Holographic imaging of full-color real-existing three-dimensional objects with computer-generated sequential kinoforms

Huadong Zheng (郑华东), Tao Wang (王 涛), Linmao Dai (代林茂), and Yingjie Yu (于瀛洁)*

Department of Precision Mechanical Engineering, Shanghai University, Shanghai 200072, China

**Corresponding author: yingjieyu@staff.shu.edu.cn*

Received November 10, 2010; accepted December 14, 2010; posted online March 23, 2011

We propose a computational method for generating sequential kinoforms of real-existing full-color three-dimensional (3D) objects and realizing high-quality 3D imaging. The depth map and color information are obtained using non-contact full-color 3D measurement system based on binocular vision. The obtained full-color 3D data are decomposed into multiple slices with RGB channels. Sequential kinoforms of each channel are calculated and reconstructed using a Fresnel-diffraction-based algorithm called the dynamic-pseudorandom-phase tomographic computer holography (DPP-TCH). Color dispersion introduced by different wavelengths is well compensated by zero-padding operation in the red and green channels of object slices. Numerical reconstruction results show that the speckle noise and color-dispersion are well suppressed and that high-quality full-color holographic 3D imaging is feasible. The method is useful for improving the 3D image quality in holographic displays with pixelated phase-type spatial light modulators (SLMs).

OCIS codes: 090.1760, 090.1705, 090.2870, 100.6890.
doi: 10.3788/COL201109.040901

Holography is one of the most important approaches for three-dimensional (3D) imaging due to its capability of recording and reconstructing the wavefronts of 3D objects or 3D scenes. Full-color holographic imaging with panchromatic photosensitive material is now available to record a full-color hologram in one single-layer film[1].

Recently, some electro-holographic displays (EHDs) based on spatial light modulators (SLMs) have been proposed to realize real-time holographic display[2−9]. They are characterized by their capability to load video holograms. When the EHDs are used to display a full-color object, it is necessary to obtain digital holograms containing information of RGB channels. Digital holography is one approach that has been used to obtain full-color holograms[10−16]. Another important approach to generate full-color holograms is computer holography. Lee *et al.* generated synthetic phase holograms based on iterative Fourier transform algorithm (IFTA) for autostereoscopic full-color 3D imaging[4]. Yamaguchi *et al.* proposed a method for calculating full-color image-plane holograms[9]. Yoshikawa *et al.* invented a method to calculate full-color rainbow holograms[17,18]. Sando *et al.* calculated full-color holograms based on the 3D Fourier spectra generated from multiple projection images of a full-color 3D object[19]. Makowski *et al.* proposed a method based on multi-plane iterative algorithm (MPIA) to calculate a single phase hologram, which contains information on the RGB channels of a full-color two-dimensional (2D) object[20,21].

Among the holograms calculated using the aforementioned methods, phase holograms (e.g., kinoform) have higher diffraction efficiency than amplitude holograms or complex amplitude holograms. This is particularly true with multilevel-phase kinoform, in which the zero-order and conjugated image can be suppressed during the reconstruction process. This intrinsic characteristic is important to improve the image quality in holographic imaging. Recently, we proposed a computational method based on multiple fractional Fourier transform to generate the kinoforms of 3D objects[22].

Another factor which influences the quality of reconstructed image by use of kinoform is speckle noise. Some optimization algorithms, such as MPIA[21], IFTA (also referred to as G-S algorithm)[23], and simulated annealing algorithm (SAA)[24], have been proposed to generate low-noise kinoforms. However, generating speckle-free kinoforms of a complex full-color 3D object using these optimization algorithms is still difficult.

Color dispersion introduced by different wavelengths in full-color digital holography and full-color computer holography is another important factor influencing the quality of reconstructed image. Some methods have been proposed to suppress this kind of color dispersion, such as image-scaling operation[4], zero-padding operation[10,12,13], recording/reconstruction distance modification[12,14], and reconstructing wave-front modification[14].

In this letter, we introduce a method using dynamic-pseudorandom-phase tomographic computer holography (DPP-TCH) and zero-padding operation to calculate the sequential multilevel-phase kinoforms of the RGB channels of real-existing full-color 3D objects in order to realize high-quality holographic 3D imaging. The qualities of the numerically reconstructed images are also evaluated.

The simulated model of DPP-TCH for RGB kinoform generation of full-color 3D objects is shown in Fig. 1. A full-color 3D object can be decomposed into multiple slices with RGB channels. For simplicity, we take a full-color 3D object composed of two slices as an example (a lotus and a lily placed at $d_1 = 300$ mm and $d_2 = 450$ mm, respectively) to demonstrate the proposed method. Note that the model could be extended easily to generate kinoforms of complex 3D objects composed of more than two slices.

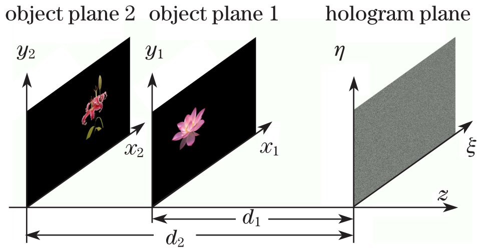


Fig. 1. Simulated model for RGB kinoform generation of full-color 3D objects based on DPP-TCH.

According to the Fresnel diffraction theory, the complex amplitude in the hologram plane contributed by the wavefront of a certain color channel of the ith object plane is described as

$$E_{i,t}(\xi,\eta;\lambda)=\frac{\exp(\mathrm{j}kd_i)}{\mathrm{j}\lambda d_i}\iint A_i(x_i,y_i;\lambda)\exp[\mathrm{j}\varphi_t(x_i,y_i)] \times\exp\{\frac{\mathrm{j}k}{2d_i}[(\xi-x_i)^2+(\eta-y_i)^2]\}\mathrm{d}x_i\mathrm{d}y_i, \quad (1)$$

where λ is the wavelength used for generating sequential kinoforms of the corresponding channel, $k=2\pi/\lambda$; d_i denotes the recording distance between the ith object plane and hologram plane; $i=1, 2, 3, \cdots, L$ (L is the total number of object slices); $A_i(x_i,y_i;\lambda)$ denotes the amplitude of the corresponding channel of the ith object plane; $\varphi_t(x_i,y_i)$ is the pseudorandom phase used for smoothening the spectrum of object wave in generating the tth kinoform; $t=1, 2, 3, \cdots, T$ (T is the total number of kinoforms in each channel). $\varphi_t(x_i,y_i)$ is randomly assigned in the range $[0, 2\pi]$ and follows a uniform distribution. $\varphi_t(x_i,y_i)$ is invariant in the process of calculating one kinoform, but is generated randomly again in another kinoform calculating process.

The whole complex amplitude in the hologram plane contributed by all the slices of the full-color 3D object during the tth kinoform calculation is expressed as

$$E_t(\xi,\eta;\lambda)=\sum_{i=1}^{L}E_{i,t}(\xi,\eta;\lambda), \quad (2)$$

where L is the total number of slices of the full-color 3D object. The tth kinoform of the corresponding color channel is described as

$$H_t(\xi,\eta;\lambda)=\arg[E_t(\xi,\eta;\lambda)], \quad (3)$$

where arg[·] denotes the argument of the complex amplitude. The reference wave is not necessary in the process of kinoform calculation. Under this setting, the zero-order in the reconstructed image introduced by the reference wave can be suppressed. In the reconstruction process, the complex amplitude in the ith image plane from the tth kinoform is expressed as

$$O'_{i,t}(x_i,y_i;\lambda)= \frac{\exp(-\mathrm{j}kd_i)}{-\mathrm{j}\lambda d_i}\iint R(\xi,\eta;\lambda)\exp[\mathrm{j}H_t(\xi,\eta;\lambda)] \times\exp\left\{-\frac{\mathrm{j}k}{2d_i}[(\xi-x_i)^2+(\eta-y_i)^2]\right\}\mathrm{d}\xi\mathrm{d}\eta, \quad (4)$$

where $R(\xi,\eta;\lambda)$ denotes the reference wave for reconstruction. Moreover, in the reconstruction process, the reference wave is required to illuminate the kinoform but does not introduce zero-order in the reconstruction process because the kinoform only contains the argument of complex amplitude of object wave in the hologram plane. The complex amplitudes reconstructed from the T kinoforms of the corresponding color channel are superposed to form a speckle-free image according to

$$I_i(x_i,y_i;\lambda)=\left|\frac{1}{T}\sum_{t=1}^{T}O'_{i,t}(x_i,y_i;\lambda)\right|^2. \quad (5)$$

Compared with digital filtering methods for suppressing speckle noise of digital images, the averaging approach is effective in suppressing speckle noise in both numerical reconstruction and optoelectronic reconstruction.

In the process of numerical reconstruction or optoelectronic reconstruction based on pixelated SLMs, the kinoform calculation formula described in Eq. (1) can be discretized as

$$E_{i,t}(u,v;\lambda)=\frac{\exp(\mathrm{j}kd_i)}{\mathrm{j}\lambda d_i}\sum_{m=1}^{M}\sum_{n=1}^{N}A_i(m,n;\lambda) \times\exp\left\{\frac{\mathrm{j}k}{2d_i}[(u\Delta\xi-m\Delta x_i)^2+(v\Delta\eta-n\Delta y_i)^2]\right\}, \times\exp[\mathrm{j}\varphi_t(m,n)] \quad (6)$$

and Eq. (4) can be rewritten as

$$O'_{i,t}(m,n;\lambda)=\frac{\exp(-\mathrm{j}kd_i)}{-\mathrm{j}\lambda d_i}\sum_{u=1}^{M}\sum_{v=1}^{N}R(u,v;\lambda) \times\exp\left\{-\frac{\mathrm{j}k}{2d_i}[(u\Delta\xi-m\Delta x_i)^2+(v\Delta\eta-n\Delta y_i)^2]\right\}, \times\exp[\mathrm{j}H_t(u,v;\lambda)] \quad (7)$$

where $(\Delta x_i\times\Delta y_i)$ and $(\Delta\xi\times\Delta\eta)$ are the pixel sizes of the ith object plane and hologram plane, respectively; $(\Delta x'_i\times\Delta y'_i)$ denotes the pixel size of the ith image plane; and $M\times N$ is the sampling number of the hologram plane.

The relation between the resolutions of the ith object plane (or image plane) and the hologram plane (SLM) is described as[25]

$$\Delta x_i=\Delta x'_i=\lambda d_i/(M\Delta\xi), \quad (8)$$

and

$$\Delta y_i=\Delta y'_i=\lambda d_i/(N\Delta\eta). \quad (9)$$

Thus, the width and height of the reconstructed image in the corresponding color channel are determined by $\lambda d_i/\Delta\xi$ and $\lambda d_i/\Delta\eta$, respectively.

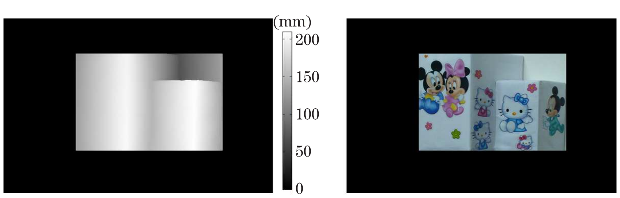
To superpose the three-channel reconstructed images perfectively and form a dispersion-free full-color 3D image, and for uniform detail resolutions in the RGB channels of 3D object, the pixel sizes ($\Delta x_i\times\Delta y_i$, i.e., the sampling intervals) in the RGB channels of the ith object plane should be set the same. Hence, in this letter, we introduce zero-padding operation to make the pixel size ($\Delta x_i' \times \Delta y_i'$) of the reconstructed full-color image independent of wavelengths. The relationship between sampling numbers and corresponding wavelengths in the RGB channels can be expressed as

$$M_\mathrm{R} : M_\mathrm{G} : M_\mathrm{B} = N_\mathrm{R} : N_\mathrm{G} : N_\mathrm{B} = \lambda_\mathrm{R} : \lambda_\mathrm{G} : \lambda_\mathrm{B}. \quad (10)$$

The relationship indicates that a larger number of sampling points in the object plane is needed for the longer wavelength used, which results in redundant marginal districts in the red and green channels. These marginal districts are padded with zeros. The central valid district of full-color information is determined by the sampling number of the blue channel. In this letter, the wavelengths of the RGB channels are set as 635 nm (λ_R), 532 nm (λ_G), and 473 nm (λ_B), and we suppose that the sampling number ($M_\mathrm{R} \times N_\mathrm{R}$) of the red channel is 1024×768 (pixels). Thus, according to Eq. (10), the sampling number of the green and blue channels are 858×643 pixels and 763×572 (pixels), respectively.

To realize holographic imaging of real full-color 3D objects with the proposed method, we use a non-contact full-color 3D measurement system based on binocular vision to obtain the color information and depth map of a full-color 3D object (Fig. 2). Maximum depth value in the depth map is 210 mm. The distance of the base-plane (i.e., black background shown in the depth map) to the hologram plane is set as $d_0 = 500$ mm. The obtained depth map is then decomposed into multiple object slices with a constant small depth interval of δ ($\delta = 0.1$ mm) in the depth direction. During the RGB kinoform calculation process, the depth values of the points belonging to the ith object slice can be reset as $i\delta$ because of $\delta \ll d_0 - i\delta$. Supposing that L is the total number of object slices, then the distance d_i from the ith object slice (i = 0, 1, 2, 3,$\cdots$, L–1) to the hologram plane is $d_i - i\delta$. Color information of the object points of the ith object slice are obtained from the corresponding points in Fig. 2(b). The pixel size in the hologram plane is set as 8×8 (μm).

We can then obtain the sequential kinoforms of RGB channels of the full-color 3D object from multiple object slices according to the calculation model shown in Fig. 1. To generate a single kinoform with a pixel number of 1024×768, the calculation time is approximately 2 min using a personal computer with a central processing unit (CPU) working at 2.5 GHz and memory of 2 GB. Figure 3 shows the reconstruction results from the RGB kinoforms with different reconstruction distances when $T = 1$ and $T = 20$, respectively. When the reconstruction distance is changed slowly from 290 to 350 mm, the corresponding object slice is clearly in sharp focus when the recording distance is equal to the reconstruction distance. Specifically, a full-color 3D image of the real full-color 3D object can be reconstructed with the proposed method.



(a) (b)

Fig. 2. Original information obtained with non-contact full-color 3D measurement system. (a) Depth map (1024×768 pixels); (b) color information (1024×768 pixels).

In addition, we also note that the speckle noise is well suppressed and the detail resolution is improved when T = 20 (see Figs. 3(c) and (d)). We introduce the speckle index (SI) to evaluate the retrieval level of detail resolution of reconstructed image. The SI of the reconstructed image in the each channel is defined as

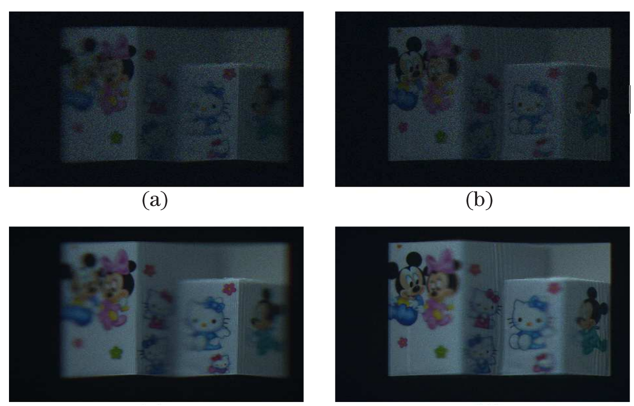
$$\mathrm{SI}(\lambda) = \frac{1}{MN} \sum_{m=1}^{M} \sum_{n=1}^{N} \frac{\sigma(m,n;\lambda)}{\mu(m,n;\lambda)}, \quad (11)$$

where $\sigma(m,n;\lambda)$ and $\mu(m,n;\lambda)$ are the standard deviation and mean value of the $p \times q$ neighborhood ($p = q = 3$) of a reconstructed image point $P(m,n;\lambda)$, respectively.

General quality of the full-color reconstructed image can be evaluated by calculating the averages of SIs in the RGB channels. It is expressed as

$$\overline{\mathrm{SI}} = \frac{1}{3}\left[\mathrm{SI(R)} + \mathrm{SI(G)} + \mathrm{SI(B)}\right]. \quad (12)$$

The $\overline{\mathrm{SI}}$ is about 0.5 in Figs. 3(a) and (b), and decreases to about 0.2 in Figs. 3(c) and (d). Little color shifts in the reconstructed images are still noticeable and are introduced mainly by the reconstruction errors from the RGB kinoforms. Image qualities of the reconstructed full-color images can be further improved when their color shifts are well suppressed by calculating more kinoforms into each channel to further suppress the effect of speckle noise. Moreover, it follows the speckle theory by supposing that the independent speckle patterns suppress speckle noise, that is, the SI of the reconstructed image decreases slowly with the increase in the number of kinoforms and tends to a state level.



(a) (b) (c) (d)

Fig. 3. Numerical reconstruction of a real full-color 3D object using the proposed method, when the reconstruction distance d_i changes slowly between 290 and 350 mm. (a) and (b) are the numerical reconstructed full-color images (763×572 pixels) when $T = 1$ with $d_i = 290$ and 350 mm, respectively; (c) and (d) are the numerical reconstructed full-color 3D images (763×572 pixels) when $T = 20$ with $d_i = 290$ and 350 mm, respectively.

With the sequential kinoforms of the RGB channel of 3D objects generated by the proposed method and the combined use of time division multiplexing (TDM) or spatial division multiplexing (SDM), we can realize high-quality optoelectronic reconstruction of full-color 3D objects based on phase-only SLMs. This type of phase-type SLMs with a refresh rate of 180 Hz or faster is available commercially. The improvement of the refresh rate of phase-type SLM will be very useful in realizing high-quality full-color electro-holographic 3D displays with the use of RGB kinoforms calculated by the proposed method.

Another important factor which influences the image quality of full-color EHD is the chromatic aberration introduced by the diffraction characteristics of pixelated SLMs and other optical elements. Chromatic aberration should be carefully compensated in order to realize high-quality full-color electro-holographic 3D displays. Our future work will be on the chromatic aberration compensation for a full-color 3D holographic display system with the RGB kinoforms calculated using the proposed method in this letter to finally realize high-quality full-color holographic imaging of full-color 3D objects.

In conclusion, a computational method is proposed for realizing high-quality full-color 3D holographic imaging. We decompose full-color 3D object into multiple slices with RGB channels, and calculate multiple kinoforms of each channel using DPP-TCH. Color dispersion introduced by different wavelengths is also compensated by zero-padding operation in the red and green channels of object slices. High-quality full-color 3D image is reconstructed with speckle noise and color-dispersion is well suppressed. The method is also useful for realizing high-quality full-color holographic 3D displays with pixelated phase-type SLMs.

This work was supported by the National Natural Science Foundation of China (No. 60772124), the International Cooperation Project of Science and Technology Commission of Shanghai Municipality (No. 09530708700), and the Shanghai University Innovation Funds for Graduates (Nos. SHUCX101060 and SHUCX102195).

References

1. H. I. Bjelkhagen and E. Mirlis, Appl. Opt. **47,** A123 (2008).
2. M. L. Huebschman, B. Munjuluri, and H. R. Garner, Opt. Express **11,** 437 (2003).
3. T. Ito and K. Okano, Opt. Express **12,** 4320 (2004).
4. K. Choi, H. Kim, and B. Lee, Opt. Express **12,** 5229 (2004).
5. K. Takano and K. Sato, Opt. Eng. **46,** 095801 (2007).
6. T. Shimobaba, A. Shiraki, N. Masuda, and T. Ito, J. Opt. A: Pure Appl. Opt. **9,** 757 (2007).
7. J. L. Martínez, A. Martínez-García, and I. Moreno, Appl. Opt. **48,** 911 (2009).
8. H. Zheng, Y. Yu, and C. Dai, Optik **120,** 431 (2009).
9. T. Yamaguchi, G. Okabe, and H. Yoshikawa, Opt. Eng. **46,** 125801 (2007).
10. I. Yamaguchi, T. Matsumura, and J. Kato, Opt. Lett. **27,** 1108 (2002).
11. J. Kato, I. Yamaguchi, and T. Matsumura, Opt. Lett. **27,** 1403 (2002).
12. J. Zhao, H. Jiang, and J. Di, Opt. Express **16,** 2514 (2008).
13. B. Javidi, P. Ferraro, S.-H. Hong, S. De Nicola, A. Finizio, D. Alfieri, and G. Pierattini, Opt. Lett. **30,** 144 (2005).
14. P. Picart, P. Tankam, D. Mounier, Z. Peng, and J. Li, Opt. Express **17,** 9145 (2009).
15. P. Ferraro, S. Grilli, L. Miccio, D. Alfieri, S. de Nicola, A. Finizio, and B. Javidi, J. Display Technol. **4,** 97 (2008).
16. D. Alfieri, G. Coppola, S. De Nicola, P. Ferraro, A Finizio, G. Pierattini, and B. Javidi, Opt. Commun. **260,** 113 (2006).
17. H. Yoshikawa and T. Yamaguchi, Chin. Opt. Lett. **7,** 1079 (2009).
18. Y. Shi, H. Wang, Y. Li, H. Jin, and L. Ma, Appl. Opt. **48,** 4219 (2009).
19. Y. Sando, M. Itoh, and T. Yatagai, Opt. Express **12,** 6246 (2004).
20. M. Makowski, M. Sypek, I. Ducin, A. Fajst, A. Siemion, J. Suszek, and A. Kolodziejczyk, Opt. Express **17,** 20840 (2009).
21. M. Makowski, M. Sypek, and A. Kolodziejczyk, Opt. Express **16,** 11618 (2008).
22. H. Zheng, Y. Yu, T. Wang, and L. Dai, Chin. Opt. Lett. **7,** 1151 (2009)
23. M. Hacker, G. Stobrawa, and T. Feurer, Opt. Express **9,** 191 (2001).
24. N. Yoshikawa and T. Yatagai, Appl. Opt. **33,** 863 (1994).
25. U. Schnars and W. P. O. Jüptner, Meas. Sci. Technol. **13,** R85 (2002).