

Enhancing entanglement generation of two atoms in a cavity with white noise using classical driving fields

Jiansong Zhang (张建松)* and Aixi Chen (陈爱喜)

Department of Applied Physics, East China Jiaotong University, Nanchang 330013, China

*Corresponding author: jszhang1981@zju.edu.cn

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We investigate the entanglement dynamics of a quantum system consisting of two two-level atoms in a cavity with classical driving fields in the presence of white noise. The cavity is initially prepared in the vacuum state. Generally, the entanglement of two atoms decreases with the intensity of the thermal fields and the coupling strength of the two-level atoms to the thermal fields. However, we find that the entanglement of the quantum system can be enhanced by adjusting the frequency and the strength of the classical driving fields in the presence of white noise.

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In the past decades, quantum entanglement has been considered an important resource for the engineering and processing of quantum information^[1]. It can exhibit non-local correlations between quantum systems that cannot be accounted for classically. The cavity quantum electrodynamics (QED) can be used to create entanglement between atoms in cavities and establish quantum communications between different optical cavities^[2]. Thus, many efforts have been devoted to the study of the manipulation of quantum entanglement with atoms and photons in cavities^[3–21].

In practice, a quantum system is inevitably influenced by its surrounding environment^[22]. The interaction between the quantum system and its environment leads to the environment-induced decoherence. As a result, a pure and entangled quantum system will generally become mixed, and the amount of its entanglement will subsequently degrade. This is perhaps the most serious problem for all entanglement manipulation in quantum information processing and quantum computation. Up to now, several protocols have been suggested to minimize the effects of decoherence, such as quantum error correction^[23], decoherence-free subspaces^[24,25], quantum feedback control^[26], and dynamical decoupling^[27].

In this letter, we propose a scheme to enhance the amount of entanglement of a quantum system consisting of two two-level atoms interacting with vacuum fields in the presence of white noise by applying and controlling classical driving fields. We find an explicit expression of the density matrix of the system and study the entanglement dynamics of the system by employing concurrence^[28]. Our calculation shows that the amount of entanglement of the system can be enhanced by controlling the classical driving fields even in the presence of white noise.

We first consider a system consisting of two two-level atoms interacting with two quantized fields with white noise. Each atom is additionally driven by a classical field. The Hamiltonian for the system can be described by^[13,14]

$$H = \omega a^\dagger a + \frac{\omega_0}{2}(\sigma_1^z + \sigma_2^z)$$

$$+ g[a(\sigma_1^+ + \sigma_2^+) + a^\dagger(\sigma_1^- + \sigma_2^-)] + \lambda[e^{-i\omega_c t}(\sigma_1^+ + \sigma_2^+) + e^{i\omega_c t}(\sigma_1^- + \sigma_2^-)], \quad (1)$$

where ω , ω_0 , and ω_c are the frequencies of the cavity, atoms, and classical field, respectively. The operators σ_i^z and σ_i^\pm are defined by $\sigma_i^z = |e_i\rangle\langle e_i| - |g_i\rangle\langle g_i|$ and $\sigma_i^+ = |e_i\rangle\langle g_i|$, where $|e_i\rangle$ and $|g_i\rangle$ are the excited and ground states of atom i and $\sigma_i^- = (\sigma_i^+)^\dagger$. Here, a and a^\dagger are annihilation and creation operators of the cavity; g and λ are the coupling constants of the interactions of each atom with the cavity and with the classical driving fields, respectively. Note that we have set $\hbar = 1$ throughout this paper. The schematic diagram of the present model is presented in Fig. 1.

In the rotating reference frame, the Hamiltonian of the system can be transformed to the Hamiltonian H_1 under a unitary transformation $U_1 = \exp[-i\omega_c t(\sigma_1^z + \sigma_2^z)/2]$ ^[11]:

$$H_1 = U_1^\dagger H U_1 - iU_1^\dagger \frac{\partial U_1}{\partial t} = H_1^{(1)} + H_1^{(2)}, \quad (2)$$

with

$$H_1^{(1)} = \omega a^\dagger a + g[e^{i\omega_c t} a(\sigma_1^+ + \sigma_2^+) + e^{-i\omega_c t} a^\dagger(\sigma_1^- + \sigma_2^-)],$$

$$H_1^{(2)} = \frac{\Delta_1}{2}(\sigma_1^z + \sigma_2^z) + \lambda[(\sigma_1^+ + \sigma_2^+) + (\sigma_1^- + \sigma_2^-)], \quad (3)$$

and $\Delta_1 = \omega_0 - \omega_c$. Clearly, the Hamiltonian $H_1^{(2)}$ can be diagonalized as

$$H_1^{(2)} = \frac{\Omega_1}{2}(\tilde{\sigma}_1^z + \tilde{\sigma}_2^z), \quad (4)$$

where $\Omega_1 = \sqrt{\Delta_1^2 + 4\lambda^2}$ and $\tilde{\sigma}_i^z$ is defined by $\tilde{\sigma}_i^z = |0_i\rangle\langle 0_i| - |1_i\rangle\langle 1_i|$. Here, $|0_i\rangle$ and $|1_i\rangle$ are dressed states:

$$|0_i\rangle = \cos \frac{\theta}{2} |e_i\rangle + \sin \frac{\theta}{2} |g_i\rangle,$$

$$|1_i\rangle = -\sin \frac{\theta}{2} |e_i\rangle + \cos \frac{\theta}{2} |g_i\rangle, \quad (5)$$

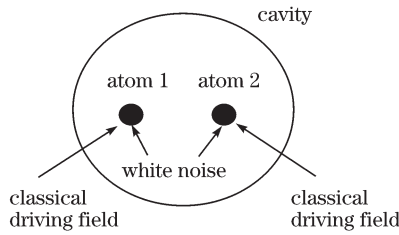


Fig. 1. Schematic diagram of the present system. The two two-level atoms are put into a cavity and separated by a large distance so that there is no direct interaction between them. The atoms are driven by two independent thermal fields and classical driving fields.

with $\theta = \arctan(\frac{2\lambda}{\Delta_1})$. We find the effective Hamiltonian H_1 in the dressed states by ignoring the terms which do not conserve energies (rotating wave approximation):

$$H_1 = \omega a^\dagger a + \frac{\Omega_1}{2}(\tilde{\sigma}_1^z + \tilde{\sigma}_2^z) + g \cos^2 \frac{\theta}{2} [e^{i\omega_c t} a(\tilde{\sigma}_1^+ + \tilde{\sigma}_2^+) + e^{-i\omega_c t} a^\dagger(\tilde{\sigma}_1^- + \tilde{\sigma}_2^-)], \quad (6)$$

with $\tilde{\sigma}_i^+ = |0_i\rangle\langle 1_i|$ and $\tilde{\sigma}_i^- = |1_i\rangle\langle 0_i|$. The Hamiltonian in Eq. (6) can be diagonalized by a final unitary transformation U_2 with $U_2 = \exp[\frac{i\omega_c t}{2}(\tilde{\sigma}_1^z + \tilde{\sigma}_2^z)]$. We can rewrite the Hamiltonian of the system in the rotating reference frame:

$$H_2 = U_2^\dagger H U_2 - i U_2^\dagger \frac{\partial U_2}{\partial t} = \omega a^\dagger a + \frac{\omega'}{2}(\tilde{\sigma}_1^z + \tilde{\sigma}_2^z) + g'[a(\tilde{\sigma}_1^+ + \tilde{\sigma}_2^+) + a^\dagger(\tilde{\sigma}_1^- + \tilde{\sigma}_2^-)], \quad (7)$$

where $\omega' = \Omega_1 + \omega_c = \sqrt{\Delta_1^2 + 4\lambda^2} + \omega_c$ and $g' = g \cos^2 \frac{\theta}{2}$. Note that the unitary transformations U_1 and U_2 are both local unitary transformations. The entanglement of a quantum system does not change under local unitary transformations. Thus, the entanglement of the system considered here will not change by applying local unitary transformations U_1 and U_2 . Hereafter, unless specified otherwise, we work in the rotating reference frame.

In the dispersive limit $|\Delta_2| = |\omega' - \omega| \gg \sqrt{n+1}g$, the interaction Hamiltonian $g'[a(\tilde{\sigma}_1^+ + \tilde{\sigma}_2^+) + a^\dagger(\tilde{\sigma}_1^- + \tilde{\sigma}_2^-)]$ can be regarded as a small perturbation. We assume the

cavity to be initially prepared in the vacuum states. Using the method of Refs. [9–11,13–15], we can recast the effective Hamiltonian given in Eq. (7) in the dispersive limit as

$$H_e = \Omega[|1_1\rangle\langle 1_1| + |1_2\rangle\langle 1_2| + (\tilde{\sigma}_1^+ \tilde{\sigma}_2^- + \tilde{\sigma}_1^- \tilde{\sigma}_2^+)], \quad (8)$$

with $\Delta_2 = \omega' - \omega$ and $\Omega = \frac{(g \cos^2 \frac{\theta}{2})^2}{\Delta_2}$. Note that in the case of the dispersive limit and the initial state of the cavity being the vacuum state, the cavity mode will stay in the vacuum state during the evolution. Thus, we can disregard the cavity mode. The master equation for the density matrix ρ of the two atoms is

$$\frac{d\rho}{dt} = -i[H_e, \rho] + \mathcal{L}_a \rho, \quad \mathcal{L}_a \rho = (m+1)\gamma \sum_{i=1}^2 (2\tilde{\sigma}_i^- \rho \tilde{\sigma}_i^+ - |1_i\rangle\langle 1_i| \rho - \rho |1_i\rangle\langle 1_i|) + m\gamma \sum_{i=1}^2 (2\tilde{\sigma}_i^+ \rho \tilde{\sigma}_i^- - |0_i\rangle\langle 0_i| \rho - \rho |0_i\rangle\langle 0_i|), \quad (9)$$

where γ denotes the coupling strength of the two-level atom to the external fields and $m\gamma$ is the transition rate due to the thermal field. Here, we assume that the two atoms are driven by two independent thermal fields with the same intensity. In addition, the spectral width of the thermal fields is large compared with that of the atomic transition so that the influence of the thermal fields can be considered as white noise. Note that the parameter m can be interpreted as an effective photon number, and the spontaneous decay of the atom outside the cavity is described by the parameter $(m+1)\gamma$.

We assume that the two atoms are initially prepared in the state $|1_1\rangle \otimes |0_2\rangle$. Thus, the explicit analytical solution of the above master equation is

$$\rho(t) = a_{11}(t)|1_1\rangle\langle 1_1| \otimes |1_2\rangle\langle 1_2| + a_{22}(t)|1_1\rangle\langle 1_1| \otimes |0_2\rangle\langle 0_2| + a_{33}(t)|0_1\rangle\langle 0_1| \otimes |1_2\rangle\langle 1_2| + a_{44}(t)|0_1\rangle\langle 0_1| \otimes |0_2\rangle\langle 0_2| + [a_{23}(t)|1_1\rangle\langle 0_1| \otimes |0_2\rangle\langle 1_2| + h.c.], \quad (10)$$

with $h.c.$ denoting the Hermitian conjugate and

$$a_{11}(t) = \frac{m}{(2m+1)^2} [m + e^{-2(2m+1)\gamma t} - (m+1)e^{-4(2m+1)\gamma t}], \quad a_{22}(t) = \frac{m(m+1)}{(2m+1)^2} \left[e^{-4(2m+1)\gamma t} + 1 \right] + \frac{1}{2} \left[\frac{1}{(2m+1)^2} + \cos(2\Omega t) \right] e^{-2(2m+1)\gamma t}, \quad a_{33}(t) = \frac{m(m+1)}{(2m+1)^2} \left[e^{-4(2m+1)\gamma t} + 1 \right] + \frac{1}{2} \left[\frac{1}{(2m+1)^2} - \cos(2\Omega t) \right] e^{-2(2m+1)\gamma t}, \quad a_{44}(t) = \frac{m+1}{(2m+1)^2} [m+1 - e^{-2(2m+1)\gamma t} - m e^{-4(2m+1)\gamma t}], \quad a_{23}(t) = \frac{i}{2} e^{-2(2m+1)\gamma t} \sin(2\Omega t). \quad (11)$$

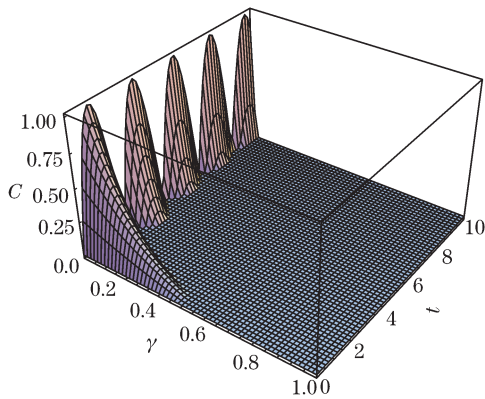


Fig. 2. Concurrence C as a function of time t and coupling constants γ of the atoms and the fields with $g = 0.01, \omega = 2, \omega_0 = 1, \omega_c = \lambda = 0, m = 0.5$.

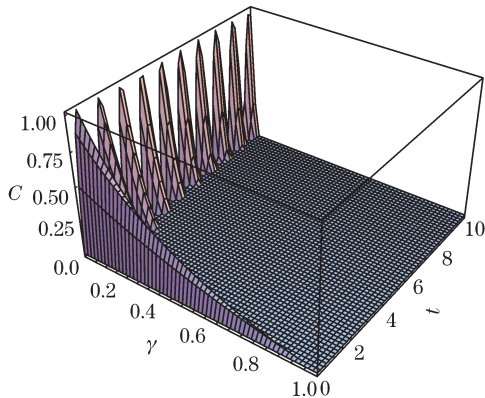


Fig. 3. Concurrence C a function of time t and coupling constants γ of the atoms and the fields with $g = 0.01, \omega = 2, \omega_0 = 1, \omega_c = \lambda = 2, m = 0.5$.

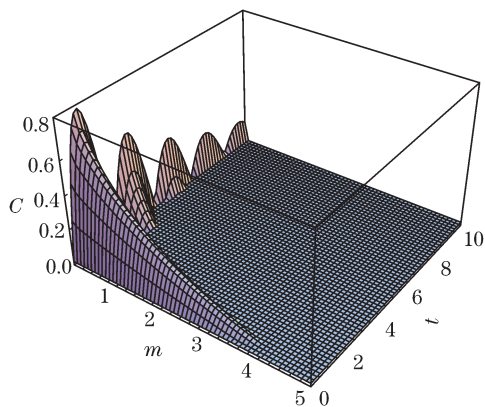


Fig. 4. Concurrence C as a function of time t and intensity of the thermal fields m with $g = 0.01, \omega = 2, \omega_0 = 1, \omega_c = \lambda = 0, \gamma = 0.1$.

Now, we calculate the entanglement of two atoms by using the explicit analytical solution (see Eq. (10)) of the master equation given above. In order to study the entanglement of the above system described by density matrix ρ , we adopt the measure concurrence which is defined by^[28]

$$C = \max \{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \tag{12}$$

where λ_i ($i=1,2,3,4$) are the square roots of the eigenvalues in decreasing order of the magnitude of the “spin-flipped” density matrix operator $R = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$, with the asterisk indicating complex conjugation. Here, σ_y is the Pauli Y matrix defined by

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}.$$

Clearly, the density matrix ρ of Eq. (10) belongs to the class of the “X” states. Explicitly, if the density matrix of a quantum state is of the form

$$\begin{pmatrix} a_{11} & 0 & 0 & a_{14} \\ 0 & a_{22} & a_{23} & 0 \\ 0 & a_{23}^* & a_{23} & 0 \\ a_{14}^* & 0 & 0 & a_{44} \end{pmatrix}, \tag{13}$$

then it belongs to the class of the X states. The concurrence of the quantum states of the X states is

$$C = 2 \max \{0, |a_{23}| - \sqrt{a_{11}a_{44}}, |a_{14}| - \sqrt{a_{22}a_{33}}\}. \tag{14}$$

Combing the above equation and the density matrix of Eq. (10), we obtain the concurrence of two atoms as follows:

$$C(t) = 2 \max \{0, |a_{23}(t)| - \sqrt{a_{11}(t)a_{44}(t)}\}, \tag{15}$$

where $a_{23}(t)$, $a_{11}(t)$, and $a_{44}(t)$ are defined by Eq. (11). Here, we have used the fact that $a_{14}(t) = 0$.

In order to see the influence of the coupling constant γ and the classical driving fields on the entanglement dynamics of the atoms, we plot the concurrence $C(t)$ as a function of time t and parameter γ in Figs. 2 and 3. From the above figures, one can see that in the case of $\gamma = 0$, the concurrence is a periodic function of time t . The period of the concurrence can be adjusted by controlling the classical driving fields as seen in the two figures. Comparing Figs. 2 and 3, we find that the classical driving fields can improve the entanglement of the two atoms significantly. For example, in Fig. 2, the concurrence of the two atoms is zero in the case of $\gamma > 0.5$, that is, the two atoms are disentangled. However, their concurrence is larger than zero in Fig. 3 in the case of $0.5 < \gamma < 0.9$, indicating that the two atoms can be entangled by applying the classical driving fields.

Now, we present the influence of the intensity of the thermal fields m and the classical driving fields. We plot the concurrence of the atoms as a function of time t and

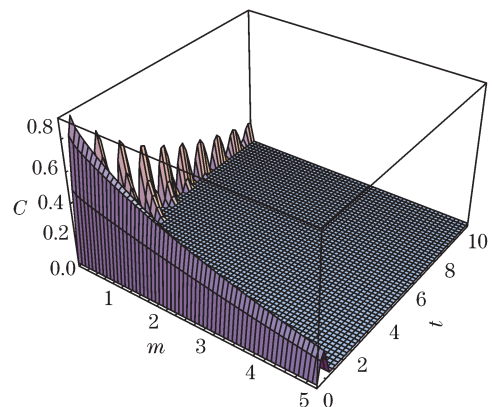


Fig. 5. Concurrence C as a function of time t and intensity of the thermal fields m with $g = 0.01, \omega = 2, \omega_0 = 1, \omega_c = \lambda = 2, \gamma = 0.1$.

the intensity of the thermal fields in Figs. 4 and 5. Generally, the entanglement of two atoms decreases with the intensity of the thermal fields^[10] as shown in Figs. 4 and 5. Comparing these two figures, we observe that the classical driving fields can be used to enhance the entanglement of the two atoms.

In conclusion, we have proposed a scheme to improve the amount of entanglement of a quantum system consisting of two two-level atoms interacting with vacuum fields in the presence of white noise by applying and controlling the classical driving fields. We derive the effective Hamiltonian of the present system and find an explicit analytical expression of the density matrix of the system. We also study the entanglement dynamics of the system by employing concurrence. The influences of the coupling constants γ , the intensity of the thermal fields m , and the classical driving fields are discussed. In general, the entanglement of the two atoms decreases with the increase of the parameter γ and m . Our calculation shows that the amount of entanglement of the system can be enhanced by controlling the classical driving fields. In other words, the scheme suggested in this letter could be used to generate entangled states under the influence of white noise more efficiently, which is important for quantum information processing.

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