Analysis of speckle reduction based on theory of optical eigenfunction

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The principle of optical eigenfunction is used in the analysis of speckle reduction in laser projection display. When the moving diffuser is used in the speckle reduction approach, the speckle contrast is decided by both the degree of freedom (DOF) of the projector and the area of resolving cell of the eyes on the screen. It can gain high DOF of the projector by increasing the space-bandwidth product, i.e., adopting a projection lens with a high numerical aperture or a large viewing field. The best scheme is equalizing the DOF of the scattering wave from a moving diffuser to that of the projection lens. The experimental results are in accordance with the conclusion drawn by optical eigenfunction.

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To acquire extensive color coverage and high luminance, lasers are used in projectors as illumination sources. However, there is a major obstacle: the image quality is affected by the speckle produced by coherent light. Different methods of speckle reduction have been introduced in Refs. [1–8], such as polarization diversity, temporal averaging with a moving diffuser, wavelength and angle diversity, and temporal coherence to destroy spatial coherence. Speckle contrast is adopted to evaluate speckle reduction, which is defined as the intensity fluctuation ratio of the average intensity. Following Goodman[9], the speckle can be reduced by superimposing uncorrelated speckle patterns.

In this letter, the principle of optical eigenfunction[10] is introduced to analyze the regime of speckle reduction using a moving diffuser in laser projection display. Firstly, under the condition of pure scattering, we analyze the eigenequation of the imaging- integral operator of the projection system and deduce the eigenequation of the speckle-correlation operator on screen based on Fourier optics and the van Cittert-Zernike theorem. Thus, the speckle contrast is determined with both the degree of freedom (DOF) of the projection system and the area of resolving cell of the eyes on the screen. To gain lower speckle contrast, the DOF of projection system, namely, the space-bandwidth product of the projection system should be increased. Secondly, speckle contrast versus size of illumination spot and the numerical aperture (NA) of projection lens are respectively studied by experiments. The experimental results are in accordance with our qualitative analysis, which denotes that optical eigenfunction is a convenient and feasible method for analyzing the speckle reduction in laser projection display. It provides the best scheme of equalizing the DOF of the scattering wave from a moving diffuser to that of the DOF of the projection lens.

A conventional optical system is presented in Fig. 1. A phase object illuminated with coherent light is laid in front of the focal plane of the projection lens and forms a real image. Suppose the phase object is pure scattering and the transmission light is δ correlated. The optical system is a linear space shift invariance system.

According to the theory of optical eigenfunction[10], the integral eigen equation of the projection system for the image plane can be expressed as follows:

$$\iint\limits_{A_s} \tilde{h}(x^{(\mathrm{i})}, y^{(\mathrm{i})}; M_x^{(\mathrm{o})}, M_y^{(\mathrm{o})})\varphi_n\left(\mathrm{SW}; M_x^{(\mathrm{o})}, M_y^{(\mathrm{o})}\right) \mathrm{d}(Mx^{(\mathrm{o})})\mathrm{d}(My^{(\mathrm{o})}) = \lambda_n\varphi_n(\mathrm{SW}; x^{(\mathrm{i})}, y^{(\mathrm{i})}), \quad (1)$$

where A_s is the area of illumination on the phase object, M is the magnification of the projection lens, $\varphi_n\left(\mathrm{SW}; x^{(\mathrm{i})}, y^{(\mathrm{i})}\right)$ is the eigenfunction, λ_n is the eigenvalue that denotes the capability of transferring the eigenfunction by the system, $\tilde{h}(x^{(\mathrm{i})}, y^{(\mathrm{i})}; M_x^{(\mathrm{o})}, M_y^{(\mathrm{o})})$ is the point spread function of the projection lens for the

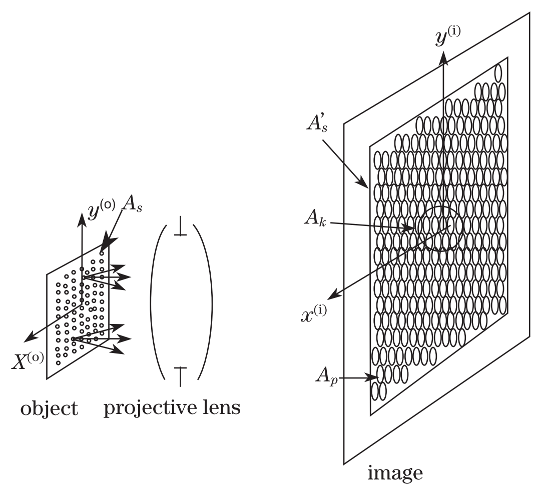

Fig. 1. Optical schematic diagram of the analysis of optical eigenfunction.

image plane, and SW is the space-bandwidth product of the projection system. It can be written as

$$\mathrm{SW} = \frac{\pi\alpha A_s}{2\lambda}, \quad (2)$$

where λ is the wavelength of illumination light, and α is the NA of the projection lens. According to Fourier optics, $\tilde{h}(x^{(\mathrm{i})}, y^{(\mathrm{i})}; M_x^{(\mathrm{o})}, M_y^{(\mathrm{o})})$ is the Fourier transform of the amplitude of the pupil plane, which can be expressed as

$$\tilde{h}(x^{(\mathrm{i})}, y^{(\mathrm{i})}; M_x^{(\mathrm{o})}, M_y^{(\mathrm{o})}) = F[P(x^{(\mathrm{p})}, y^{(\mathrm{p})})]_{(f_x = x^{(\mathrm{i})}/\lambda z_i, f_y = y^{(\mathrm{i})}/\lambda z_i)}, \quad (3)$$

where F denotes the Fourier transform, $P(x^{(\mathrm{p})}, y^{(\mathrm{p})})$ is the pupil function, and z_i is the distance between the pupil plane and the scattering screen. Transforming the point spread function of the projective system into a canonical equation is easy based on Eq. (1):

$$\tilde{h}(x^{(\mathrm{i})}, y^{(\mathrm{i})}; M_x^{(\mathrm{o})}, M_y^{(\mathrm{o})}) = \sum_{n=1}^{N} \lambda_n \varphi_n(x^{(\mathrm{i})}, y^{(\mathrm{i})}) \varphi_n^*(M_x^{(\mathrm{o})}, M_y^{(\mathrm{o})}), \quad (4)$$

where N is the DOF of the projective system, which is decided by the space-bandwidth product of the projection system and can be written as

$$N = \frac{A_s'}{A_p}, \quad (5)$$

where A_s' is the area of image, and A_p is the area of the point spread function of the projection lens.

When the illuminated area on the object is much larger compared with the correlation area of the scattered light on the object, that the object is δ correlated is assured. In a projective system, the order of the illuminated region is millimeter, and the order of the correlation area is micron, in accordance with the assumption of δ correlation. As the order of projective lens resolution cell is micron, the illuminated region in the object is much larger than that in the extent of the projective lens resolution cell on the object. The correlation function of the image plane can be acquired by Fourier transform of the intensity distribution across the pupil plane of a projective system in terms of the generalized van Cittert-Zernike theorem:

$$\mu_a(x_1^{(\mathrm{i})}, y_1^{(\mathrm{i})}; x_2^{(\mathrm{i})}, y_2^{(\mathrm{i})}) = F[P(x^{(\mathrm{p})}, y^{(\mathrm{p})})]_{(f_x = x^{(\mathrm{i})}/\lambda z_{\mathrm{i}}, f_y = y^{(\mathrm{i})}/\lambda z_{\mathrm{i}})}. \quad (6)$$

According to Eqs. (3) and (6), $\tilde{h}(x^{(\mathrm{i})}, y^{(\mathrm{i})}; M_x^{(\mathrm{o})}, M_y^{(\mathrm{o})})$ and $\mu_a(x_1^{(\mathrm{i})}, y_1^{(\mathrm{i})}; x_2^{(\mathrm{i})}, y_2^{(\mathrm{i})})$ have the same function. Otherwise, they have the same integral area. Thus, the same eigenfunction satisfies both the integral eigenequation of the correlation function on the screen and that of the point spread function of the projective system. We have

$$\iint_{A_s'} \mu_a(x_1^{(\mathrm{i})}, y_1^{(\mathrm{i})}; x_2^{(\mathrm{i})}, y_2^{(\mathrm{i})}) \varphi_n\left(\mathrm{SW}; x_2^{(\mathrm{i})}, y_2^{(\mathrm{i})}\right) \mathrm{d}(x_2^{(\mathrm{i})}) \mathrm{d}(y_2^{(\mathrm{i})}) = \frac{I_n}{I_0} \varphi_n(\mathrm{SW}; x_1^{(\mathrm{i})}, y_1^{(\mathrm{i})}), \quad (7)$$

where I_n and I_0 are the intensity of the eigen-mode and the total intensity on the image plane, respectively. We can obtain the canonical equation of the correlation function of the image plane.

$$\mu_a(x_1^{(\mathrm{i})}, y_1^{(\mathrm{i})}; x_2^{(\mathrm{i})}, y_2^{(\mathrm{i})}) = \sum_{n=1}^{N} \frac{I_n}{I_0} \varphi_n(\mathrm{SW}; x_1^{(\mathrm{i})}, y_1^{(\mathrm{i})}) \varphi_n^*(\mathrm{SW}; x_2^{(\mathrm{i})}, y_2^{(\mathrm{i})}). \quad (8)$$

In a laser projection display, the regime of a moving diffuser is used effectively to reduce speckle contrast. When the diffuser is held still, the coherent area of scattered light on the screen is equal to the illumination area on the screen. Thus, the speckle contrast for eyes is approximately equal to 1. Generally, the size of the phase cell is in the order of micrometers. When the diffuser with a random phase distribution revolves quickly, the time of independent phase from the diffuser is far less than the integral time of the eyes; thus, the coherent area of scattered light on the screen is approximately equal to the size of the speckle. The coherent area indicates that the number of independent coherent areas on the screen is equal to the DOF of the projection system according to Eq. (8). Thus, the contrast of speckle patterns for the eyes can be expressed as[9]

$$C = \sqrt{\frac{N + K + 1}{NK}}, \quad (9)$$

where K is the number of coherent areas in the resolving cell of the eyes on the screen, which can be written as

$$K = \frac{A_k}{A_p}, \quad (10)$$

where A_k is the area of the resolving cell of the eyes on the screen. As $A_s' \gg A_k$, then $N \gg K$ and the speckle contrast is approximately expressed as

$$C \approx \sqrt{\frac{1}{K}}. \quad (11)$$

Clearly, if the size of the image is fixed, reducing the area of the correlation speckle cell is recommended to obtain low speckle contrast, which can be gained by increasing the DOF of the projective system, i.e., increasing the space-bandwidth product of the projection system. Thus, we can adopt high NA and increase the area illuminated on the diffuser to obtain a high space-bandwidth product. For a fixed NA of the projective system, equalizing the DOF of the scattering wave from a moving diffuser to the DOF of the projective system is recommended instead of immediately expanding the area illuminated on the diffuser.

To confirm these analyses, experiments were performed using the measurement setup described in Fig. 2. The 488-nm continuous-wave (CW) TEM_{00} Ar^+ laser (Spectra-Physics Inc., Model 177-G12, US) was used. An expander lens was laid in front of the revolving random phase plane (Luminit Inc., LSD, CA). The adjustable NA projection lens (Seagull Inc., F50-1.8, China) projected the enlarged image of the random phase plane

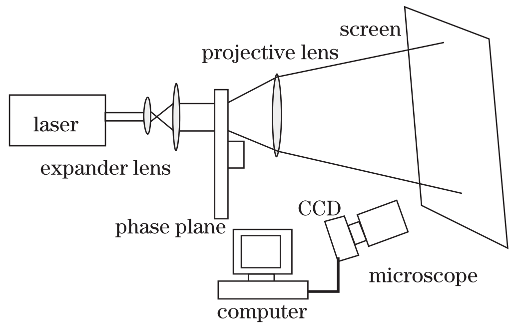

Fig. 2. Measurement setup for the speckle contrast with the speckle reduction regime of a revolving phase plane.

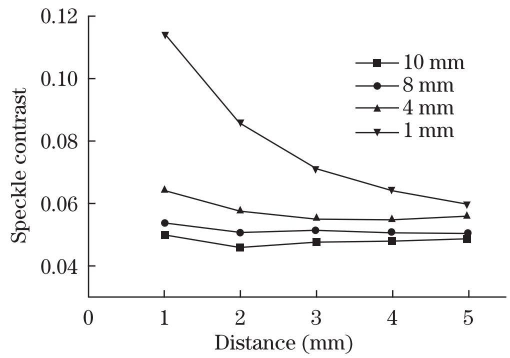

Fig. 3. Speckle contrast affected by size of the illumination spot.

on the screen. To study the speckle contrast plane distribution, a microscope (Nikon Inc., ST100, Japan) with a short depth of field was reconstructed and laid perpendicularly to the scattering screen. The magnification of the micro objective was 10× and NA was 0.25. The charge-coupled device (CCD) camera (Nikon Inc., DS-U2, Japan) replaced the microscopic ocular, and the CCD active area was adjusted to the image plane. The running cycle of the phase plane was set less than the exposure time of the CCD camera at 0.05 s.

The diameter of the projection lens was 20 mm. We obtained different sizes of the illumination spot on the phase plane by adjusting the position of the expander lens. The speckle patterns, which were produced by screen scattering light at different observation distances from the screen, were recorded by the CCD camera. The speckle contrasts were processed by computer. Figure 3 shows that the speckle produced by screen scattering light is limited near the screen. According to Eq. (2), the space-bandwidth product becomes larger when the area of illumination spot increases and the speckle contrast decreases, which agrees with the curves of the speckle contrast in Fig. 3. The curves of speckle contrast also denote that the decreasing speckle contrast further becomes indistinct when the area of illumination on the phase plane is large enough to make the DOF of the phase plane greater than that of the projection lens.

According to Eq. (2), the projection lens with a high NA yields a high space-bandwidth product, and thus obtaining a higher DOF of the projection system and a lower speckle contrast. In this experiment, the diagram of illumination spot was 1 mm, and the different diagrams of projection lens with the same focal length of 50 mm were used. In Fig. 4, the curves of the speckle contrast fit our analysis well. Thus, adopting a projection lens with a high NA is recommended to reduce speckle contrast.

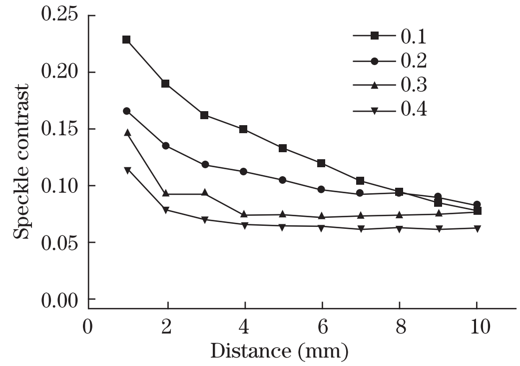

Fig. 4. Speckle contrast affected by NA of the projection lens.

In conclusion, analyzing speckle reduction in a laser projection display by optical eigenfunction is simple and convenient. When a moving diffuser is used in the speckle reduction approach, the speckle contrast is determined by both the DOF of the projector and the area of resolving cell of the eyes on the screen. We can have a low speckle contrast by adopting a projection lens with a high NA or a large viewing field. Equalizing the DOF of the scattering wave from a moving diffuser to that of the projection lens is recommended. Qualitative analysis shows that the experimental results agree well with the conclusion drawn by optical eigenfunction.

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