Time domain dispersion of underwater optical wireless communication

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A new method to count the expected value and variance of time dispersion is presented for time dispersion of underwater optical wireless communication. Instead of the typically used Gamma distribution, inverse-Gaussian distribution is suggested for underwater optical impulse response time waveform function. The expectation of this method is in good agreement with experimental data. Future works may include water absorption to the model.

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Due to underwater light scattering, there is spatial dispersion when an optical pulse propagates underwater; this phenomenon has been studied by many authors[1−4]. However, there is another problem in underwater optical wireless communication. Since photons (photon in this article is defined as the abstract photon model in optical communication but not the conventional physical photon) emitted from transmitter travel along different optical paths, they reach the receiver after different periods of time, resulting in a time domain dispersion for the pulse form (Fig. 1)[5]. This phenomenon is not remarkable when the underwater optical wireless communication transfer rate is low. However, as the system transfer rate increases (the recent published transfer rate has approached 1 Gb/s[6]), the time domain dispersion character of optical pulse underwater can no longer be ignored.

The expected value and variance of time domain dispersion of optical pulses are two important indices when studying time domain character of light underwater. Hulst *et al*. proposed their solutions to the radiative transport equation in the limit of small-angle approximation using a successive approximation method and Fourier–Laplace transformation, respectively[7,8]. Meanwhile, Zaccanti *et al*. employed the phenomenological model in small-angle approximation and approached solutions through multiple measurement of equivalent attenuation coefficient[9,10]. Stotts[11] simplified multiple scatterings into a single scattering. These diverse approaches are listed as follows:

$$\langle\Delta t\rangle \approx \frac{bz^2\langle\theta\rangle}{4c}, \text{in Refs. [8,12]};$$

$$\langle\Delta t\rangle \approx \frac{9bz^2\langle\theta\rangle}{16c}, \text{in Ref. [11]};$$

$$\langle\Delta t\rangle \approx \frac{bz^2\langle\theta\rangle}{12c}, \text{in Ref. [7]};$$

$$\langle\Delta t\rangle \approx \frac{z}{c}\left\{1-\frac{[1-\exp(-vbz)]}{v}\right\}, \text{in Ref. [13]};$$

$$\langle\Delta t\rangle \approx \log_{\langle\cos\theta\rangle}\frac{[1-bz(1-\langle\cos\theta\rangle)]}{bc}-\frac{z}{c}, \text{by author};$$

$$\mathrm{VAR}(\Delta t) \approx \frac{bz^3\langle\theta^4\rangle}{12c^2}+\frac{b^2z^4\langle\theta^2\rangle^2}{24c^2}, \text{in Ref. [8]};$$

$$\mathrm{VAR}(\Delta t) \approx \frac{7b^2z^4\langle\theta^2\rangle^2}{720c^2}, \text{in Ref. [7]};$$

$$\mathrm{VAR}(\Delta t) \approx (\frac{z}{c})^2\left\{\frac{2}{3}\frac{(w^2-3wv)[\exp(-vbz)-1+vbz]+2v^2[\exp(-wbz)-1+wbz]}{wv^2(w-v)}-\left[\frac{1-\exp(-vbz)}{v}\right]^2\right\}, \text{in Ref. [13]};$$

$$\mathrm{VAR}(\Delta t) \approx \frac{\log_{\langle\cos\theta\rangle}[1-bz(1-\langle\cos\theta\rangle)]-1}{(bc)^2}, \text{by author};$$

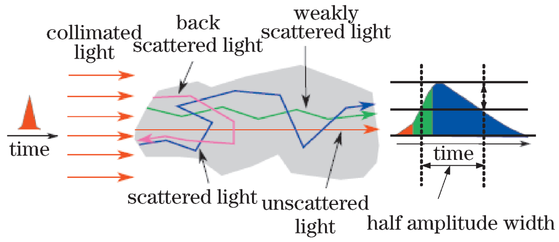

Fig. 1. Multiple paths incurring time dispersion[5].

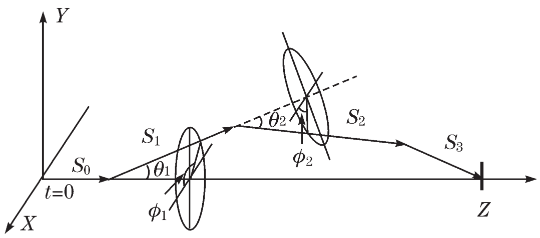

Fig. 2. Multiple scattering path along the Z-axis of an arbitrary photon propagating underwater, S_m denotes the distance between the mth and the $(m+1)$th scattering events, and θ_k and ϕ_k are polar and azimuthal angles from the kth scattering event, respectively.

where b is the volume scattering coefficient (m^{-1}), c is the speed of light underwater (m/s), z is the distance along the Z-axis (m), θ is the scattering angle, $v = 1 - \langle\cos\theta\rangle$, and $w = \frac{3}{2}(1 - \langle\cos^2\theta\rangle)$. These approaches provide different values of the two indices, but generally require that the scattering length be large while simultaneously satisfy the small-angle approximation. Lutomirski proposed statistical method-based multiple scatterings; their method required no small-angle or diffusive approximations, and allowed arbitrary volume scattering functions associate with each collision[13,14]. However, the expressions are constrained by large scattering lengths and are too complex to count. Following the multiple scatterings statistical theory proposed by Lutomirski, our analytical model is not limited by scattering length while providing simple expressions of higher accuracy. Moreover, a new impulse response waveform function is suggested. Finally, experimental validations of theoretical predictions are also presented.

Given that propagation occurs in the forward direction, it would be simple to take the absorption into account by an exponential factor. However, like many other models proposed by theorists in precise works[7−11,13,14], the absence of absorption is assumed throughout this paper.

Consider a photon launched in an infinite, nonabsorbing, multiple-scattering medium with collision geometry as illustrated in Fig. 2. For a water medium with the volume scattering coefficient b, the expected optical length between adjacent scattering events will be $1/b$[7,11,13,14], and then the expected path after the nth times of scatterings is $l_n = (n+1)/b$, whose moving distance along the Z-axis is z (Fig. 2). The expected time domain dispersion after n scatterings can be expressed as

$$\langle \Delta t_n \rangle = \left[(n+1)/b - z\right]/c, \tag{1}$$

where Δt denotes time domain dispersion, and $\langle\cdot\rangle$ represents the expected value. The scattering times of a great deal of photons yield the probability of Poisson distribution[13], so the expected value of time domain dispersion at distance z along the Z-axis is

$$\langle \Delta t \rangle = \sum_{n=0}^{\infty} P\left\{n(z) = n\right\} \cdot \langle \Delta t_n \rangle, \tag{2}$$

where the Poisson distribution $P\left\{n(z) = n\right\} = (N)^n \exp(-N)/n!$ is the probability of n times scattering occurring precisely, and N or $\langle n \rangle$ is the expected value of n.

The expected value of time domain dispersion at z depends on the expected value of n, which represents the scattering times. It is derived as follows: the random-position vector of a photon is described in the Cartesian co-ordinate (Fig. 2), and the initial position of a photon is at (0,0,0). After it is scattered precisely n times, the position vector of this photon, denoted as $\vec{L}_n$, could be expressed as

$$\begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix} = \vec{L}_n = \sum_{m=0}^{n} S_m \cdot \vec{w}_m, \tag{3}$$

where S_m denotes the distance between the mth and the $(m+1)$th scattering event, so $\langle S_m \rangle = 1/b$ and $\vec{w}_m$ denotes the traveling direction after the mth scattering (Fig. 1). From Fig. 2, we can obtain $\vec{w}_m = \left(\prod_{k=0}^{m} A_k\right)\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, where $A_k = \begin{bmatrix} \cos\phi_k\cos\theta_k & -\sin\phi_k & \cos\phi_k\sin\theta_k \\ \sin\phi_k\cos\theta_k & \cos\phi_k & \sin\phi_k\sin\theta_k \\ -\sin\theta_k & 0 & \cos\theta_k \end{bmatrix}$, A_0 is the unit matrix, and θ_k and ϕ_k are polar and azimuthal angles from the kth scattering event, respectively. θ_k and ϕ_k are statistically independent, which implies the independence of A_k; thus,

$$\left\langle \vec{L}_n \right\rangle = \sum\nolimits_{m=0}^{n} \langle S_m \rangle \langle \vec{w}_m \rangle, \tag{4}$$

$$\langle \vec{w}_m \rangle = \langle A_0 \rangle \langle A_1 \rangle \langle A_2 \rangle L \langle A_m \rangle \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \langle A \rangle^m \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \tag{5}$$

where $\langle A \rangle = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\langle\sin\theta\rangle & 0 & \langle\cos\theta\rangle \end{bmatrix}$, making $\langle \vec{w}_m \rangle = \begin{pmatrix} 0 \\ 0 \\ \langle\cos\theta\rangle^m \end{pmatrix}$. Then Eq. (3) will be

$$\langle x_n \rangle = \langle y_n \rangle = 0, \tag{6}$$

$$\langle z_n \rangle = \frac{1}{b}\frac{1 - \langle\cos\theta\rangle^{n+1}}{1 - \langle\cos\theta\rangle}. \tag{7}$$

For a small angle of light scattering underwater, we can take that $\theta \to 0$, so $\langle\cos\theta\rangle \approx \exp(\langle\cos\theta\rangle - 1)$. Thus, Eqs. (2) and (7) can be simplified as

$$z \approx \frac{1}{b}\frac{1 - \langle\cos\theta\rangle^{\langle n \rangle + 1}}{1 - \langle\cos\theta\rangle}. \tag{8}$$

The inverse type of Eq. (8) is given by

$$N = \langle n \rangle \approx \log_{\langle \cos\theta \rangle} \left[1 - bz\left(1 - \langle \cos\theta \rangle\right)\right] - 1. \quad (9)$$

As such, the expected time domain dispersion at distance z along the Z-axis is

$$\langle \Delta t \rangle \approx \log_{\langle \cos\theta \rangle} \left[1 - bz\left(1 - \langle \cos\theta \rangle\right)\right] / bc - z/c, \quad (10)$$

where $\langle \cos\theta \rangle$ is figured out from the Scattering Phase Function[13].

The expected optical lengths after the nth scattering are $l_n = (n+1)/b$ and $l_n^2 = (n+1)^2/b^2$. Substituting Poisson distribution, we arrive at

$$E\left(l^2\right) = \left(N^2 + 3N + 1\right)/b^2. \quad (11)$$

As $E(l) = (N+1)/b$, the variance of l is

$$\mathrm{VAR}(l) = N/b^2. \quad (12)$$

Then the variance of time dispersion is

$$\mathrm{VAR}(\Delta t) = N/(bc)^2. \quad (13)$$

In previous work, the small-angle diffusion approximation was employed which generally required that bz be large while simultaneously satisfy the small-angle approximation $(1 \gg \langle \theta \rangle \gg \langle \theta^2 \rangle \gg \langle \theta^4 \rangle \gg \cdots)$[6,7,10,11,13]. This limit permitted a Taylor expanding approximation that formed basic analytic solutions to the radiative transport equation. Our model shows that the actual values of VAR(Δt) and $\langle \Delta t \rangle$ are not constrained by the large scattering length and that only the small-angle approximation $\theta \to 0$ is of importance.

The one-parameter Gamma function given as $f(t) = a^2 t \exp(-at)$, where $a = 2/\langle \Delta t \rangle$[10], was used as a pulse waveform function in many previous works. Mooradian *et al.*[15] first employed it to empirically fit mass experimental data when he studied blue-green pulsed propagation through fog. Zaccanti *et al.*[9], meanwhile, investigated the temporal spreading of a light pulse propagating in water; he introduced it and found that the Gamma function gave a very good fit with his experimental data. After that, the Gamma function has been used as the distribution of choice to water[9,10,16]. At present, since the second moment (variance) of time dispersion has already been obtained, it is possible to define a new impulse response waveform of higher accuracy using this tool.

The central limit theorem should assure that a Gaussian distribution (normal distribution) is asymptotically approached for a large number of scattering events. Thus, the time dispersion may be similarly distributed at the limit of many scattering events[14]. The inverse-Gaussian probability density function then becomes an equivalent for the normal distribution when the variable is positively definite. This corresponds to the time dispersion as the forward scattering becomes dominant underwater. Furthermore, the inverse-Gaussian originally appeared as the distribution of the time spent by molecules on travelling a given distance in Brownian motion (while the Gaussian distribution describes the range distribution at a given time).

Brownian motion, as a simple diffusion in photon-propagation problems, is frequently considered as the limiting process obtained when a scattering particle experiences a large number of isotropic scattering events that are separated by small path lengths. This similarity leads us to believe that the inverse-Gaussian distribution is a likely candidate for the impulse response waveform of water. A probability density function could be a normalized energy density function, so we assign the inverse-Gaussian distribution as the optical impulse response waveform function for water as

$$h(t) = \left[\frac{\alpha}{2\pi t^3}\right]^{\frac{1}{2}} \exp\left[\frac{-\alpha (t-\beta)^2}{2\beta^2 t}\right], \quad (14)$$

where $\langle \Delta t \rangle = \alpha$, and $\mathrm{VAR}(\Delta t) = \beta^3/\alpha$.

For practical laser pulses, time domain dispersion is tested as the algebraic difference of pulse half-amplitude width between transmitted pulse and received pulse[10,16]. Zhang *et al.*[10] range-gated a high-speed image intensified CCD and obtained experimental results for time domain dispersion of optical pulses propagating underwater (Table 1). Compared with his experimental findings, theory results from previous work[7,8,11−15] and our theory results (Wei method) are shown in Fig. 3, where curves of 'Turchin', 'Lutomirski', 'Stotts', and 'Hulst' are plotted in Gamma distribution, and 'Wei' in inverse-Gaussian distribution. The parameters are the same with those in Ref. [10]: laser unit pulse energy E_0=25 mJ, pulse half-amplitude width T_0=8.8 ns, $b \approx$ 0.27 m^{-1} when k_{ext}=0.3 m^{-1} from Ref. [17] and Lorentz line shape was used as the emitting laser pulse shape (Fig. 4):

$$I(t) = \frac{E_0 (T_0/2)^2}{(t-T_0)^2 + (T_0/2)^2}. \quad (15)$$

All results show that time domain dispersion increases with the rise of range z. In contrast with the methods of Stotts and Hulst, our method is closed to the experimental results for the whole tested range. The methods of Turchin, Ishimaru, and Lutomirski show a reasonable agreement but are a little lower for small range (z<14 m) than experimental data and begin to deviate significantly for large range (z>14 m). In contrast, our method is nearly faultlessly matched with experimental results for small range, and deviates more slowly for large range.

In our opinion, the inverse Gaussian appears to match experimental results better than the Gamma distribution for two reasons. First, function defined by the first and

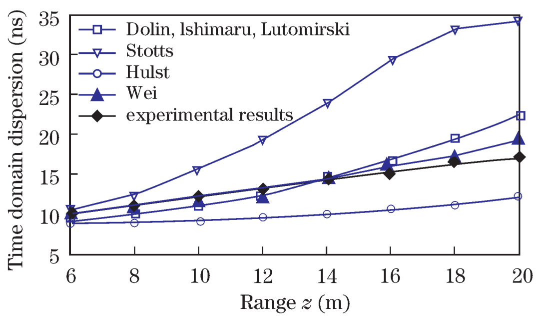

Fig. 3. Comparison between theoretical and experimental data.

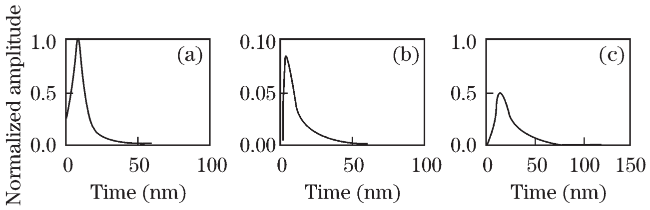

Fig. 4. Time-domain dispersion simulation of optical pulse propagating underwater where $z = 20$ m; (a) emitting optical pulse shape $I(t)$, the pulse amplitude peak value is normalized to unit amplitude; (b) time-domain dispersion simulation of ideal pulse propagating underwater $h(t)$; (c) time-domain dispersion simulation of optical pulse propagating underwater, which is obtained by $I(t) \otimes h(t)$.

Table 1. Experimental Data[9]

z Range (m)	6	8	10	12	14	16	18	20
Experimental Data by ICCD (ns)	10.1	10.7	11.9	13.2	14.2	15.2	16.4	17.3

second-order moments of time dispersion is of higher accuracy than those by first-order moment only. Second, in the same case of water, the skewness of the inverse Gaussian distribution is more left than that of the Gamma distribution[18], and it varies the half-amplitude width only slightly for the whole range, which matched the variation of experimental data. In a small propagating range (z<14 m), the kurtosis of the inverse Gaussian distribution is larger than that of the Gamma distribution[18], which makes the half-amplitude width larger than that of the Gamma distribution. This characteristic may also explain why the inverse Gaussian distribution seems more matched with experimental results in small range, where the un-scattering and low-order scattering light dominate.

In conclusion, we provide a numerical method based on a multiple scattering model of photons while not being constrained by large scattering lengths; the result is a better description of the time domain dispersion of underwater light propagation than previous methods. In our research work to be published in another paper, the spatial dispersion for light propagation underwater is suggested to be Gaussian distribution theoretically and experimentally, which complements past results due to the internal relations between time and spatial dispersion for underwater light in the future.

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