Effects of high-order dispersions on dark-bright vector soliton propagation and interaction

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The dynamics of dark-bright vector solitons is investigated in a birefringent fiber with the high-order dispersions, and their effects on vector soliton propagation and interaction are analyzed using the numerical method. The combined role of the high-order dispersions, such as the third-order dispersion (TOD) and the fourth-order dispersion (FOD), may cause various deformation of the vector soliton and enhance interaction. These effects depend strictly on the sign of the high-order dispersions. Results indicate that the disadvantageous effects can be reduced effectively via proper mapping of the high-order dispersions.

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Optical solitons in fibers are formed by a balance of group dispersion and Kerr nonlinearity. Numerical simulation and experiments have demonstrated that solitons can propagate an extended distance without distortion. So, they may be the ideal message carriers in long distance communication[1].

Numerous perturbations exist in a practical soliton system; these lead to fluctuations in amplitude or pulse width of the soliton, and deform soliton shape[2,3]. Specially, an optical fiber exhibits high-order dispersions because of the short pulse width (such as sub-picosecond pulse width soliton) in the fiber. When the high-order dispersions cannot be disregarded during soliton propagation, the traveling pulses experience waveform distortion, and a dispersive wave radiation appears in the system, resulting in the decrease in transmission capacity[4].

Dark (bright) solitons propagate in the normal (abnormal) dispersion region. Techniques for generating and detecting dark (bright) soliton pulses have been developed, and the dynamics of the dark (bright) solitons propagating in a birefringent optical fiber has been investigated[5,6]. For example, the bright and dark solitons can be generated by a pulsed laser in a cold three-state medium[7], and numerical and analytical theoretical studies have demonstrated their evolution in a fiber. The coupled dark-bright vector soliton, in which a bright optical solitary wave exists in a system with defocusing nonlinearity because it is trapped within a co-propagating dark soliton, possesses interesting and distinguishing dynamics, which differs from that of bright and dark solitons. Nonlinear effects and the high-order perturbations (such as the third-order dispersion (TOD) and the fourth-order dispersion (FOD)) may modify the effects of the short-ranged interactions between the neighboring solitons[8,9].

Polarization-division multiplexing, in which pulses travel along two orthogonal polarizations of the fiber, doubles the transmission rate compared with the launching of pulses along the same polarizations; transmission capacity can be further increased using a combination of polarization and other multiplexing techniques (such as time-division and wavelength-division multiplexings)[10]. However, the high-order dispersions, including TOD and FOD, may become leading limiting factors in the combination of polarization and other multiplexing systems, with increasing communication capacity.

Recent numerical and analytical theoretical studies have demonstrated both stable and unstable evolution of dark-bright vector solitons in birefringent fiber, and have shown that dispersion plays a crucial role in the physical features of the dark-bright vector solitons. These features include modulation instability and transmission capacity. In this letter, the effects of the high-order dispersions are investigated on the dark-bright vector soliton propagation and interaction in a birefringent fiber. The combined role of the high-order dispersions is discussed and novel results are obtained.

When the effects of the high-order dispersions, such as TOD and FOD, are considered on vector soliton propagation and interaction, the envelop of the field can be described in a normal dispersion fiber by the coupled nonlinear Schrödinger equation (CNLSE) as

$$\begin{aligned}
&\mathrm{j}\frac{\partial u}{\partial Z}+\mathrm{j}\eta\frac{\partial u}{\partial \tau}-\frac{1}{2}\frac{\partial^2 u}{\partial \tau^2}+\left(|u|^2+\frac{2}{3}|v|^2\right)u\\
&\qquad+\mathrm{j}\beta_3\frac{\partial^3 u}{\partial \tau^3}+\beta_4\frac{\partial^4 u}{\partial \tau^4}=0,\\
&\mathrm{j}\frac{\partial v}{\partial Z}-\mathrm{j}\eta\frac{\partial v}{\partial \tau}-\frac{1}{2}\frac{\partial^2 v}{\partial \tau^2}+\left(|v|^2+\frac{2}{3}|u|^2\right)v\\
&\qquad+\mathrm{j}\beta_3\frac{\partial^3 v}{\partial \tau^3}+\beta_4\frac{\partial^4 v}{\partial \tau^4}=0,
\end{aligned}\tag{1}$$

where u and v are normalized elliptically polarized components along two orthogonal directions, respectively. $Z=z/z_{\mathrm{d}}$, $z_{\mathrm{d}}=\tau_0^2/\left|\bar{d}\right|$, and $\tau=t/\tau_0$. z and t are actual distance coordinates and time, respectively. τ_0, z_{d}, $\bar{d}$, η, β_3, and β_4 represent the pulse width, the dispersion length, the average second-order dispersion, the group-velocity delay, and TOD and FOD coefficients, respectively.

Although Eq. (1) has a Hamiltonian structure, it is not exactly integrable because the envelop of the polarized components becomes inhomogeneous due to the high-order dispersions along the fiber (Z dependent coefficient). Such behaviors of the normalized elliptically polarized components u and v may be obtained as their responses are averaged over the high-order dispersions (the average high-order dispersion). However, simply taking the average high-order dispersion fails to provide the proper response because of the correlations with variations of u, v, β_3, and β_4. Thus, the behaviors of dark-bright vector soliton propagation and interaction are investigated using numerical simulation to provide the proper dynamics in the birefringent fiber.

In the calculation models, four maps of the high-order dispersions are considered.

Map (a): The signs of the TOD and FOD coefficients are the same, and both are negative in the fiber.

Map (b): The signs of the TOD and FOD coefficients are the same, and both are positive in the fiber.

Map (c): The TOD coefficient is positive whereas the FOD coefficient is negative along the fiber.

Map (d): The TOD coefficient is negative whereas the FOD coefficient is positive along the fiber.

Equation (1) can be solved numerically using the split-step Fourier algorithm, and the group-velocity delay is $\eta = 0.5$ for a general birefringent fiber in the figures below. The second-order dispersion of the birefringent fiber is $\overline{d} = -1.00$ ps^2/km as the perfect second-order dispersion map for the dark-bright vector solitons[9], and the dispersion length is about 1 km corresponding to the second-order dispersion for the soliton of initial pulse width $\tau_0 = 1$ ps. Figure 1 shows the normalized intensity of a vector soliton in the same polarizing direction versus the propagation distance for only TOD or FOD, and the initial soliton pulses are $u(\tau, 0) = \tanh(\tau)$ and $v(\tau, 0) = \mathrm{sech}(\tau)$. We can see that the individual roles of the high-order dispersions lead to the disintegration of the bright soliton and submergence of the dark soliton. The disintegration distance or submergence distance, which is defined as the distance until disintegration

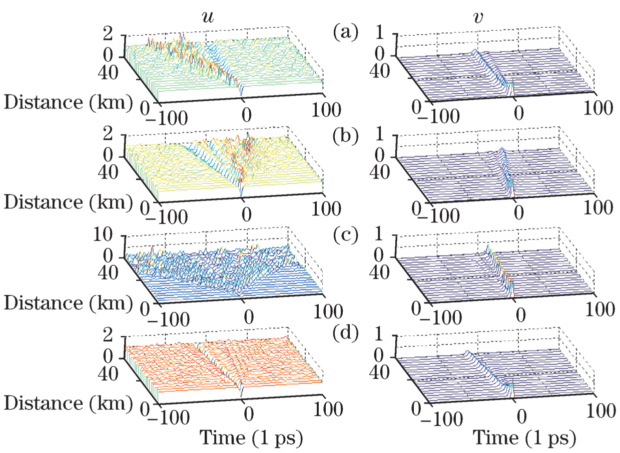

Fig. 1. Normalized intensity of a vector soliton in the same polarizing direction versus the propagation distance for different dispersion maps. (a) $\beta_3 = -0.8$ and $\beta_4 = 0$; (b) $\beta_3 = 0.8$ and $\beta_4 = 0$; (c) $\beta_3 = 0$ and $\beta_4 = -0.3$; (d) $\beta_3 = 0$ and $\beta_4 = 0.3$.

or submergence, is reduced. The effective propagating distance is defined as the shorter distance between the disintegration distance and submergence distance. For example, the effects of TOD are smaller than those of FOD when they have negative signs, but the opposite occurs when they have positive signs. The effective propagating distance strictly depends on the amplitude and sign of each high-order dispersion.

Figure 2 shows the normalized intensity of a vector soliton in the same polarizing direction versus the propagation distance for different dispersion maps. Figure 3 illustrates the effective propagation (interaction) distance versus the TOD coefficient for different FOD coefficients. The initial soliton pulses are the same as those in Fig. 1, and the TOD and FOD coefficients are chosen in different dispersion maps. From Figs. 2 and 3, we can see that the effects of the high-order dispersion in maps (b) and (d) are smaller than those in maps (a) and (c), which have almost the same effects, with a corresponding propagation distance far longer than those in maps (a) and (c). The disintegration of the bright soliton and the submergence of the dark soliton are principally caused by the combined role of the high-order dispersions. However, the combined role can be reduced effectively if the signs of the high-order dispersions are organized properly. For example, the combined role of the high-order dispersions can be reduced if the FOD coefficient is positive along the fiber.

Figure 4 shows the normalized intensity of four vector solitons in the same polarizing direction versus the propagation distance for different dispersion maps. The initial inputs of four dark soliton pulses are

$$u(\tau,0) = \begin{cases} \tanh(\tau - 3\Delta/2) & \text{for} \quad \Delta \leq \tau < \infty, \\ \tanh(\tau - \Delta/2) & \text{for} \quad 0 \leq \tau < \Delta, \\ \tanh(\tau + \Delta/2) & \text{for} \quad -\Delta \leq \tau < 0, \\ \tanh(\tau + 3\Delta/2) & \text{for} \quad -\infty < \tau < -\Delta. \end{cases} \tag{2}$$

The initial input of four bright soliton pulses is

$$v(\tau,0) = \mathrm{sech}(\tau - 3\Delta/2) + \mathrm{sech}(\tau - \Delta/2) + \mathrm{sech}(\tau + \Delta/2) + \mathrm{sech}(\tau + 3\Delta/2), \tag{3}$$

where Δ is the separation between two neighboring solitons in the same polarizing direction, and $\Delta = 10$ (about 6 times of the initial soliton width), as shown in Fig. 3. The interaction between neighboring solitons can be suppressed effectively when initial separation between solitons in the same polarizing direction is larger than five times of the soliton width in the average dispersion soliton system[1]. We can see that the high-order dispersions enhance the interaction between the vector solitons even if the separation is larger than five times of the soliton width. The interaction distance is defined as the shorter distance where the timing shifts of the neighboring solitons exceed half of their full-width at half-maximum (FWHM) of the vector solitons between two polarizing directions. We find that the interaction distances relate to the signs of the high-order dispersions and the separation between two neighboring solitons. Additionally, the effects of the high-order dispersion in maps (b) and (d) are smaller than those in maps (a) and (c). Moreover, the combined role has an

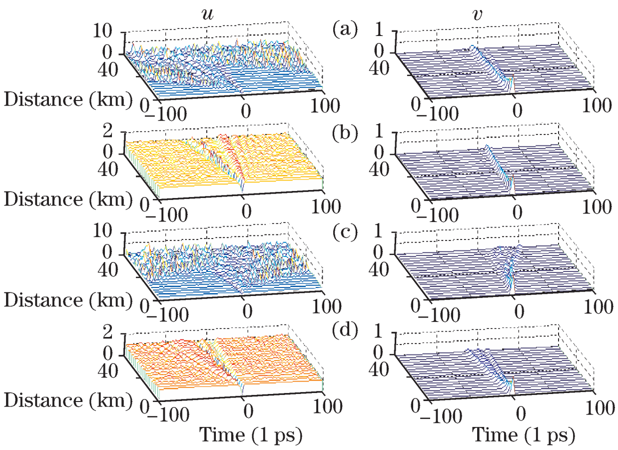

Fig. 2. Normalized intensity of a vector soliton in the same polarizing direction versus the propagation distance for different dispersion maps. (a) $\beta_3 = -0.8$ and $\beta_4 = -0.3$; (b) $\beta_3 = 0.8$ and $\beta_4 = 0.3$; (c) $\beta_3 = 0.8$ and $\beta_4 = -0.3$; (d) $\beta_3 = -0.8$ and $\beta_4 = 0.3$.

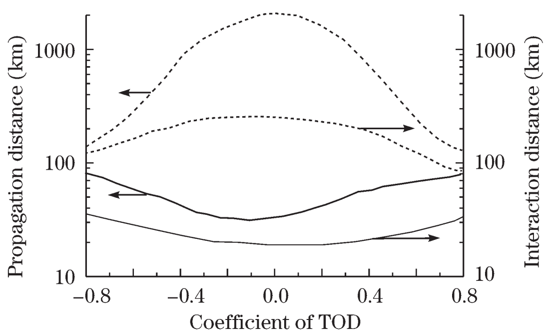

Fig. 3. Effective propagation (interaction) distance versus the coefficient of TOD. Solid line: $\beta_4 = -0.3$; dashed line: $\beta_4 = 0.3$.

important function in the soliton interaction in the same polarizing direction, and the proper high-order dispersion map may increase interaction distance.

Results in Figs. 1−4 show that the effects of the high-order dispersions can be reduced using the proper high-order dispersion maps. Furthermore, we find that the bright solitons have robust features compared with the dark solitons in the presence of the high-order dispersions. These features include good resistance to the combined role of the high-order dispersions. The result is different from that previously obtained[9,11], in which the dark solitons generally have robust features, such as good resistance to the randomly varying birefringence, compared with the bright solitons. This is attributed to the fact that the high-order dispersions modify the effects of the soliton interactions between the neighboring dark-bright vector solitons in two orthogonal directions.

To reduce the effect of the combined role of the high-order dispersions, we can change their signs in a special material fiber (such as metamaterial waveguide fibers), in which organizing the proper high-order dispersion maps to stabilize the propagation and interaction of the vector soliton pulses is possible. The study of the modulation instability of the nonlinear polarizing soliton propagation in metamaterials shows the probability of the

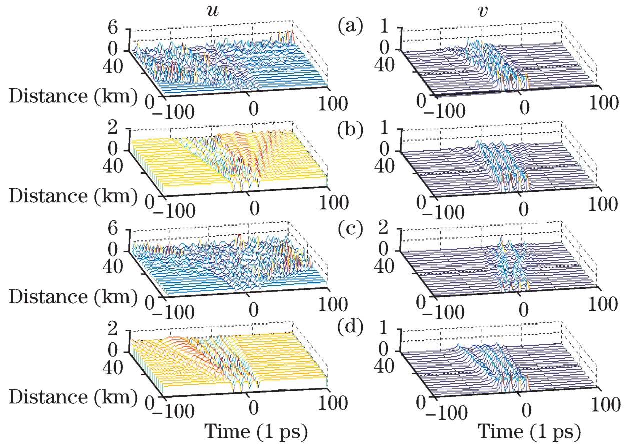

Fig. 4. Normalized intensity of four vector solitons in the same polarizing direction versus the propagation distance for different dispersion maps. (a) $\beta_3 = -0.8$ and $\beta_4 = -0.3$; (b) $\beta_3 = 0.8$ and $\beta_4 = 0.3$; (c) $\beta_3 = 0.8$ and $\beta_4 = -0.3$; (d) $\beta_3 = -0.8$ and $\beta_4 = 0.3$.

reduction of the effects by incorporating the dispersive permeability of metamaterials and the high-order dispersion map. Borrowing the derivation of the nonlinear pulse propagation equation in ordinary material, the general nonlinear propagation equation close to Eq. (1) for ultrashort pulse (such as sub-picosecond pulse width soliton) in metamaterials can be obtained[12].

The high-order dispersions may become the leading limiting factors because of the use of ultrashort pulses, and an available means to surmount these difficulties is managing the second, third, and other higher-order dispersions. Utilizing the higher-order dispersion management, the asymmetric broadening that takes place for ultrashort optical pulses in single channel systems with conventional dispersion system is almost exactly compensated for. Furthermore, the higher-order dispersion management makes it possible to compensate for dispersion over many neighboring frequency channels simultaneously. Advancements in fiber manufacturing techniques have enabled the incorporation of this idea into new optical fibers, called dispersion slope compensating fibers. Recent experiments have yielded impressive results, and the higher-order dispersion management of optical pulses in a fiber poses promising prospects for high speed communication[13].

In conclusion, the coupled dark-bright vector solitons are considered using different high-order dispersion maps, and the combined role of the high-order dispersions on the dark-bright vector soliton propagation and interaction is investigated using the numerical method. The combined role may lead to the rapid disintegration of the bright soliton and the submergence of the dark soliton, as well as enhance the neighboring soliton interaction. The effects depend strictly on the signs of the high-order dispersions, and are principally caused by the combined role of the high-order dispersions. The effects may be reduced using the proper high-order dispersion maps. The bright solitons may have robust features compared with the dark solitons in the presence of the high-order dispersions, attributed to the fact that the high-order dispersions modify the effects on the interactions between the neighboring dark-bright vector solitons in two orthogonal directions. The results show that it is possible to reduce the disadvantageous effects by incorporating the dispersive permeability of the metamaterial waveguide and the high-order dispersion map.

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