure 1(a) presents the evolution of a 2π chirp-free ($\beta = 0$) optical pulse with $\tau_\mathrm{p} = 40$ fs at different propagation distances. According to the area theorem, the pulse is a standard SIT soliton, which could propagate through the medium with no reshaping. For an incident nonlinear chirped pulse with chirp parameters of $\beta = 3$ and $\sigma = 3T$, as can be seen from Fig. 1(b), a strong reshaping effect occurs: the incident nonlinear chirped pulse splits into two sub-pulses, including an oscillatory structure and a soliton pulse. The soliton is a SIT soliton since the calculated area is appropriately 2π, and a complete Rabi oscillation (red dashed line) can be induced as shown in the inset of Fig. 1(b). SIT solitons are stable and have slower velocity than the light velocity c in vacuum. Thus, the separation between the front of the oscillatory structure and the soliton becomes larger with the increase in propagation distance. Furthermore, the duration of the split SIT soliton is about 57.2 fs, which is longer than that in the chirp-free case ($\tau_\mathrm{p} = 40$ fs). Moreover, its velocity is slower than that in the chirp-free case, as can be seen when we compare Figs. 1(a) and (b).

In order to explore the physical essence of the phenomenon, we treat the problem from the frequency domain since chirp will generally generate a relatively broader Fourier spectrum. The spectrum of the incident nonlinear chirped pulse with $\beta = 3$ and $\sigma = 3T$ is shown in Fig. 2(a). The spectrum contains not only the central frequency components but also the separated high-frequency and low-frequency sidebands. Furthermore, the spectra of the delayed SIT soliton (red line) and the oscillatory tail (blue line) before the soliton at $z = 350$ μm are presented in Fig. 2(b). This indicates that the central resonant frequency components contribute to the formation of the SIT soliton, while the separated high-frequency and low-frequency sidebands result in the oscillatory structure. The oscillatory structure is the result of the interference of the Sommerfeld precursor (high-frequency components) and the Brillouin precursor (low-frequency components)[17,18]. It is indicated that precursors are contributed from the far-off resonance spectral components. Thus, it can be concluded that a 2π nonlinear chirped pulse can split into optical precursors and a 2π SIT soliton during propagation. On the other hand, because only the central resonant frequency components contribute to the formation of the SIT soliton, the area will change to 2π through pulse broadening. This is the reason why the duration of the split SIT soliton is longer than that in the chirp-free case.

We now consider the effects of chirp parameters on pulse evolution. Figure 3(a) shows the normalized electric fields at the propagation distance $z = 350$ μm for different chirp strengths of $\beta = 2.8$, 3.0, and 3.2 with the same chirp width of $\sigma = 3T$. The velocity of the split SIT soliton increases with the increase in the chirp strength β. By comparing the corresponding spectra as shown in Fig. 3(c), we can see that as the chirp strength β increases, the intensity of the central resonant frequency components also increases. As a result, a larger SIT soliton is obtained for larger initial chirp strength β, and the separation between the optical precursors and the SIT soliton becomes smaller. The normalized electric fields at $z = 350$ μm and the corresponding spectra for the incident pulses for the different chirp widths of $\sigma = 2.0T$, $\sigma = 3.0T$, and $\sigma = 4.0T$ with the same chirp strength of $\beta = 3$ are shown in Figs. 3(b) and (d). As the width σ increases, the intensity of the central resonant frequency components decreases. Thus, a smaller SIT soliton is obtained for larger chirp width σ, and the separation between the optical precursors and the SIT soliton becomes larger.

As mentioned above, the propagation dynamics lies on the Fourier spectra of nonlinear chirped pulses containing not only the central resonant components but also the separated high-frequency and low-frequency sidebands. If the chirp strength is small enough ($|\beta| < 0.1$), the pulse will evolve into a bare SIT soliton as in the chirp-free case

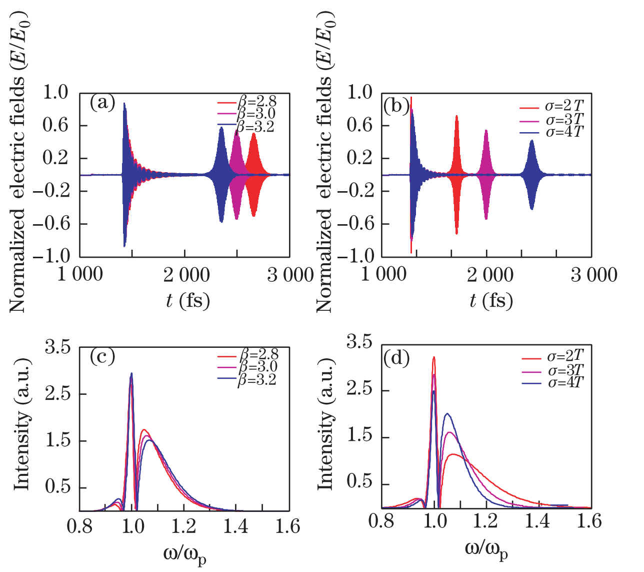

Fig. 3. (Color online) Normalized electric fields at the propagation distance of $z = 350$ μm for (a) $\beta = 2.8$, $\beta = 3.0$, $\beta = 3.2$ and (b) $\sigma = 2.0T$, $\sigma = 3.0T$, $\sigma = 4.0T$; corresponding spectra of incident pulses with different initial chirp parameters for (c) $\beta = 2.8$, $\beta = 3.0$, $\beta = 3.2$ and (d) $\sigma = 2.0T$, $\sigma = 3.0T$, $\sigma = 4.0T$.

($\beta = 0$). If the chirp strength is large enough ($|\beta| > 5$) or the chirp width is sufficiently large ($\sigma > \tau_p/3$), the SIT soliton character is completely destroyed because the resonant frequency components almost disappear.

From the above analysis, it can be seen that the phenomenon could extend to a wider class of pulses whose Fourier spectra contain central resonant components and separated high-frequency and low-frequency sidebands, such as $\phi_1(t) = \alpha t \Big/ \sqrt{1+\gamma t^2}$ and $\phi_2(t) = \eta \arctan(t/\kappa)$.

In conclusion, we demonstrate that an incident 2π nonlinear chirped pulse with moderate chirp strength and width will split into optical precursors and a SIT soliton because its Fourier spectrum contains not only central resonant frequency components but also separate high-frequency and low-frequency sidebands. It is found that a larger chirp strength $|\beta|$ or smaller chirp width σ results in a larger 2π SIT soliton and a slight separation between optical precursors and the SIT soliton. Additionally, the phenomenon can be extended to a wider class of pulse shapes with different chirp forms, provided that their Fourier spectra contain not only the central resonant frequency components but also separated high-frequency and low-frequency sidebands.

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