Output nonlocality and nonclassicality in a two-mode entanglement laser

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Output nonlocality and nonclassicality for the two modes are investigated in an entanglement laser system. Within the framework of a quantum theory of multiwave mixing, nonlocality and nonclassicality are discussed according to the violations of Bell inequality and Cauchy-Schwarz inequality. It is found that both nonlocality and nonclassicality can be fulfilled in the outside cavity fields under certain conditions. It is also shown that there are some nonclassical states that do not show nonlocality.

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Quantum nonlocality, one of the most remarkable aspects of quantum theory, can be described by the violation of Bell inequality[1]. Recently, violations of Bell inequality in different physical systems have been investigated[2−6]. Cauchy-Schwarz (CS) inequality is the mathematical relations between the cross-correlation function and the autocorrelation function of two intensity-correlated light beams[7], and its violation is a nonclassical characteristic of light fields[8].

On the other hand, entanglement is arguably the central concept in quantum information processing[9−12]. Recently, continuous-variable entanglement is proposed as an alternative to discrete level systems for performing quantum information tasks[13,14]. The appearance of laser greatly changed the world nearly in every aspect of daily life. Thus, the entangled lights in laser systems must take an important effect in the process of quantum information, especially in quantum communications. In some recent studies, the entanglement amplifiers based on laser systems using three-level atoms interacting with two modes of the cavity field were theoretically proposed[15−17]. However, few attention has been devoted to the output nonlocal and nonclassical characteristics in these laser systems. For instance, Tan *et al.* investigated the output entanglement in the correlated emission laser[18]. Zhou *et al.* studied the output squeezing and entanglement in a single-atom laser[19]. Our previous work also investigated the output entanglement in a two-mode three-level atomic system[20].

In this letter, based on the input-output theory, we investigate the output nonlocality and nonclassicality in a two-mode entanglement laser system. Under the framework of a quantum theory of multiwave mixing, the measurements of nonlocality and nonclassicality are discussed according to the violations of Bell inequality and CS inequality, respectively. It is shown that there are some nonclassical states that do not show nonlocality.

We consider a two-photon three-level cascade configuration, as shown in Fig. 1. The upper level a and the bottom level c have the same parity, but the intermediate level b has an opposite one. The dipole-allowed transitions a⟷b and b⟷c with frequencies ν_1 and ν_3, respectively, are considered weak and treated quantum mechanically up to the second order in coupling constant. The transition a⟷c requires two pump photons of frequency ν_2. Strong pump field is treated classically up to all orders. We assume that the one-photon pump detuning $\omega_{\rm bc}-\nu_2$ is sufficiently large that the dipole transition c⟷b with pump frequency ν_2 is negligible. The pump frequency ν_2 is exactly one half the atomic transition frequency $\omega_{\rm ac}\equiv\omega_{\rm a}-\omega_{\rm c}$. The side-mode frequencies ν_1 and ν_3 are assumed to satisfy the conservation condition $\nu_1+\nu_3=2\nu_2$, which gives the relation between the side-mode detuning Δ' and the beat frequency $\Delta\equiv\nu_2-\nu_1$ as $\Delta'=(\omega_{\rm bc}-\nu_2)-\Delta$.

The Hamiltonian for the atom-field system is[21]

$$H=H_0+V, \tag{1}$$

where the unperturbed part of the Hamiltonian is

$$H_0=\sum_{i={\rm a,b,c}}\hbar\omega_i|i\rangle\langle i|+\sum_{j=1}^{3}\hbar\nu_j a_j^\dagger a_j, \tag{2}$$

and the perturbed part is

$$V=\sum_{j=1}^{3}\hbar g_j a_j U_j\sigma_j^\dagger+H.c., \tag{3}$$

where a_1 and a_3 are the annihilation operators for the field modes 1 and 3, a_2 is the effective two-photon

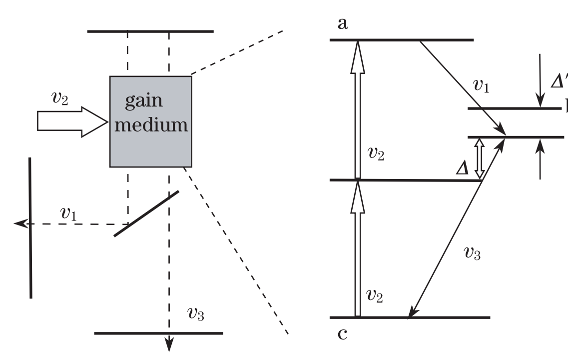

Fig. 1. Systematic diagram for a two-mode three-level entanglement laser system.

annihilation operator for the pump mode, $U_j = U_j(r)$ is the spatial mode factor for the jth field mode, and g_j is the corresponding atom-field coupling constant. The matrices $\sigma_j^\dagger$ are

$$\sigma_1^\dagger = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \sigma_2^\dagger = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \sigma_3^\dagger = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}. \tag{4}$$

The time dependence of the atom-field density operator $\rho_{\mathrm{a-f}}$ can be obtained from the basic density operator equation of motion, as

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{\mathrm{a-f}} = -\frac{\mathrm{i}}{\hbar}[H, \rho_{\mathrm{a-f}}] + r, \tag{5}$$

where r denotes the relaxation processes. By considering the slowly varying field modes and taking traces over the atomic states, the density matrix equation of motion for the field modes, as obtained in Ref. [21] is

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\rho = & - A_1(\rho a_1 a_1^\dagger - a_1^\dagger \rho a_1) - (B_1 + \kappa_1)(a_1^\dagger a_1 \rho - a_1 \rho a_1^\dagger) \\ & - A_3(\rho a_3 a_3^\dagger - a_3^\dagger \rho a_3) - (B_3 + \kappa_3)(a_3^\dagger a_3 \rho - a_3 \rho a_3^\dagger) \\ & + C_3(a_3^\dagger a_1^\dagger \rho - a_1^\dagger \rho a_3^\dagger)\mathrm{e}^{-\mathrm{i}\phi} \\ & + D_1(\rho a_3^\dagger a_1^\dagger - a_1^\dagger \rho a_3^\dagger)\mathrm{e}^{-\mathrm{i}\phi} + H.c., \end{aligned} \tag{6}$$

with $\kappa_j (j = 1, 3)$ is the damping constant of each mode. Different coefficients are given by

$$A_1 = \frac{N g_1^2 \mathscr{D}_1}{1 + I_2^2} \frac{f_\mathrm{a} + I_2^2 \mathscr{D}_3^* D_2/4T_1T_2}{1 + I_2^2 \mathscr{D}_1 \mathscr{D}_3^*/4T_1T_2}, \tag{7a}$$

$$B_1 = \frac{N g_1^2 \mathscr{D}_1}{1 + I_2^2} \frac{f_\mathrm{b}}{1 + I_2^2 \mathscr{D}_1 \mathscr{D}_3^*/4T_1T_2}, \tag{7b}$$

$$A_3 = \frac{N g_3^2 \mathscr{D}_3}{1 + I_2^2} \frac{f_\mathrm{b}}{1 + I_2^2 \mathscr{D}_1^* \mathscr{D}_3/4T_1T_2}, \tag{7c}$$

$$B_3 = \frac{N g_3^2 \mathscr{D}_3}{1 + I_2^2} \frac{f_\mathrm{c} - I_2^2 \mathscr{D}_1^* D_2/4T_1T_2}{1 + I_2^2 \mathscr{D}_1^* \mathscr{D}_3/4T_1T_2}, \tag{7d}$$

$$C_3 = \frac{\mathrm{i} N g_3^2 \mathscr{D}_3}{1 + I_2^2} \frac{I_2}{2(T_1T_2)^{1/2}} \frac{-f_\mathrm{a} \mathscr{D}_1^* + D_2}{1 + I_2^2 \mathscr{D}_1^* \mathscr{D}_3/4T_1T_2}, \tag{7e}$$

$$D_1 = \frac{\mathrm{i} N g_1^2 \mathscr{D}_1}{1 + I_2^2} \frac{I_2}{2(T_1T_2)^{1/2}} \frac{f_\mathrm{c} \mathscr{D}_3^* + D_2}{1 + I_2^2 \mathscr{D}_1 \mathscr{D}_3^*/4T_1T_2}. \tag{7f}$$

The complex Lorentzian for the field modes 1 and 3 is $\mathscr{D}_{1,3} = 1/(\gamma_{1,3} + \mathrm{i}\Delta_{1,3})$, where $\Delta_1 = \omega_\mathrm{a} - \omega_\mathrm{b} - \nu_1 = -\Delta'$ and $\Delta_3 = \omega_\mathrm{b} - \omega_\mathrm{c} - \nu_3 = \Delta'$. $D_2 = 1/\gamma_2$, where $\gamma_2 \equiv 1/T_2$ is the two-photon coherent decay rate between the levels a and c. The dimensionless pump intensity I_2 is defined by $I_2 = 2|V_2|(T_1T_2)^{1/2}$, where $V_2 = g_2 U_2 (n_2)^{1/2}$ is the effective two-photon interaction energy. The population difference decay time T_1 is

$$T_1 = \frac{1}{\Gamma_\mathrm{a}}\left[1 + \frac{\Gamma_1}{2\Gamma_3}\right], \tag{8}$$

where $\Gamma_\mathrm{a} = \Gamma_1 + \Gamma_2$ is the upper, level decay rate to the lower levels b and c. Γ_1 and Γ_3 are the decay constants for the a→b and b→c transitions, and Γ_2 allows for non-radiative decay of level a to c. The probability factors f_k are

$$f_\mathrm{a} = \frac{\Gamma_3}{\Gamma_1 + 2\Gamma_3} I_2^2, \tag{9a}$$

$$f_\mathrm{b} = \frac{\Gamma_1}{\Gamma_1 + 2\Gamma_3} I_2^2, \tag{9b}$$

$$f_\mathrm{c} = 1 + f_\mathrm{a}. \tag{9c}$$

Also, ϕ is the phase of the classical pump field, which can be obtained from the relation $V_2 = |V_2|\mathrm{e}^{-\mathrm{i}\phi}$, and N is the total number of interacting atoms.

In order to look for a violation of Bell inequality when dealing with a driven two-mode three-level cascade atomic system, we use the generalization of Bell inequality proposed by Ansari[6]. The Bell inequality can be rewritten in the form

$$|B| < 2, \tag{10}$$

where

$$B = 1.414 \frac{\langle a_1^{\dagger 2} a_1^2 \rangle + \langle a_3^{\dagger 2} a_3^2 \rangle - 4\langle a_1^\dagger a_3^\dagger a_1 a_3 \rangle}{\langle a_1^{\dagger 2} a_1^2 \rangle + \langle a_3^{\dagger 2} a_3^2 \rangle + 2\langle a_1^\dagger a_3^\dagger a_1 a_3 \rangle}. \tag{11}$$

In order to characterize the statistical properties of light beams in one state, we introduce the following function[22],

$$G_{ij} = \frac{\langle a_i^\dagger a_j^\dagger a_i a_j \rangle}{\langle a_i^\dagger a_i \rangle \langle a_j^\dagger a_j \rangle}, i, j = 1, 3, \tag{12}$$

where G_{ii} define the degrees of the second-order coherence in the modes and G_{ij} (i≠j) describes the degree of intermode correlation. The CS inequality can be violated in systems where the correlation between the photons of different modes is larger than the correlation of the photon of the same mode. Hence, the CS inequality is violated when

$$G_{ij}^2 > G_{ii} G_{jj}. \tag{13}$$

For convenience, we discuss the quantity

$$\lambda = \frac{\langle a_1^\dagger a_3^\dagger a_1 a_3 \rangle}{\langle a_1^\dagger a_1 \rangle \langle a_3^\dagger a_3 \rangle} - \frac{(\langle a_1^{\dagger 2} a_1^2 \rangle \langle a_3^{\dagger 2} a_3^2 \rangle)^{1/2}}{\langle a_1^\dagger a_1 \rangle \langle a_3^\dagger a_3 \rangle}. \tag{14}$$

The CS inequality is violated if $\lambda > 0$.

Using the same methods in Ref. [6], the fourth-order moments can be calculated from the steady-state solution in the Q representation in antinormally ordered form. Considering the commutation relation $[a_i, a_i^\dagger] = 1$, we can obtain

$$\langle a_1^{\dagger 2} a_1^2 \rangle = 2\langle a_1^\dagger a_1 \rangle^2, \tag{15}$$

$$\langle a_3^{\dagger 2} a_3^2 \rangle = 2\langle a_3^\dagger a_3 \rangle^2, \tag{16}$$

$$\langle a_1^\dagger a_3^\dagger a_1 a_3 \rangle = \langle a_1^\dagger a_1 \rangle \langle a_3^\dagger a_3 \rangle + |\langle a_1 a_3 \rangle|^2. \tag{17}$$

After substituting Eqs. (15)–(17) into Eq. (14), the following simple expression for λ is obtained:

$$\lambda = \frac{|\langle a_1 a_3 \rangle|^2}{\langle a_1^\dagger a_1 \rangle \langle a_3^\dagger a_3 \rangle} - 1. \tag{18}$$

By means of the input-output theory[23], the steady-state expressions for the spectral density of the second-order moments outside the cavity can be calculated along the same lines as discussed by Holm *et al.*[24,25]. The resulting expressions are

$$\langle a_1^\dagger a_1\rangle_{\rm out} = 2\kappa\frac{(\alpha_3-{\rm i}\omega)(\alpha_3^*+{\rm i}\omega)A_1+|D_1|^2A_3-(\alpha_3^*+{\rm i}\omega)D_1^*C_3+c.c.}{|(\alpha_1+{\rm i}\omega)(\alpha_3^*+{\rm i}\omega)+D_1C_3^*|^2}, \tag{19}$$

$$\langle a_3^\dagger a_3\rangle_{\rm out} = 2\kappa\frac{(\alpha_1-{\rm i}\omega)(\alpha_1^*+{\rm i}\omega)A_3+|C_3|^2A_1+(\alpha_1^*+{\rm i}\omega)C_3^*C_3+c.c.}{|(\alpha_3+{\rm i}\omega)(\alpha_1^*+{\rm i}\omega)+C_3D_1^*|^2}, \tag{20}$$

$$\langle a_1a_3\rangle_{\rm out} = 2\kappa\frac{(\alpha_3^*+{\rm i}\omega)C_3(A_1+A_1^*)-(\alpha_1^*-{\rm i}\omega)D_1(A_3+A_3^*)+(\alpha_1^*-{\rm i}\omega)(\alpha_3^*+{\rm i}\omega)C_3-D_1|C_3|^2}{|(\alpha_1+{\rm i}\omega)(\alpha_3^*+{\rm i}\omega)+D_1C_3^*)|^2}, \tag{21}$$

where $\alpha_j = B_j - A_j + \kappa_j (j = 1, 3)$ and ω is the frequency deviation from the central frequency.

In Fig. 2, we plot the value of $|B|_{\rm out}$ and $\lambda_{\rm out}$ as a function of $\widetilde{\omega}$ with different values of the cooperativity parameter C, where $C = Ng^2/2\kappa\gamma$ (for simplicity, we assume $g_1 = g_3 = g$ and $\kappa_1 = \kappa_3 = \kappa$). It can be clearly seen from the figures that Bell inequality and CS inequality are both violated in different frequency regions. At the central frequency, the degree of violation is decreased with the increasing of C. In the larger frequency regions, the degree of violation is enhanced with the increasing of C. In addition, for a fixed value of C, the degree of violation is enhanced with the increasing of frequency.

In order to investigate the relation between violations of the Bell inequality and CS inequality, we plot $|B|_{\rm out}$ and $\lambda_{\rm out}$ as a function of I_2 with different values of C in Fig. 3. It is not difficult to see from Fig. 3(a), that for the small pump intensities, the values of $|B|_{\rm out}$ are smaller than 2, i.e., Bell inequality is not violated in these regions. With the increasing of I_2, the values of $|B|_{\rm out}$ are bigger than 2. Thus, Bell inequality is violated in these regions, i.e., the output states display some nonlocal characteristics. In addition, we need much larger I_2 to obtain the nonlocality for the bigger C. However, violation of CS inequality occurred even in the small areas of I_2 (see Fig. 3(b)). There are some nonclassical states that do not show nonlocality in this system. Nonlocality is more difficult to fulfill in comparison with nonclassicality. If we want to achieve the nonlocality and nonclassicality at the same time, large pump intensity is necessary. These results have also been predicted recently in a different three-level atomic system[26].

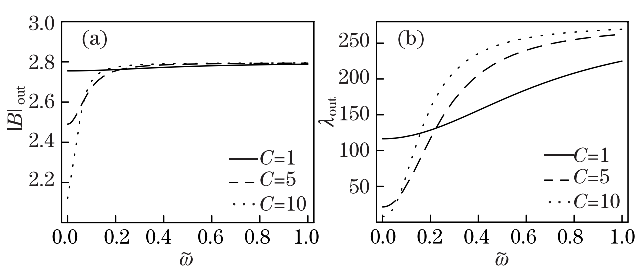

Fig. 2. Variance (a) $|B|_{\rm out}$ and (b) $\lambda_{\rm out}$ versus $\widetilde{\omega}$ for ϕ=$\pi/2$; I_2=50; C=1, 5, and 10; $\Gamma_{\rm a}$ = 1; $\Gamma_1 = \Gamma_3 = 1$; $\gamma_1 = \gamma_3 = \gamma_2 =$ 1. All frequencies are in units of γ_2.

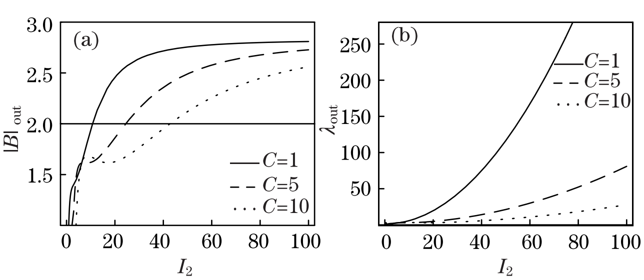

Fig. 3. Variance (a) $|B|_{\rm out}$ and (b) $\lambda_{\rm out}$ versus I_2, the other parameters are the same as in Fig. 2.

Considering the output entanglement in this system[20], we find that the variations of entanglement, nonclassicality, and nonlocality are different. Degree of entanglement and violations of such inequalities look contradictory. Generally speaking, the more violations of Bell inequality and CS inequality, the more nonlocal and nonclassical effects we can obtain. However, better entanglement was obtained at the central frequency and at a certain value of pump intensity[20]. The physical reason for this result is not very clear at this point. One possible explanation is that the definition of continuous-variable entanglement's degree remains an open question and must be further investigated.

Before we conclude, it may be profitable to give a brief discussion on the experimental realization of the present work. The generation of entanglment has been experimentally implemented in a single trapped ion[27] and in a cavity[28]. In this study, the quantities in inequalities (11 or 18) can be measured in a relatively straightforward way in experiments[29]. We predict that the results obtained in this letter can be verified in the future experiments.

In conclusion, we investigate the output nonlocality and nonclassicality in an entanglement laser system. The nonlocality and nonclassicality are discussed by the violations of Bell inequality and CS inequality, respectively. It is found that both Bell inequality and CS inequality can be violated outside cavity fields under certain conditions. It is also shown that there are some nonclassical states that do not show nonlocality. We hope our work can enhance the understanding of the basic features of quantum fields.

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