New transfer matrix method for long-period fiber gratings with coupled multiple cladding modes

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A new transfer matrix method for long-period fiber gratings with coupled multiple cladding modes is proposed and numerically characterized. The transmission spectra of uniform and non-uniform long-period fiber gratings are numerically characterized. The theoretical results excellently agree with the experimental measurements. Compared with commonly used methods, such as using the fourth-order adaptive step size control of the Runge-Kutta algorithm in solving the coupled mode equation, the new transfer matrix method exhibits a faster calculation speed.

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Long-period fiber gratings (LPFGs) have found many applications in optical communication systems and optical sensor systems because of their capability to induce coupling between core and cladding modes at resonant wavelengths[1−4]. LPFG can be used as mode converters[5], rejection filters[6], gain-flattening filters for erbium-doped fiber amplifiers[7], and optical fiber sensors for strain[8], temperature[9], and refractive index measurements[10].

The commonly methods used in the analysis of the spectral characteristics of LPFGs and fiber Bragg gratings (FBGs) include the transmission matrix method (TMM) and solving the coupled-mode equations (SCMEs)[11−13]. The traditional TMM enables the analysis of uniform and non-uniform LPFGs and FBGs when only two modes are considered[14,15]. SCME using the fourth-order adaptive step size control of the Runge-Kutta algorithm enables the determination of the spectra of uniform and non-uniform-structure LPFGs when multiple modes are taken into account. In this letter, a new TMM for LPFGs with coupled multiple cladding modes is proposed and numerically characterized. In this new type of TMM, we can analyze the uniform and non-uniform coupled modes observed between the core mode and the multiple cladding modes. Additionally, the proposed method is simple to implement, almost always sufficiently accurate, and generally faster than SCME.

The coupled-mode equations that describe the co-propagating interactions in a LPFG are given as[14]

$$\begin{cases} \frac{\mathrm{d}A^{\mathrm{co}}}{\mathrm{d}z} = \mathrm{j}k_{01-01}^{\mathrm{co-co}}A^{\mathrm{co}} + \mathrm{j}\sum\limits_{v}\frac{m}{2}k_{1v-01}^{\mathrm{cl-co}}A_v^{\mathrm{cl}}\mathrm{e}^{-\mathrm{j}2\delta_{1v-01}^{\mathrm{cl-co}}z} \\ \sum\limits_{v}\left[\frac{\mathrm{d}A_v^{\mathrm{cl}}}{\mathrm{d}z} = \mathrm{j}\frac{m}{2}k_{1v-01}^{\mathrm{cl-co}}A^{\mathrm{co}}\mathrm{e}^{\mathrm{j}2\delta_{1v-01}^{\mathrm{cl-co}}z}\right] \end{cases}, \quad (1)$$

where A^{co} is the amplitude for the core mode, A^{cl} denotes the amplitude for cladding mode HE_{1v}, $k_{01-01}^{\mathrm{co-co}}$ is the coupling constant for core mode–core mode, m is the induced-index fringe modulation ($0 \leqslant m \leqslant 1$), and $k_{1v-01}^{\mathrm{cl-co}}$ represents the coupling constant for core mode–cladding mode. Then, we define

$$\delta_{1v-01}^{\mathrm{cl-co}} = \frac{1}{2}\left(\beta_{01}^{\mathrm{co}} - \beta_{1v}^{\mathrm{cl}} - \frac{2\pi}{\Lambda}\right), \quad (2)$$

where β_{01}^{co} and β_{1v}^{cl} are the propagation constants of the core mode and cladding mode, and Λ is the grating period.

If S_v is defined as

$$S_v = A_v^{\mathrm{cl}}\mathrm{e}^{-\mathrm{j}2\delta_{1v-01}^{\mathrm{cl-co}}z}, \quad (3)$$

the following equations should be obtained:

$$\begin{cases} \frac{\mathrm{d}A^{\mathrm{co}}}{\mathrm{d}z} = \mathrm{j}k_{01-01}^{\mathrm{co-co}}A^{\mathrm{co}} + \sum\limits_{v}\mathrm{j}\frac{m}{2}k_{1v-01}^{\mathrm{cl-co}}S_v \\ \sum\limits_{v}\left[\frac{\mathrm{d}S_v}{\mathrm{d}z} = \mathrm{j}\frac{m}{2}k_{1v-01}^{\mathrm{cl-co}}A^{\mathrm{co}} - \mathrm{j}2\delta_{1v-01}^{\mathrm{cl-co}}S_v\right] \end{cases}. \quad (4)$$

Then, the matrix form of Eq. (3) can be expressed as

$$\begin{bmatrix} \frac{\mathrm{d}A^{\mathrm{co}}}{\mathrm{d}z} \\ \frac{\mathrm{d}S_1}{\mathrm{d}z} \\ \vdots \\ \frac{\mathrm{d}S_v}{\mathrm{d}z} \end{bmatrix} = \mathbf{F}\begin{bmatrix} A^{\mathrm{co}} \\ S_1 \\ \vdots \\ S_v \end{bmatrix}, \quad (5)$$

where

$$\mathbf{F} = \mathrm{j}\begin{bmatrix} k_{0,1-01}^{\mathrm{co-co}} & \frac{m}{2}k_{1,1-01}^{\mathrm{cl-co}} & \frac{m}{2}k_{1,2-01}^{\mathrm{cl-co}} & \cdots & \frac{m}{2}k_{1,v-1-01}^{\mathrm{cl-co}} & \frac{m}{2}k_{1,v-01}^{\mathrm{cl-co}} \\ \frac{m}{2}k_{1,1-01}^{\mathrm{cl-co}} & -2\delta_{1,1-01}^{\mathrm{cl-co}} & 0 & \cdots & 0 & 0 \\ \frac{m}{2}k_{1,2-01}^{\mathrm{cl-co}} & 0 & -2\delta_{1,2-01}^{\mathrm{cl-co}} & 0 & \cdots & 0 \\ \vdots & \vdots & 0 & \ddots & 0 & \vdots \\ \frac{m}{2}k_{1,v-1-01}^{\mathrm{cl-co}} & 0 & \vdots & 0 & -2\delta_{1,v-1-01}^{\mathrm{cl-co}} & 0 \\ \frac{m}{2}k_{1,v-01}^{\mathrm{cl-co}} & 0 & 0 & \cdots & 0 & -2\delta_{1,v-01}^{\mathrm{cl-co}} \end{bmatrix}. \quad (6)$$

Thus, the transmission matrix of the uniform LPFG can be expressed as

$$\mathbf{T} = \mathrm{e}^{\mathbf{F}L}, \quad (7)$$

where L is the grating length.

For non-uniform LPFGs, the grating can be divided into hundreds of segments, and every segment can be considered uniform. The transmission matrix of the ith segment can be expressed as

$$\mathbf{T}_i = \mathrm{e}^{\mathbf{F}L_i}, \quad (8)$$

where L_i is the length of the ith segment. The total transmission matrix of the grating can be expressed as

$$\mathbf{T} = \mathbf{T}_0\ \mathbf{T}_1 \cdots \mathbf{T}_N. \quad (9)$$

We analyze the transmission spectra of LPFGs using the TMM described above. The fiber considered has a step-index profile and a three-layer structure. The parameters of the fiber are as follows: core radius $r_1 = 2.625\ \mu$m, cladding radius $r_2 = 62.5\ \mu$m, core index $n_1 = 1.458$, cladding index $n_2 = 1.45$, and air index $n_3 = 1.0$.

To verify the accuracy of the new TMM, a relatively weak uniform LPFG is analyzed. The transmission spectrum of this grating is shown in Fig. 1. The grating parameters are grating period $\Lambda = 396\ \mu$m, grating length $L = 50$ mm, and a peak induced index change of 0.00019. The single resonance that is visible within the plotted wavelength range is associated with coupling to the $v = 1$ cladding mode. Using Fig. 11 in Ref. [14] for comparison, we can conclude that the theoretical results calculated by the new TMM excellently agrees with the experimental measurements.

Figure 2 shows the calculated transmission spectrum

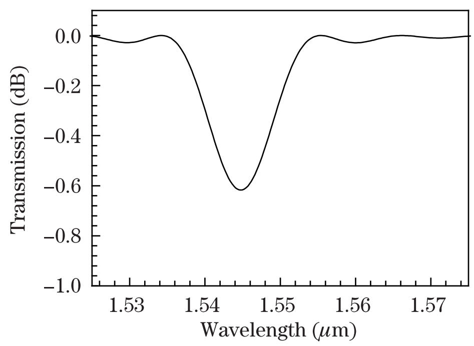

Fig. 1. Theoretically calculated transmission spectrum using a relatively weak uniform grating.

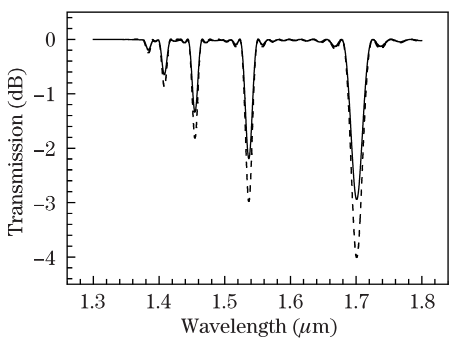

Fig. 2. Theoretical transmission spectrum calculated using the proposed TMM (solid line) and traditional TMM (dashed line) using uniform grating.

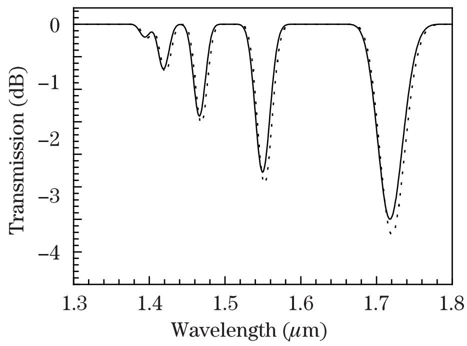

Fig. 3. Theoretical transmission spectrum calculated using the proposed TMM (solid line) and SCME (dotted line) using a Blackman apodization grating.

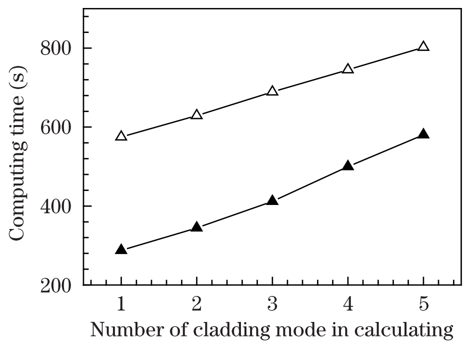

Fig. 4. Computing times for the proposed TMM (▲) and SCME (△).

(solid line) for the relatively weak uniform grating with a length of 25 mm and a peak induced index change of 0.0001. The five main dips observed in the spectrum correspond to coupling to the v =1, 3, 5, 7, and 9 cladding modes. In this case, the grating period is adjusted to 470 μm to induce coupling at 1 550 nm between the LP_{01} core mode and the v =7 cladding mode.

Compared with the traditional TMM, the advantage of the proposed TMM is that coupled multiple cladding modes can be calculated to analyze the transmission characteristics of LPFGs. For comparison, we calculate the transmission spectra using the traditional TMM (Fig. 2, dashed line). On the basis of the figure, we conclude that the accuracy of the coupled wavelength of the new TMM matches that of the traditional TMM. The value in the dotted line is calculated five times, and each time only one cladding mode is considered because of the drawback of the traditional TMM.

Compared with the method in which the coupled mode equation is solved using the fourth-order step size of the Runge-Kutta algorithm, the advantage of the proposed TMM is that it enables the analysis of the transmission characteristics of non-uniform LPFGs at a faster speed. The theoretical results calculated using a Blackman apodization LPFG are shown in Fig. 3. It indicates that the two methods exhibit almost the same accuracy. The comparison of the calculation speeds of the two methods is shown in Fig. 4; computation times at different cladding modes are considered. These results show that the calculation speed using the proposed TMM is faster than that of SCME.

The theoretical spectrum calculated using the proposed TMM and by SCMEs for the non-uniform LPFG is

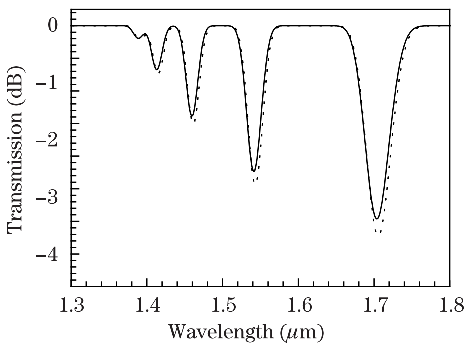

Fig. 5. Theoretical transmission spectrum calculated using the new TMM (solid line) and SCME (dotted line) using non-uniform period grating.

illustrated in Fig. 5, in which the grating period linearly changes from 460 to 480 μm along the grating length. In this figure, the results computed using the new TMM agree with those computed by SCME. The fiber and grating parameters in Figs. 3 and 5 are the same as those in Fig. 2.

In conclusion, a new transfer matrix method for LPFGs with coupled multiple cladding modes is proposed. The transmission spectra of uniform and non-uniform LPFGs are numerically characterized. The theoretical results excellently agree with the experimental measurements. Compared with SCME using the fourth-order adaptive step size of the Runge-Kutta algorithm, the new TMM exhibits a faster calculation speed. We anticipate the proposed method to serve an important function in the design and fabrication of LPFG filters and sensors.

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References

1. A. Harhira, F. Guay, M. Daigle, J. Lapointe, and R. Kashyap, Opt. Commun. **283,** 4633 (2010).
2. C. L. Eggen, Y. S. Lin, T. Wei, and H. Xiao, Sensor. Actuator. B Chem. **150,** 734 (2010).
3. S. H. Nam, J. Lee, and S. Yin, Opt. Commun. **284,** 961 (2011).
4. M. Jiang, D. Feng, and Q. Sui, Chin. Opt. Lett. **7,** 112 (2009).
5. N. Andermahr and C. Fallnich, Opt. Express **18,** 4411 (2010).
6. J. M. Baptista, S. F. Santos, G. Rego, O. Frazao, and J. L. Santos, Opt. Commun. **260,** 8 (2006).
7. N. Ni, C. C. Chan, K. M. Tan, S. C. Tjin, and X. Y. Dong, Opt. Commun. **271,** 377 (2007).
8. K. Li and Z. Zhou, Chin. Opt. Lett. **7,** 191 (2009).
9. T. Venugopalan, T. Sun, and K. T. V. Grattan, Sensor. Actuator. A Phys. **160,** 29 (2010).
10. A. Martinez-Rios, D. Monzon-Hernandez, and I. Torres-Gomez, Opt. Commun. **283,** 958 (2010).
11. G. Wang and B. Xie, Optik **122,** 557 (2011).
12. X. Dong, S. Feng, O. Xu, S. Lu, and L. Pei, Optik **120,** 855 (2009).
13. L. Shao, A. Laronche, M. Smietana, P. Mikulic, W. J. Bock, and J. Albert, Opt. Commun. **283,** 2690 (2010).
14. T. Erdogan, J. Opt. Soc. Am. A **14,** 1760 (1997).
15. T. Erdogan, J. Lightwave Technol. **15,** 1277 (1997).