Research on the Moiré fringes formed by circular and linear grating

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The features of Moiré fringe generated by overlapping a circular and a linear grating are studied. Given that the pitch of circular grating is a and that of linear grating is P, the shapes of the Moiré fringe that they form are hyperbola, parabola, and ellipse when $a > P$, $a = P$, and $a < P$, respectively. As the pitches of these two gratings become close to each other, the magnification of the Moiré fringe is over 100, which is useful for the measurement of small displacement. This letter also discusses how the fill factor influences Moiré fringe visibility.

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Given that the Moiré fringe has high rate of amplification, metrology gratings that can form a Moiré fringe have been widely applied in many fields, such as precision instruments, super finishing, CNC machine, and so on[1]. In particular, the system composed of two parallel gratings is often used for displacement measurement. Moiré fringe technology has been widely applied to machining, laboratory, and photoelectric instruments[2]. In recent years, grating interferometers used in the field of urban construction have become low-cost, miniaturized, and portable, making them easy to use for outdoor measurements. Constructing a sensor monitoring network on the building surface can effectively help construction engineers learn about the change of the stress inside the building and displacement in the plane by analyzing Moiré interference patterns[3]. In addition, when fringe image as shadows of the grid is analyzed by wavelet transform, micro-sized products can be measured using this three-dimensional (3D) measurement technique[4].

Nowadays, the most widely applied kind of metrological grating is formed by two circular or two linear gratings. The shape of this kind of Moiré fringe is simple and fixed, and is typically used in measuring tiny relative displacement. In this letter, a new grating composite system is proposed (Fig. 1 (a)). This system consists of a circular grating and a linear grating. The shapes of the Moiré fringe are hyperbola, parabola, or ellipse depending on the different pitches of two gratings. This kind of Moiré fringe contains the amplification effect and has much more shape facility than the previous ones. Thus, it can be used in the new field.

In the past, using metrological gratings to acquire and analyze data depends mainly on photoelectric detectors, integrated circuit, and MCU. However, because the Moiré fringe consisting of circular and linear gratings is more manifold and complicated, it is very difficult to analyze it in the traditional way. Fortunately, with the development of charge coupled device (CCD) and digital imaging processing technology, this new grating system can be better studied.

By exploring the relationship between grating parameters containing pitch and fill factor, as well as the shape, visibility and size of the Moiré fringe, the choice of grating parameters can be analyzed and summarized to achieve an ideal observation result. The ordinal equation method is used to study the general performance of the grating. At the same time, along with the change of fill factor and pitch of grating, the Moiré fringe possesses different properties that provide significant theoretical foundation for the application of this kind of Moiré fringe.

We establish a rectangular coordinate system whose origin is the center of the circular grating (Fig. 1 (b)). Taking the initial phases of the two gratings as zero, we will have grating equations of circular grating and linear grating given by

$$x^2 + y^2 = (ma)^2, \tag{1}$$

$$x = nP, \tag{2}$$

where m and n are grating ordinals, a is the pitch of circular grating, and P is the pitch of linear grating. Using the relationship $m - n = \pm N$, where N is Moiré fringe ordinal, the Moiré cluster equation can be obtained as follows[5]:

$$(P^2 - a^2)x^2 \pm 2aPNx + P^2y^2 = a^2P^2N^2. \tag{3}$$

From Eq. (3), the hyperbola, parabola, and ellipse are represented by $a > P$, $a = P$, and $a < P$, respectively, which are shown as

$$\left[\sqrt{(a^2 - P^2)}x \mp \frac{aPN}{\sqrt{(a^2 - P^2)}}\right]^2 - P^2y^2 = \frac{a^2P^2N^2}{(a^2 - P^2)} - a^2P^2N^2, \tag{4}$$

$$y^2 = \pm 2PNx + P^2N^2, \tag{5}$$

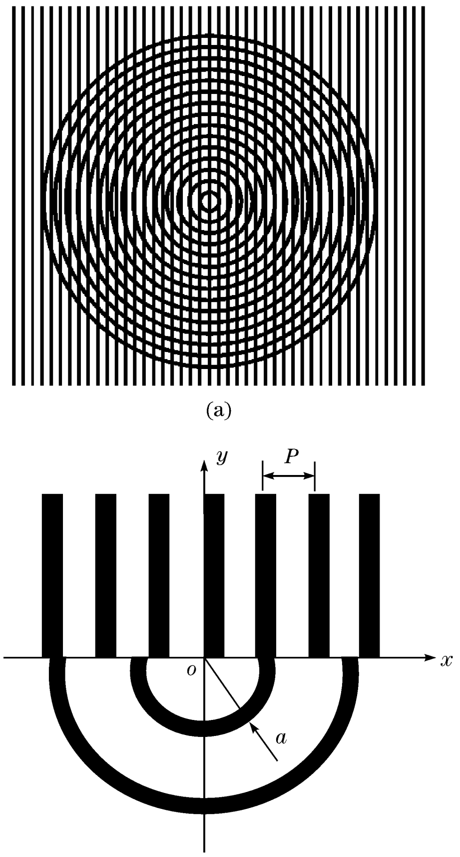


Fig. 1. (a) Overlap of a circular grating and a linear grating; (b) the establishment of grating coordinate.

$$\left[\sqrt{(P^2-a^2)}x \pm \frac{aPN}{\sqrt{(P^2-a^2)}}\right]^2 + P^2y^2$$
$$= a^2P^2N^2 + \frac{a^2P^2N^2}{(P^2-a^2)}. \quad (6)$$

Using a software, we drew a large number of circular and linear gratings with different parameters. Through a series of observations and comparisons, we found that the grating parameters influencing the Moiré fringe were phase shift, pitch, and fill factor. The key that determines the relative position between the Moiré fringe and the grating is the phase difference between these two gratings. Periodic changes in phase are shown as the movement of the Moiré fringe.

When $P>a$, the figure of the Moiré fringe is ellipse. When we put the center of circular grating at the origin of the coordinate, the Moiré fringe at $y=0$ (i.e., x-axis) approximates to vertical stripes (i.e., Moiré fringes formed by two parallel linear gratings with different pitches, and whose direction is consistent with that of the gratings). Thus, for facilitating derivation and observation, we only studied the Moiré fringe adjacent to the x-axis.

The grating equations of the two linear gratings are given by:

$$x = nP, \quad (7)$$

$$x = ma. \quad (8)$$

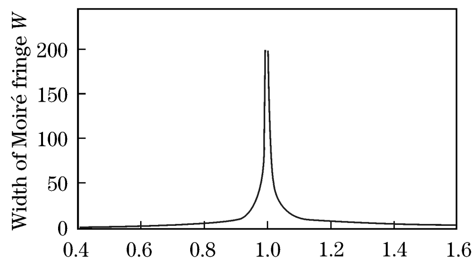


Fig. 2. Plots of fringe width with pitch ratio P/a.

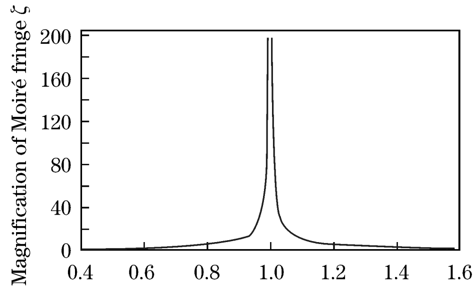


Fig. 3. Plots of magnification with pitch ratio P/a.

Table 1. Observed Results under the Different Pitch Ratios

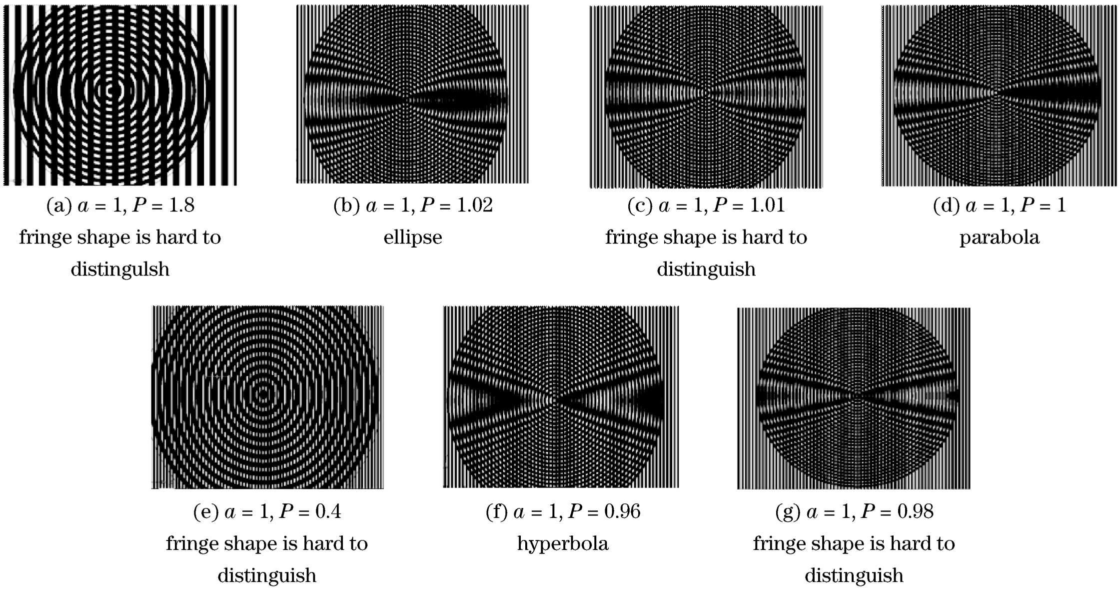
P	a	W	ζ	Result
1.8	1	2.25	4.05	fringe shape is hard to distinguish
1.02	1	51	52.02	ellipse
1.015	1	67.7	68.68	barely discernible ellipse
1.01	1	101	102.01	fringe shape is hard to distinguish
1	1	∞	∞	parabola
0.98	1	49	48.02	fringe shape is hard to distinguish
0.97	1	32.3	31.36	barely discernible hyperbola
0.96	1	24	23.04	hyperbola
0.8	1	4	3.2	hyperbola
0.4	1	0.67	0.27	fringe shape is hard to distinguish

Using ordinal equation $m-n=\pm N$, we can solve the equation of the Moiré fringe cluster as

$$x = \pm\frac{NaP}{P-a}. \quad (9)$$

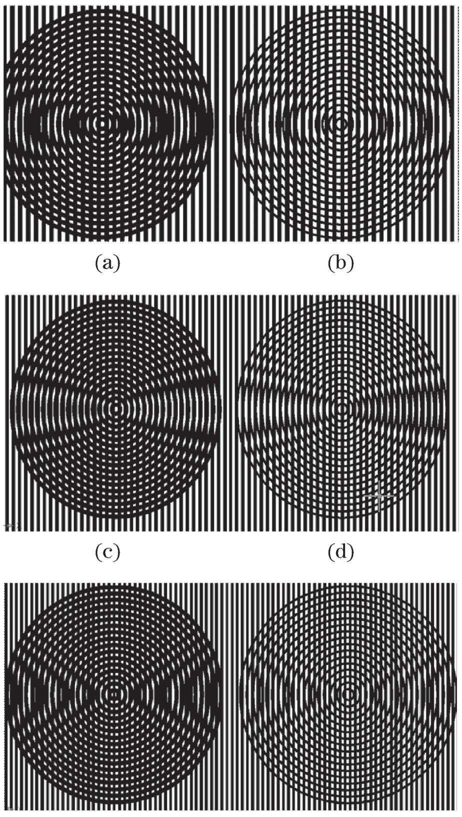
Thus, the width of the Moiré fringe W and the magnification of the grating pair ξ are

$$W = \left|\frac{aP}{P-a}\right|, \quad (10)$$



(a) $a = 1, P = 1.8$ fringe shape is hard to distingulsh
(b) $a = 1, P = 1.02$ ellipse
(c) $a = 1, P = 1.01$ fringe shape is hard to distinguish
(d) $a = 1, P = 1$ parabola
(e) $a = 1, P = 0.4$ fringe shape is hard to distinguish
(f) $a = 1, P = 0.96$ hyperbola
(g) $a = 1, P = 0.98$ fringe shape is hard to distinguish

Fig. 4. Moiré fringes with different P/a values.



(a) (b)
(c) (d)
(e) (f)

Fig. 5. Moiré fringes with different fill factors.

$$\xi = \frac{W}{P} = \left| \frac{1}{P/a - 1} \right|. \quad (11)$$

When we only consider one side of the parabola, i.e., $y^2 = 2PNx + P^2N^2 = 2PN(x + \frac{PN}{2})$, the Moiré fringes comprise a series of parabolas that share a common focus at zero. The vertex of the Nth parabola is $x^N = -\frac{PN}{2}$. When the linear grating moves at a distance of P, the movement of the Moiré fringes reaches half of P. In the case of parabolic Moiré fringes, there is no intersection on the x-axis. On the other hand, when they are elliptical or hyperbolical near the intersections with the x-axis, they can be considered to be approximately perpendicular stripes.

From Eq. (11), it can be seen that the smaller the difference between the circular grating pitch and the linear grating pitch is the better the Moiré fringe amplification effect becomes. According to Eqs. (10) and (11), the curves of W versus P/a and ξ versus P/a are plotted in Figs. 2 and 3 by MATLAB software, respectively. The magnification is especially high when the pitches of the two gratings are very close (a is approximately equal to P), whereas it significantly drops when there is a huge difference between a and P.

In the current study, we chose some examples for visual observation and theoretical calculation. Table 1 shows the shape of the Moiré fringes and the calculated values of the magnification and fringe width, as the pitch of circular grating is constant and that of linear grating is various. The gratings were drawn by AUTOCAD and their fill factors were all 0.5.

We can obtain higher magnification when the value of P/a is close to one. However, along with the magnification, the obscurity of the shape of the Moiré fringe also increases (Fig. 4).

1) When P is much bigger or smaller than a (second and eleventh lines in Table 1), the group of Moiré fringes become sparser, and it is harder to identify their shape. This is not beneficial to observation and measurement.

2) When P is quite close to a (fifth and seventh lines in Table 1), the magnification is very high (usually reaching 100–200). The Moiré fringe looks both elliptical and parabolic if P is a little larger than a, whereas it looks both hyperbolic and parabolic when P is a bit smaller

than a. The size of gratings is limited, so we cannot observe enough number of fringes when P is close to a; in this case, it is also difficult to recognize their shapes. Therefore, P and a are usually not very close to each other, although the magnification can be high.

3) To obtain clear elliptical or hyperbolic Moiré fringes, the difference between P and a must be about 20%–50% of the smaller of the two pitches.

From the formulas discussed above, the fill factor of gratings does not affect the shape and width of the Moiré fringe. Here are some examples of Moiré fringes formed by gratings with different fill factors (Fig. 5). The fill factors of circular grating and linear grating are all 0.4 and 0.5 in Figs. 5 (a), (c), and (e), whereas they are all 0.7 and 0.5 in Figs. 5 (b), (d), and (f). The pitch ratios of circular grating and linear grating are 1:1.2, 1:1, and 1:0.8 in Figs. 5 (a) and (b), Figs. 5 (c) and (d), and Figs. 5 (e) and (f), respectively.

The fill factor affects the contrast rather than the width of the fringes. When the fill factors of the two gratings are both small and close to each other, the Moiré fringes can obtain high contrast and a smoother border, which is beneficial to observation and measurement.

To obtain clear Moiré fringes, the fill factors of the two gratings should be less than 0.5, and the difference between them must be less than 30% of the smaller one.

The larger the width of the Moiré fringe is, the more ideal the observation result is. Therefore, we should abbreviate appropriately the pitch difference between circular grating and linear grating. However, if the gap between the pitches of the two gratings is too small, both the distinctness of the shape of the Moiré fringe and the number of observed fringes become diminished. Thus, the rational choice of grating pitch has great significance for observing the Moiré fringe.

In addition, to achieve clearer observation result, the choice of the fill factor of the gratings is also very important. The fill factors of the two gratings should be slightly smaller on the premise of ensuring adequate transmittance. At the same time, the gap between the pitches of two gratings should not be too large.

The Moiré fringe formed by circular grating and linear grating has some unique features compared with the traditional Moiré fringe formed by two circular or two linear gratings. The former is in the shape of quadratic curve, whereas the latter is usually circular or linear. The shape of a traditional Moiré fringe is fixed; on the contrary, the shape of a Moiré fringe formed by circular grating and linear grating can consistently change from an ellipse to a parabola or hyperbola according to the change of grating pitches. Therefore, if we use circular grating and linear grating, which share the same pitch and are separated by lenses, we can obtain a Moiré fringe by overlapping one grating with the image of the other. For example, we can find the focus of a convex lens by adjusting the position of the two gratings until we get parabolic Moiré fringe. Moreover, we can also use the Moiré fringe to evaluate lens distortion. Given that distortion can make the pitch of the image of the grating unstable, the width and the shape of the Moiré fringe in the margin of the visual field become different from those in the center.

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