Novel temperature modeling and compensation method for bias of ring laser gyroscope based on least-squares support vector machine

Xudong Yu (于旭东)[1*], Yu Wang (王 宇)[2], Guo Wei (魏 国)[1], Pengfei Zhang (张鹏飞)[1], and Xingwu Long (龙兴武)[1]

[1]*College of Opoelectric Science and Engineering, National University of Defense Technology, Changsha 410073, China*

[2]*School of Mechanical Engineering, Nanjing University of Science and Technology, Nanjing 210094, China*

[*]*Corresponding author: wind0909@163.com*

Received October 22, 2010; accepted January 20, 2011; posted online April 11, 2011

Bias of ring-laser-gyroscope (RLG) changes with temperature in a nonlinear way. This is an important restraining factor for improving the accuracy of RLG. Considering the limitations of least-squares regression and neural network, we propose a new method of temperature compensation of RLG bias−building function regression model using least-squares support vector machine (LS-SVM). Static and dynamic temperature experiments of RLG bias are carried out to validate the effectiveness of the proposed method. Moreover, the traditional least-squares regression method is compared with the LS-SVM-based method. The results show the maximum error of RLG bias drops by almost two orders of magnitude after static temperature compensation, while bias stability of RLG improves by one order of magnitude after dynamic temperature compensation. Thus, the proposed method reduces the influence of temperature variation on the bias of the RLG effectively and improves the accuracy of the gyro scope considerably.

OCIS codes: 120.0120, 120.5790, 140.3370, 280.4788.
doi: 10.3788/COL201109.051201.

The ring-laser-gyroscope (RLG), based on the principle of Sagnac effect, is a kind of core sensor in inertial systems which has been widely used in numerous cutting-edge military fields[1,2]. For gyroscopes in practical engineering, the general requirement is wide range of working temperature. However, RLG bias is sensitive to environmental temperature variation[3,4]. To improve further the accuracy and performance of RLG, the most effective way is to establish a practical and effective temperature model for RLG bias based on experimental data, and subsequently formulate real-time compensation.

At present, the commonly used method for temperature compensation of RLG bias is least-squares regression. However, for the complex non-linear relationship, the fitting precision of this traditional modeling method has been limited. In recent years, the neural network technique has attracted considerable scientific interest particularly in temperature modeling and compensation of sensors for its parallel processing, self-learning, and self-adaptive capabilities in non-linear modeling. However, the neural network does not have the support of complete mathematical theory. Thus, the danger of over-learning and jumping into a local minimum of mean square error exists which can lead to poor generalization capability of the network.

In 1995, Vapnik *et al.* studied and developed support vector machine (SVM), a machine learning algorithm based on statistical learning theory (SLT)[1]. To obtain the best generalization capability, SVM seeks the best compromise between complexity and learning ability of a model according to limited sample information[5,6]. SVM has intuitional geometric interpretation and perfect mathematical form, and it implements the structural risk minimization (SRM) principle unlike most of the neural network models which implement the empirical risk minimization. The training of the SVM is a uniquely solvable quadratic optimization problem, which means that the solution of SVM is unique, optimal, and absent from local minima. With the above-mentioned advantages, SVM has been applied in the areas of pattern recognition, regression forecast, and nonlinear control. However, in application of the standard SVM algorithm, matrix size is impacted greatly by a number of the training samples while solving quadratic optimization problem. This causes the problem of over-large solving scale. Least-squares SVM (LS-SVM)[7] starts with loss function of machine learning, applies 2-norm to the objective function in optimization process, and replaces the inequality constraint conditions of standard SVM algorithm with equality constraint conditions. This facilitates conversion of the optimization problem toward solving a set of linear equations[7,8]. As a new expansion of the standard SVM, LS-SVM reduces computational complexity and keeps high fitting accuracy at the same time. We apply LS-SVM into temperature compensation for the bias of RLG to obtain better compensation effect and improve further the accuracy of RLG.

To reflect the influence law of temperature variation on the bias of RLG completely, it is necessary to study the output characteristics of RLG in cases of certain fixed temperature spot and changing temperature. Therefore, temperature experiments of RLG should include both static and dynamic tests. The steps in the static temperature test method are as follows: temperature of temperature box is kept constant, the gyro is started, and output data are collected at a certain sampling frequency until RLG is in a stable working state. Subsequently, the setting temperature is changed and static tests at different temperature spots are conducted in accordance with the above method. Meanwhile, the dynamic temperature measurement method is performed by setting a stable variation rate for temperature box to rise or drop to a preset value. Consequently, the output data of RLG, temperature value of temperature box, and temperature value of gyro during the temperature changing process are recorded.

We suppose that the given training sample set is $\{(\mathbf{x}_1, y_1), (\mathbf{x}_2, y_2), \cdots, (\mathbf{x}_l, y_l)\}$, $x_i \in \mathbf{R}^n$, $y_i \in \mathbf{R}$, where $i = 1, 2, \cdots, l$, $\mathbf{x}_i$ denotes input patterns, y_i denotes the targets, and l denotes the total number of training samples. For non-linear regression estimation problems, SVM maps the input sample from the original space into a high-dimensional feature space through a non-linear mapping function $\boldsymbol{\Phi}(\mathbf{x})$, which can convert the problem of non-linear function estimation into that of linear function estimation in high-dimensional feature space. The optimal decision function of this estimation problem is denoted as

$$f(\mathbf{x}) = \boldsymbol{\omega}^{\mathrm{T}} \cdot \boldsymbol{\Phi}(\mathbf{x}) + b, \tag{1}$$

where ω^{T} and b are two unknown variables. For standard SVM, the convex constrained optimization problem is described as[5]

$$\begin{aligned} &\min_{\boldsymbol{\omega}, b, \xi, \xi^*} \frac{1}{2}\boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{\omega} + c\sum_{i=1}^{l}(\xi_i + \xi_i^*) \\ &\text{subject to} \begin{cases} y_i - [\boldsymbol{\omega}^{\mathrm{T}} \cdot \Phi(\mathbf{x}_i) + b] \le \varepsilon + \xi_i \\ [\boldsymbol{\omega}^{\mathrm{T}} \cdot \boldsymbol{\Phi}(\mathbf{x}_i) + b] - y_i \le \varepsilon + \xi_i^* \end{cases}, \end{aligned} \tag{2}$$

where $\xi_i \ge 0$, $\xi_i^* \ge 0$, ξ_i^* denotes the upper (lower) training error at data point $(\mathbf{x}_i, y_i)$, c denotes penalty factor, and ε denotes the coefficient of regression estimation accuracy.

According to the duality theory of Wolf, a Lagrange function L is built according to the following equation[8]:

$$\begin{aligned} L =& \frac{1}{2}\boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{\omega} + c\sum_{i=1}^{l}(\xi_i + \xi_i^*) - \sum_{i=1}^{l}(\boldsymbol{\eta}_i\xi_i + \boldsymbol{\eta}_i^*\xi_i^*) \\ & - \sum_{i=1}^{l} a_i\{\varepsilon + \xi_i + y_i - [\boldsymbol{\omega}^{\mathrm{T}} \cdot \Phi(\mathbf{x}_i) + b]\} \\ & - \sum_{i=1}^{l} a_i^*\{\varepsilon + \xi_i^* + y_i - [\boldsymbol{\omega}^{\mathrm{T}} \cdot \Phi(\boldsymbol{x}_i) + b]\}, \end{aligned} \tag{3}$$

where $\mathbf{a}$ and $\boldsymbol{\eta}$ are the non-negative Lagrange multiplier vectors. Then, according to the Karush-Kuhn-Tucker (KKT) conditions, $\mathbf{a}$ and b in Eq. (3) are solved. Finally, the nonlinear estimation function achieves the following explicit form:

$$f(\mathbf{x}) = \sum_{i=1}^{l}(\mathbf{a}_i - \mathbf{a}_i^*) < \boldsymbol{\Phi}(\mathbf{x}_i), \Phi(\mathbf{x}) > +b. \tag{4}$$

The inequality constraints in the standard SVM algorithm are replaced with equality constraints in LS-SVM. Thus, the optimization problem described by Eq. (2) is reduced to the problem described as[9]

$$\begin{aligned} &\min_{\boldsymbol{\omega}, b, \xi,} \frac{1}{2}\boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{\omega} + \frac{c}{2}\sum_{i=1}^{l}\xi_i^* \\ &\text{subject to} \quad y_i - [\boldsymbol{\omega}^{\mathrm{T}} \cdot \Phi(\mathbf{x}_i) + b] = \xi_i. \end{aligned} \tag{5}$$

We obtained the $\boldsymbol{\omega}$ value in dual space according to objective function and constraints described in Eq. (5) and built the Lagrange solving equation as

$$\begin{aligned} L =& \frac{1}{2}\boldsymbol{\omega}^{T}\boldsymbol{\omega} + c\sum_{i=1}^{l}\xi_i^2 \\ & - \sum_{i=1}^{l} a_i\{\xi_i + [\boldsymbol{\omega}^{\mathrm{T}} \cdot \boldsymbol{\Phi}(\mathbf{x}_i) + b] - y_i\}. \end{aligned} \tag{6}$$

The optimal Lagrange multiplier vector $\mathbf{a} = [a, \cdots, a_l]^{\mathrm{T}}$ can be analyzed and reduced according to KKT conditions. The reduction procedure is given by:

$$\begin{cases} \dfrac{\partial L}{\partial \boldsymbol{\omega}^{\mathrm{T}}} = 0 \to \boldsymbol{\omega}^{\mathrm{T}} = \sum\limits_{i=1}^{l} a_i \boldsymbol{\Phi}(\mathbf{x}_i) \\ \dfrac{\partial \mathbf{L}}{\partial b} = 0 \to \sum\limits_{i=1}^{l} a_i = 0 \\ \dfrac{\partial L}{\partial \xi_i} = 0 \to \xi_i = \dfrac{a_i}{c} \\ \dfrac{\partial L}{\partial a_i} = 0 \to \boldsymbol{\omega}^{\mathrm{T}} \cdot \boldsymbol{\Phi}(\mathbf{x}_i) + b + \xi_i = y_i \end{cases}. \tag{7}$$

According to the Mercer's condition, there are many possible forms of Kernel function $K(\mathbf{x}, \mathbf{x}_i)$ that meets $K(\mathbf{x}_i, \mathbf{x}_j) = \boldsymbol{\Phi}^{\mathrm{T}}(\mathbf{x}_i) \cdot \boldsymbol{\Phi}(\mathbf{x}_j)$. As the most commonly used kernel function, Gaussian Kernel function was applied in this study, and its expression is

$$\mathbf{K}(\mathbf{x}, \mathbf{x}_i) = \exp(-||\mathbf{x} - \mathbf{x}_i||^2/2\sigma^2), \tag{8}$$

where σ denotes the Kernel width. Finally, the optimization problem is transformed to solve the following linear equations:

$$\begin{bmatrix} 0 & 1 & \cdots & 1 \\ 1 & K(\mathbf{x}_1, \mathbf{x}_1) + 1/c & \cdots & K(\mathbf{x}_1, \mathbf{x}_l) \\ \vdots & \vdots & \ddots & \vdots \\ 1 & K(\mathbf{x}_l, \mathbf{x}_1) & \cdots & K(\mathbf{x}_l, \mathbf{x}_l) + 1/c \end{bmatrix} \begin{bmatrix} b \\ a_1 \\ \vdots \\ a_l \end{bmatrix} = \begin{bmatrix} 0 \\ y_1 \\ \vdots \\ y_l \end{bmatrix}. \tag{9}$$

The regression function determined by LS-SVM can be obtained by solving

$$f(\mathbf{x}) = \sum_{i=1}^{l} a_i K(\mathbf{x}, \mathbf{x}_i) + b. \tag{10}$$

Model identification is building a function relationship between input and output of the system. Considering the relationship of temperature and RLG output bias as a black box, we can mine the non-linear relationship between them using the method of LS-SVM. The whole progress is divided into two steps: training and testing. After the training of LS-SVM shown in Fig. 1(a), the black-box model is established. Subsequently, we used the LS-SVM model shown in Fig. 1(b) to describe the relationship between RLG bias and temperature. Z represents the transformation from low dimension space to high space using the Kernel function $K(\mathbf{x}, \mathbf{x}_i)$.

Table 1. Static Temperature Test Data of Mechanically Dithered RLG and Bias Modeling Error

Temperature Box (°C)	−40	−30	−20	−10	0	10	20	30	40	50	60
Gyroscope Body (°C)	−34.80	−24.89	−14.97	−4.94	5.09	14.80	24.75	33.74	45.05	54.07	63.59
Gyroscope Output (°/h)	−7.0771	−7.0864	−7.0926	−7.0987	−7.1040	−7.1066	−7.1096	−7.1128	−7.1150	−7.1182	−7.1184
Quadratic Model Error ($\times 10^{-4}$°/h)	−13.8	7.89	5.53	8.33	11.0	−5.56	−11.9	−6.15	−8.86	9.83	3.67
LS-SVM Model Error ($\times 10^{-4}$°/h)	−1.67	5.01	−4.91	−1.77	7.05	−2.08	−3.32	2.65	−5.34	7.28	−2.81

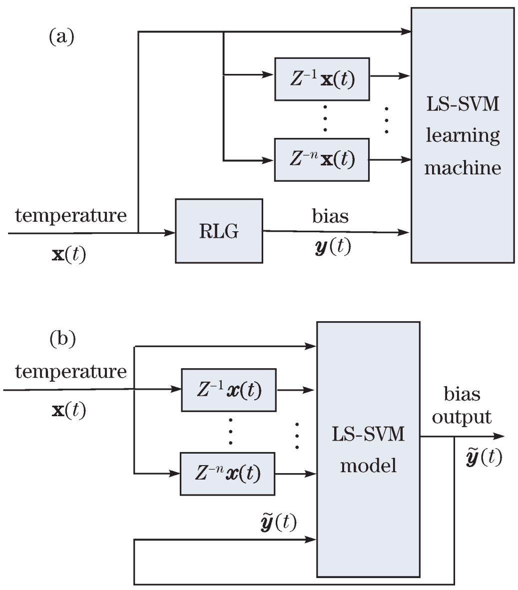

Fig. 1. Schematic diagram of (a) training and (b) testing of LS-SVM.

Mechanically dithered RLG was installed into temperature box. The temperature measurement of RLG was conducted by fixing a platinum resistance on the RLG. A temperature spot was taken every 10 °C from −40 to 60 °C, and data were recorded for an hour after each temperature spot was kept for two hours. During the measurement, the data sampling frequency was 1 Hz, and temperature and bias of RLG were both recorded.

Table 1 shows that the body temperature of RLG is a few Celsius degrees higher than the setting temperature of temperature box because of gyro scope's self-heating. First, we fit the data in Table 1 by establishing least-squares quadratic model, and the result is shown in Fig. 2(a). Then, we fit the data again with the method of LS-SVM regression and obtained the result, as shown in Fig. 2(b). The errors between the predictive values and the actual values at fixed temperature spots are shown in Table 1. From Table 1, the bias values of RLG at high and low temperature spots differ by 0.0413°/h, which is intolerance in the practical applications. The maximum error of least-squares quadratic model is less than 0.0014°/h and the maximum error of LS-SVM model is less than 0.00073°/h. Obviously, the LS-SVM model outperforms the least-squares quadratic model.

To investigate the influence of variable temperature on the bias of mechanically dithered RLG, the dynamic temperature experiment was completed as follows: the temperature changing rate of temperature box was set as 1 °C per minute and high temperature at 65 °C and low temperature at 10 °C, respectively, were kept for 3 h. The average of every 100 points was given to observe the variation tendency of bias with the temperature more clearly.

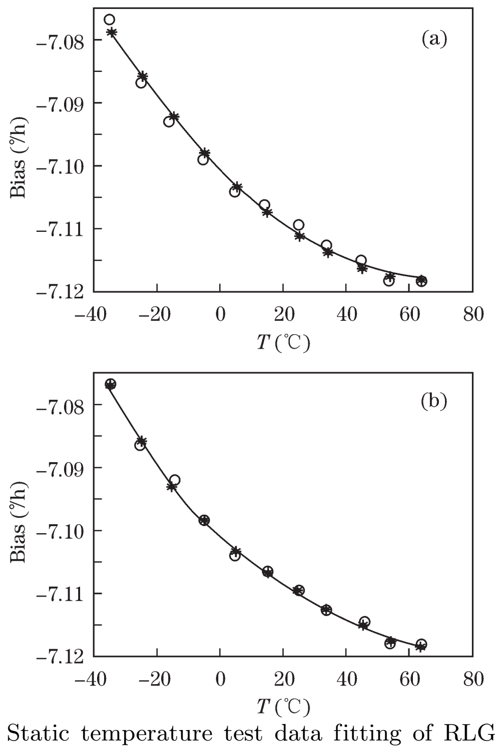

Fig. 2. Static temperature test data fitting of RLG's bias. (a) Least-squares quadratic fitting; (b) LS-SVM fitting.

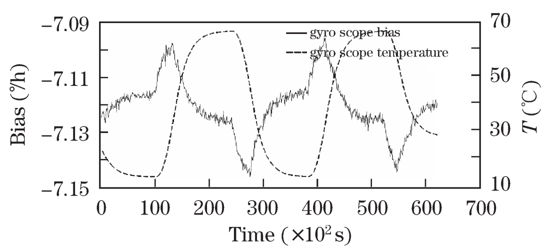

Fig. 3. Sampling curves of gyro bias and gyro scope temperature.

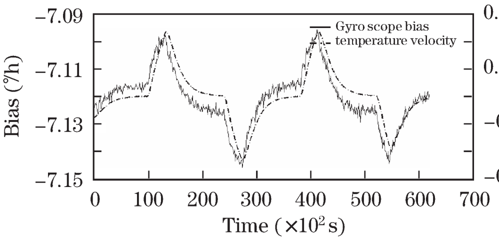

Fig. 4. Sampling curves of gyro bias and gyro temperature changing rate.

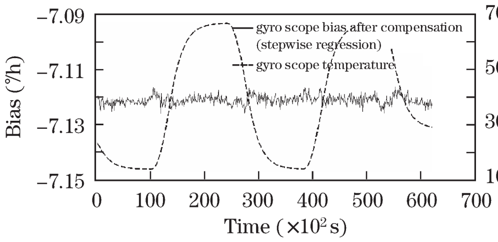

Fig. 5. Bias dynamic temperature compensation of stepwise regression model.

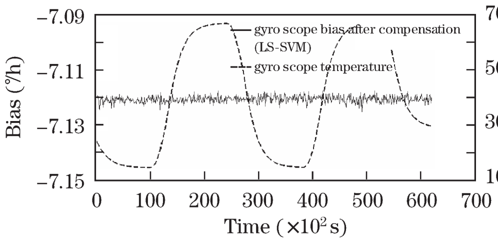

Fig. 6. Bias dynamic temperature compensation of LS-SVM model.

Figure 3 shows the sampling curves of RLG's output and body temperature of RLG. Figure 4 shows the sampling curves of RLG's output and body temperature changing rate of RLG based on the results under the settings of temperature box mentioned above.

In Fig. 3, the bias of RLG shows better repeatability in the cycle period, which allowes temperature compensation for the bias of RLG. In addition, a complex nonlinear relationship between the bias and temperature is shown. In Fig. 4, a strong correlation between the bias and temperature changing rate is evident. Thus, the influence of temperature changing rate on gyro scope's bias is not negligible. Stepwise regression analysis is carried out using three-order variables, regarded as follows[10]: $x_1 = T$, $x_2 = T^2$, $x_3 = T^3$, $x_4 = \frac{\mathrm{d}T}{\mathrm{d}t}$, $x_5 = (\frac{\mathrm{d}T}{\mathrm{d}t})^2$, $x_6 = (\frac{\mathrm{d}T}{\mathrm{d}t})^3$, $x_7 = T\frac{\mathrm{d}T}{\mathrm{d}t}$, $x_8 = T^2\frac{\mathrm{d}T}{\mathrm{d}t}$, and $x_9 = T(\frac{\mathrm{d}T}{\mathrm{d}t})^2$. Based on the degree of significance degree of variables on bias, the insignificant variables x_5, x_8, and x_9 are removed. Subsequently, the regression model of bias composed of terms of temperature related is obtained. These are the last six terms. We calibrated the gyroscope's bias using the data that have been compensated by the terms of temperature related at room temperature and obtained the first term:

$$\begin{aligned} B = &- 0.00960 + 7.1445 \times 10^{-4}T - 2.6387 \times 10^{-5}T^2 \\ &+ 2.1772 \times 10^{-7}T^3 + 2.1572 \times \frac{\mathrm{d}T}{\mathrm{d}t} - 4.2769 \\ &\times 10^3(\frac{\mathrm{d}T}{\mathrm{d}t})^3 + 1.9163 \times 10^{-5}T\frac{\mathrm{d}T}{\mathrm{d}t}. \end{aligned} \quad (11)$$

In LS-SVM regression model, the training sample is set as $\mathbf{x} = [T\frac{\mathrm{d}T}{\mathrm{d}t}]$, and bias variation is the training objective. The data in the first temperature cycle period are used to train LS-SVM, and the others are used to test the model. The bias of RLG that has been compensated by the trained LS-SVM regression model is shown in Fig. 6. As a comparison, the bias of RLG compensated by the regression model described by Eq. (11) is shown in Fig. 5.

Comparing Figs. 5 with 6, the compensation effect of LS-SVM model is better than that of stepwise regression model. When the 100 s variance, a commonly used measuring indicator of RLG's accuracy is derived, and the value for the raw data is 0.0102°/h. The accuracy of RLG can reach 0.0016°/h when the raw data are compensated by least-squares stepwise regression model. This represents an increase of 6.375 times than the accuracy before compensation. It can reach 0.0011°/h after compensation through the proposed LS-SVM model, or an increase of 9.273 times. This strongly proves that the proposed LS-SVM model is more feasible and effective in the dynamic temperature compensation of RLG bias than least-squares stepwise regression model.

In conclusion, to reduce the influence of temperature variation on the bias of RLG, we investigate and apply the LS-SVM, a novel learning machine algorithm to compensate RLG bias. The LS-SVM regression model, with the advantages of quick modeling and high modeling accuracy, overcomes the least-squares regression model's disadvantage of poor nonlinear fitting capacity. It also avoids the over-large solving scale, the weakness of standard SVM. The experimental results show that the proposed method is useful in reflecting the non-linear relationship between mechanically dithered RLG bias and temperature, and that the compensation effect is better than the conventional least-squares regression method. The LS-SVM model improves considerably the accuracy of mechanically dithered RLG. Thus, it has great importance for engineering practice.

References

1. X. Long, J. Tang, Y. Wang, X. Yu, and P. Zhang, in *Proceedings of the 6th Annual Conference of Chinese Society of Inertial Technology* (in Chinese) 15 (2008).
2. S. Qin, Z. Huang, and X. Wang, Chin. Opt. Lett. **5,** 138 (2007).
3. G. Wei and X. Long, Chinese J. Lasers (in Chinese) **37,** 1275 (2010).
4. Z. Wei, X. Long, and K. Yang, Chin. Opt. Lett. **8,** 181 (2010).
5. V. N. Vapnik, *The Nature of Statistical Learning Theory* (Springer, NewYork, 1999).
6. X. Zhang, Acta Automatica Sinica (in Chinese) **26,** 32 (2000).
7. J. A. K Suykens, L. Lukas, and J. Vandewalle, in *Proceedings of IEEE International Symposium on Circuits and Systems* 757 (2000).
8. J. A. K Suykens, in *Proceedings of IEEE Instrumentation and Measurement Technology Conference* 287 (2001).
9. L. Sun and S. Yang, Journal of Applied Sciences—Electronics and Information Engineering (in Chinese) **27,** 616 (2009).
10. P. Zhang, X. Long, J. Tang, Y. Wang, G. Li, and G. Xu, Chinese Journal of Sensors and Actuators (in Chinese) **20,** 1279 (2007).