Coupled-mode analysis for single-helix chiral fiber gratings with small core-offset

Li Yang (杨 利)*, Linlin Xue (薛林林), Jue Su (苏 觉), and Jingren Qian (钱景仁)

Department of Electronic Engineering and Information Science, University of Science and Technology of China, Hefei 230027, China

*Corresponding author: yanglil@ustc.edu.cn

Received December 8, 2010; accepted January 28, 2011; posted online May 26, 2011

Using conventional coupled-mode theory, a set of coupled-mode equations are formulated for single-helix chiral fiber long-period gratings. A helical-core fiber is analyzed as an example. The analysis is simple in mathematical form and intuitive in physical concept. Based on the analysis, the polarization independence of mode coupling in special fiber gratings is revealed. The transmission characteristics of helical-core fibers are also simulated and discussed.

OCIS codes: 060.2340, 050.2770.

doi: 10.3788/COL201109.070603.

Fiber optic sensors based on conventional fiber gratings have been intensively developed for various physical measurements. Recently, a novel kind of fiber gratings, called chiral fiber grating, has been proposed and demonstrated, showing advantages over conventional ones in many applications[1−6]. A chiral fiber is actually a spun specific fiber (e.g., a spun high birefringence fiber or a spun eccentric core fiber), in which the twist pitches are less than 1 mm. Due to their distinctive spectral properties compared with those of previous spun fibers with pitches of millimeters, as well as their similarities to those of conventional fiber gratings, they are given a new name, called chiral fiber or Chiral fiber grating[1,7]. There are two kinds of structures, namely, double- and single-helix structures. The first is formed by twisting a birefringent fiber with a 180° rotation symmetry, whereas the latter is formed by twisting a fiber with a nonconcentric core. The twist for the two structures may be either left- or right-handed. Mode coupling in a double-helix structure is polarization-selective and has relations to the twist handedness. For example, a right-handed twisting chiral fiber long-period grating (CLPG) may support the coupling of the right circularly polarized core mode to a left circularly polarized cladding mode at a certain resonant wavelength[8,9]. However, a single-helix structure does not support the polarization-dependent coupling. This is because the coupling mechanism and spectral properties are different and must be thoroughly understood for further study and wide application.

Oh *et al*. have experimentally obtained single-helix CLPGs and analyzed them using the conventional coupled-mode method[6]. However, the reason why their single-helix structures are polarization-insensitive has not been explained since the expression they used to describe the permittivity of the single-helix structure is that for the double-helix one. In this letter, starting from the coupled-mode theory that has been popularly used in the analysis of conventional fiber gratings[7,10], we use the ideal mode approach[11] to study single-helix CLPGs. The ideal mode approach is one of the methods in coupled-mode analysis, namely, to expand fields in the fiber cross-section in terms of ideal modes that are eigenmodes in a specific reference fiber selected appropriately[10]. A helical-core fiber is taken as a typical example of single-helix structures in the analysis where the polarization-independence of the mode coupling is revealed. Attributed to the superiority of the conventional coupled-mode theory, the whole analysis is simple in mathematical form and intuitive physical concept. Moreover, the proposed analysis can be easily extended to study chiral fiber gratings with shorter periods or with double-helix structures. The transmission characteristics of helical-core fibers with different core-offsets are also simulated. The results coincide with those published in previous work, thereby validating the effectiveness and accuracy of the present theoretical model. The special spectral characteristics of the structures are also discussed.

We consider the single-helix CLPG which is formed by twisting a fiber with an eccentric core. This is almost a standard single-mode fiber except that its core follows a helical path in the cladding[5]. Thus, this helical core can be considered as a deformation of a straight round core, as shown in the schematic diagram of the fiber cross-section in Fig. 1. We suppose that the cladding radius is finite and the refractive index of the outside material is less than that of the cladding to support discrete cladding modes. We use linearly polarized (LP) modes for simplicity since it is a weak-guide fiber.

If the offset of the eccentric core is very small compared with the core diameter, the perturbation of the dielectric

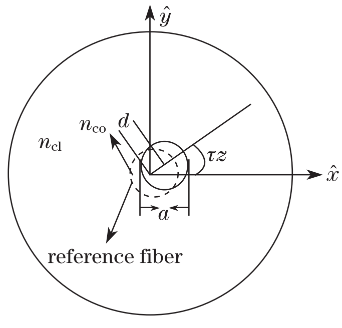

Fig. 1. Schematic diagram of the cross-section in a helical-core fiber.

constant in the core region can be expressed as[4]

$$\Delta\varepsilon(r,\varphi) = \left(n_{\mathrm{co}}^2 - n_{\mathrm{cl}}^2\right)\delta\left(r - r_0\right) d\cos\left(\varphi - \tau z\right), \quad (1)$$

where the twist rate $\tau = 2\pi/p$ (p is the twist pitch), n_{co} and n_{cl} are refractive indices of the core and cladding, respectively, z is the longitudinal coordinate variable of helical-core fiber, d is the offset, and r_0 is the equivalent radius, which is the zero order coefficient of the trigonometrical series of the eccentric core with a diameter of a.

With $\Delta\varepsilon(r,\varphi)$ expressed in Eq. (1), the core mode couples with the $\mathrm{LP}_{1\mathrm{m}}$ cladding modes, which is manifested in the expression of the coupling coefficients. According to the distribution of its electromagnetic fields, a normal fiber $\mathrm{LP}_{1\mathrm{m}}$ mode has four forms: x- and y-polarized fields with azimuthal variation of cosφ (two even modes) and sinφ (two odd modes), respectively. Since the eccentricity of the fiber is very small, the ideal mode approach proposed by Marcuse[11] is applicable. Thus, all these modes including the core mode, are defined in a reference fiber with a concentric round core in the fixed coordinate system ($\hat{x}$,$\hat{y}$,z). The pair of core modes (LP_{01}^x and LP_{01}^y modes) couple with both the two pairs of even and odd modes in this helical core fiber. In brief, the coupled-mode equations for the coupling with the two even modes will be given first. Those for the coupling with the odd modes will follow, after which all of these equations will be combined together. Such separate treatment is valid because there are no couplings between the even and odd modes. According to the theory of Marcuse[11], the coupling between the core modes and even $\mathrm{LP}_{1\mathrm{m}}$ modes can be described by the following coupled-mode equations:

$$\frac{\mathrm{d}}{\mathrm{d}z}\begin{bmatrix} A_{01}^x \\ A_{01}^y \\ A_{1\mathrm{m}}^{x,\mathrm{e}} \\ A_{1\mathrm{m}}^{y,\mathrm{e}} \end{bmatrix} = \begin{bmatrix} -\mathrm{j}\beta_{\mathrm{co}} & 0 & -\mathrm{j}C_1 & 0 \\ 0 & -\mathrm{j}\beta_{\mathrm{co}} & 0 & -\mathrm{j}C_2 \\ -\mathrm{j}C_1 & 0 & -\mathrm{j}\beta_{\mathrm{cl}} & 0 \\ 0 & -\mathrm{j}C_2 & 0 & -\mathrm{j}\beta_{\mathrm{cl}} \end{bmatrix} \begin{bmatrix} A_{01}^x \\ A_{01}^y \\ A_{1\mathrm{m}}^{x,\mathrm{e}} \\ A_{1\mathrm{m}}^{y,\mathrm{e}} \end{bmatrix}, \quad (2)$$

where A_{01}^x and A_{01}^y denote the amplitudes of x- and y-polarized core modes, respectively; $A_{1\mathrm{m}}^{x,\mathrm{e}}$ and $A_{1\mathrm{m}}^{y,\mathrm{e}}$ denote those of x- and y-polarized even cladding modes, respectively; and β_{co} and β_{cl} are the phase constants of LP_{01} and $\mathrm{LP}_{1\mathrm{m}}$ modes in the reference fiber, respectively. The coupling coefficients C_1 and C_2 can be expressed as[12]

$$C_1 = C_2 = \omega\varepsilon_0 \iint \Delta\varepsilon(r,\varphi)\, \mathbf{e}_{01}\cdot\mathbf{e}_{1\mathrm{m}}\mathrm{d}s, \quad (3)$$

where $\mathbf{e}_{01}$ and $\mathbf{e}_{1\mathrm{m}}$ are the modal fields of the LP_{01} and $\mathrm{LP}_{1\mathrm{m}}$ modes, respectively. $\Delta\varepsilon(r,\varphi)$ is defined in Eq. (1). Substituting Eq. (1) to Eq. (3), we have

$$C_1 = C_2 = C\ \cos(\tau z), \quad (4)$$

where

$$C = \frac{\pi\omega\varepsilon_0(n_{\mathrm{co}}^2 - n_{\mathrm{cl}}^2) r_0\ d\ \mathbf{e}_{01}(r_0)\cdot\mathbf{e}_{1\mathrm{m}}\,(r_0)}{4}. \quad (5)$$

Then, the coupled-mode equations for the pair of core modes and the pair of the even cladding modes is rewritten as

$$\frac{\mathrm{d}}{\mathrm{d}z}\begin{bmatrix} A_{01}^x \\ A_{01}^y \\ A_{1\mathrm{m}}^{x,\mathrm{e}} \\ A_{1\mathrm{m}}^{y,\mathrm{e}} \end{bmatrix} = -\mathrm{j}\begin{bmatrix} \beta_{\mathrm{co}} & 0 & C\cos\tau z & 0 \\ 0 & \beta_{\mathrm{co}} & 0 & C\cos\tau z \\ C\cos\tau z & 0 & \beta_{\mathrm{cl}} & 0 \\ 0 & C\cos\tau z & 0 & \beta_{\mathrm{cl}} \end{bmatrix} \begin{bmatrix} A_{01}^x \\ A_{01}^y \\ A_{1\mathrm{m}}^{x,\mathrm{e}} \\ A_{1\mathrm{m}}^{y,\mathrm{e}} \end{bmatrix}. \quad (6)$$

The coupled-mode equations for the pair of core modes and the pair of odd cladding modes can be derived in the same way. They have the same form as Eq. (6), except that:

$$C_1 = C_2 = -C\sin(\tau z). \quad (7)$$

From these two sets of coupled-mode equations, i.e., Eq. (6) and the similar equations except that the matrix element of C cos (τz) is replaced by C sin (τz), x- and y-polarized modes are not coupled with each other. Thus, for both even and odd modes, we have two independent equation groups for the x- and y-polarized modes, respectively. The two equation groups for different polarizations also have the same forms. This means that light passing through the helical-core fiber is polarization-independent. Then, for an arbitrary polarization, we have

$$\frac{\mathrm{d}}{\mathrm{d}z}\begin{bmatrix} A_{01} \\ A_{1\mathrm{m}}^{\mathrm{e}} \end{bmatrix} = -\mathrm{j}\begin{bmatrix} \beta_{\mathrm{co}} & C\cos\tau z \\ C\cos\tau z & \beta_{\mathrm{cl}} \end{bmatrix}\begin{bmatrix} A_{01} \\ A_{1\mathrm{m}}^{\mathrm{e}} \end{bmatrix} \quad (8)$$

for the even cladding mode, and

$$\frac{\mathrm{d}}{\mathrm{d}z}\begin{bmatrix} A_{01} \\ A_{1\mathrm{m}}^{\mathrm{o}} \end{bmatrix} = -\mathrm{j}\begin{bmatrix} \beta_{\mathrm{co}} & -C\sin\tau z \\ -C\sin\tau z & \beta_{\mathrm{cl}} \end{bmatrix}\begin{bmatrix} A_{01} \\ A_{1\mathrm{m}}^{\mathrm{o}} \end{bmatrix} \quad (9)$$

for the odd cladding mode.

Combining Eqs. (8) and (9) for an arbitrary polarization, the coupled-mode equation group is thus obtained. This group describes the coupling between the core mode and $\mathrm{LP}_{1\mathrm{m}}$ modes with the same polarization state in a single-helix structure as

$$\frac{\mathrm{d}}{\mathrm{d}z}\begin{bmatrix} A_{01} \\ A_{1\mathrm{m}}^{\mathrm{e}} \\ A_{1\mathrm{m}}^{\mathrm{o}} \end{bmatrix} = -\mathrm{j}\begin{bmatrix} \beta_{\mathrm{co}} & C\cos\tau z & -C\sin\tau z \\ C\cos\tau z & \beta_{\mathrm{cl}} & 0 \\ -C\sin\tau z & 0 & \beta_{\mathrm{cl}} \end{bmatrix}\begin{bmatrix} A_{01} \\ A_{1\mathrm{m}}^{\mathrm{e}} \\ A_{1\mathrm{m}}^{\mathrm{o}} \end{bmatrix}. \quad (10)$$

Using a transformation of $A_{1\mathrm{m}}^{+} = (A_{1\mathrm{m}}^{\mathrm{e}} + \mathrm{j}A_{1\mathrm{m}}^{\mathrm{o}})/\sqrt{2}$ and $A_{1\mathrm{m}}^{-} = (A_{1\mathrm{m}}^{\mathrm{e}} - \mathrm{j}A_{1\mathrm{m}}^{\mathrm{o}})/\sqrt{2}$, Eq. (10) becomes

$$\frac{\mathrm{d}}{\mathrm{d}z}\begin{bmatrix} A_{01} \\ A_{\mathrm{m}}^{+} \\ A_{1\mathrm{m}}^{-} \end{bmatrix} = -\mathrm{j}\begin{bmatrix} \beta_{\mathrm{co}} & C/\sqrt{2}\mathrm{e}^{\mathrm{j}\tau z} & C/\sqrt{2}\mathrm{e}^{-\mathrm{j}\tau z} \\ C/\sqrt{2}\mathrm{e}^{-\mathrm{j}\tau z} & \beta_{\mathrm{cl}} & 0 \\ C/\sqrt{2}\mathrm{e}^{\mathrm{j}\tau z} & 0 & \beta_{\mathrm{cl}} \end{bmatrix}\begin{bmatrix} A_{01} \\ A_{1\mathrm{m}}^{+} \\ A_{1\mathrm{m}}^{-} \end{bmatrix}. \quad (11)$$

Equation (11) is a typical coupled-mode equation for periodical coupling[13]. Under the phase matching condition, $\beta_{co}-\beta_{cl}+\tau=0$ or $\beta_{co}-\beta_{cl}-\tau=0$, it can be further reduced to a 2×2 matrix equation expressed as

$$\frac{\mathrm{d}}{\mathrm{d}z}\begin{bmatrix} A_{01} \\ A_{1\mathrm{m}}^{\pm} \end{bmatrix} = -\mathrm{j}\begin{bmatrix} \beta_{co} & C/\sqrt{2}\mathrm{e}^{\pm \mathrm{j}\tau z} \\ C/\sqrt{2}\mathrm{e}^{\mp \mathrm{j}\tau z} & \beta_{\mathrm{cl}} \end{bmatrix} \begin{bmatrix} A_{01} \\ A_{1\mathrm{m}}^{\pm} \end{bmatrix}, \quad (12)$$

where τ is assumed to be positive or negative for right- or left-handed helical structures, respectively. Then, since that β_{co} is always larger than β_{cl}, the + and − of $A_{1\mathrm{m}}^{\pm}$ in Eq. (12) correspond to the left- and right-handed structures, respectively. This implies that different forms of LP_{1m} modes ($A_{1\mathrm{m}}^{+}$ or $A_{1\mathrm{m}}^{-}$) are coupled out for different handed structures. When the initial condition is $\mathrm{A}_{01}=1$, $A_{1\mathrm{m}}^{\pm}=0$ at the input end (z=0), Eq. (12) can be easily solved when the phase matching condition is satisfied

$$\begin{aligned} |A_{01}|^2 &= \cos^2(Cz/\sqrt{2}) \\ |A_{1\mathrm{m}}^{\mathrm{e}}|^2 &= |A_{1\mathrm{m}}^{\mathrm{o}}|^2 = \sin^2(Cz/\sqrt{2})/2 \end{aligned} \quad . \quad (13)$$

Hence, this kind of chiral fiber is polarization-independent and has the same features as those of conventional fiber gratings[2,5,6]. It needs to be noted, a helical-core fiber with large eccentricity must be analyzed by using the local mode approach, from which the similar results can be obtained.

The propagation of an initial core mode of an arbitrary polarization through a helical-core fiber with a right- or left-handed structure is simulated. The following parameters of the CLPG are chosen to be the same as those in Ref. [4]: a round cladding with a diameter of 125 μm and a cladding refractive index of 1.4432, and a round core with a diameter $2r_0 = 8.3$ μm and a core refractive index of 1.4490, displaced by $d = 1$ μm. The working wavelength is set at 1.55 μm. A pitch of 564 μm is chosen to satisfy the phase matching condition. Only two modes are included: the core and the LP_{11} mode with the same polarization. The total power transfer periodically occurs between the core and the cladding mode with a period of 12.2 mm (Fig. 2). The results agree well with that presented in Ref. [4].

Corresponding to the coupling of the core mode with the cladding mode, there is a dip at a certain resonant

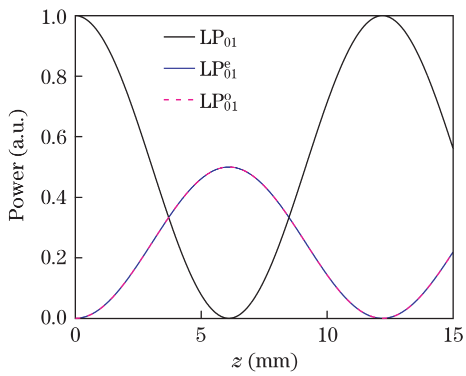

Fig. 2. Interaction of the core mode with an arbitrary polarization and the LP_{11} cladding mode with the same polarization in a single-helix CLPG.

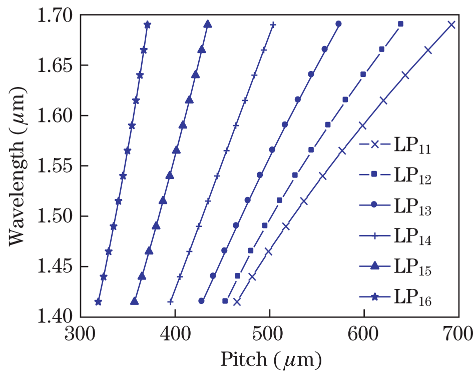

Fig. 3. Dependence of resonant wavelengths of cladding-mode coupling on the period of the helical structure.

wavelength in the transmission spectrum of the helical-core fiber. A number of dips appear in the spectrum due to the coupling to all $\mathrm{LP}_{1\mathrm{m}}$ (m = 1, 2, 3, ···) modes at different resonant wavelengths. The dependence of the resonant wavelengths on the structure period is calculated according to the phase matching condition for individual resonances (Fig. 3). These are similar to those observed in conventional long-period fiber gratings. These dependences are valid for any polarization of the core mode and are not changed for different handedness of the helix structures. Compared with the experimental results reported in Ref. [5], the variation tendencies of the two sets of results in this letter are in good agreement. However, since the design parameters of the grating used in the experiments are unknown for our simulation, there are differences between the simulation and the experimental results. Moreover, the resonant wavelength determined by the phase matching

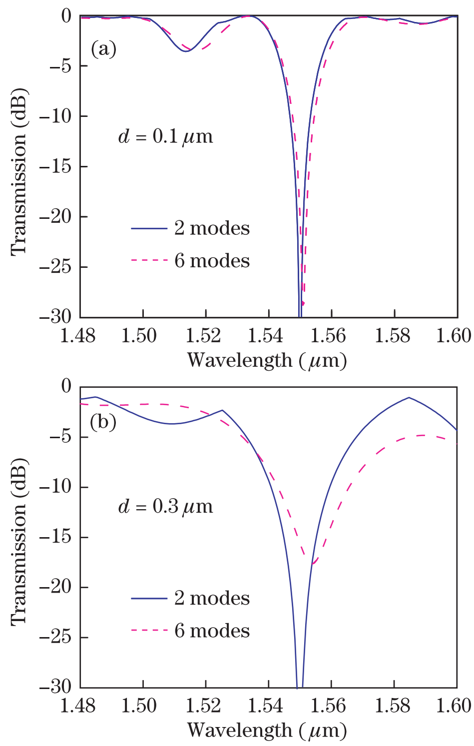

Fig. 4. Transmission spectra of the single-helix CLPG. (a) Core-offset of the eccentric core d=0.1 μm and coupling length L = 21.0 mm; (b) the core offset of the eccentric core d=0.3 μm with coupling length L = 7.1 mm.

condition of an individual resonance may shift due to the influence of adjacent resonances. These will be explained in the following example.

Figures 4(a) and (b) present the transmission spectra in the case of core-offset d=0.1 and 0.3 μm, respectively. In the entire wavelength range under consideration, there were two dips resulting from the couplings with the LP_{11} and LP_{12} modes, respectively. A pitch of 527 mm was set to allow the resonant wavelength of the coupling with the LP_{12} mode at 1.55 μm according to the phase matching conditions. The length of the minimum total power transfer of the coupling is chosen as the length of the grating. In the two figures, the solid lines represent results obtained by considering the couplings simultaneously with the four adjacent cladding modes (from LP_{13} to LP_{16} mode), whereas the dashed lines represent those considering only the couplings with the LP_{11} and LP_{12} modes. In the case of d=0.3 μm, the difference is quite obvious. The bandwidths of the resonant dip and its sidelobes are much broader than those of a conventional long-period fiber grating. Thus, to identify the spectral characteristics precisely, the couplings with the nearby cladding modes must be taken into account, especially when the coupling is stronger. Meanwhile, due to the strong influence of the couplings with the adjacent cladding mode in the case of a stronger coupling, the resonant wavelength evaluated by the phase-matching condition of two-mode coupling is expected to shift.

In conclusion, a helical-core fiber with small eccentricity has been analyzed using the ideal mode approach. The analysis reveals the property of polarization independence when light propagates through the single-helix chiral fiber gratings. The simulated transmission characteristics agree well with those obtained theoretically and experimentally. This indicates that the conventional coupled-mode theory, along with its mode approaches, is an efficient analytic method for chiral fiber gratings. The difference of the spectral characteristics from those of conventional fiber gratings are also demonstrated to be noted in practical use.

This work was supported by the National Natural Science Foundation of China (No. 60807023), the Fundamental Research Funds for the Central Universities, the Scientific Research Foundation for the Returned Overseas Chinese Scholars, the State Education Ministry, and the Anhui Key Laboratory of Optoelectronic Science and Technology.

References

1. V. I. Kopp, V. M. Churikov, J. Singer, N. Chao, D. Neugroschl, and A. Z. Genack, Science **305,** 74 (2004).
2. V. I. Kopp, V. M. Churikov, G. Y. Zhang, J. Singer, C. W. Draper, N. Chao, D. Neugroschl, and A. Z. Genack, J. Opt. Soc. Am. B **24,** A48 (2007).
3. V. I. Kopp, V. M. Churikov, and A. Z. Genack, Opt. Lett. **31,** 571 (2006)
4. G. Shvets, S. Trendafilov, V. I. Kopp, D. Neugroschl, and A. Z. Genack, J. Opt. A: Pure Appl. Opt. **11,** 074007 (2009).
5. O. V. Ivanov, Opt. Lett. **30,** 3290 (2005).
6. S. Oh, K. R. Lee, U. Paek, and Y. Chung, Opt. Lett. **29,** 1464 (2004).
7. T. Erdogan, J. Opt. Soc. Am. A **14,** 1760 (1997).
8. J. R. Qian, Acta Opt. Sin. (in Chinese) **29,** 884 (2009).
9. L. Yang, C. Li, J. Sue, and J. R. Qian, Journal of USTC (in Chinese) **40,** 1082 (2010).
10. J. R. Qian, Acta Opt. Sin. (in Chinese) **29,** 1188 (2009).
11. D. Marcuse, *Theory of Dielectric Optical Waveguides* (2nd edn) (Academic Press, San Diego, 1991).
12. J. R. Qian and W. P. Huang, J. Lightw. Tech. **4,** 619 (1986).
13. S. E. Miller, Bell Syst. Tech. J. **47**, 1801 (1968).