Weighting IFT algorithm for off-axis quantized kinoforms of binary objects

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The results of experiments on the synthesis of the off-axis quantized kinoforms of binary objects with the use of the weighting iterative Fourier transform (WIFT) algorithm are presented. Kinoforms are registered with a liquid-crystal spatial light modulator (SLM). A simple procedure to introduce the carrier frequency into the structure of an axial kinoform is proposed. An image reconstructed by an off-axis kinoform is free from the noises with the zero and close frequencies caused by the imperfection of both the phase mode of operation of the SLM and the effects of quantization of the registered phase. Data on the diffraction efficiency are also given.

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Possessing a high diffraction efficiency, the kinoform optical elements are widely used for the beam splitting (fan-out), beam shaping, and pattern or image generation[1,2].

The detailed description of the weighting iterative Fourier transform (WIFT) algorithm was given earlier[3], where we also showed that it is most efficient for the synthesis of the kinoforms of binary objects. In brief, the essence of the WIFT-algorithm consists in the following. Firstly, one or several iterations ($K_{\rm gs}$) with an input real object $f_{\rm o}$ are realized by the classical Gerchberg-Saxton (GS) scheme. Then, in all iterations with $k > K_{\rm gs}$ at the formation of the input, the amplitude $f_{\rm o}$ will be replaced by a new amplitude defined as $f_k = \alpha_k f_{\rm o}$, where the weight coefficients α_k are determined by the recurrence relation $\alpha_k = \alpha_{k-1}\beta_{k-1}$, $(k > 1)$, $\beta_{k-1} = f_{\rm o}/(|\overline{f}_{k-1}|_{+\varepsilon})$, $|\overline{f}_{k-1}|$ is the reconstructed amplitude at the $(k-1)$th iteration, and ε is a small number $\sim 10^{-10}$. The processing of the phases of an object plane and the kinoform remains the same as that in the classic GS-algorithm. From the viewpoint of optics, the system of weight coefficients α_k normalized to unity can be interpreted as some object-dependent amplitude filter which acts on the initial object $f_{\rm o}$ and varies in the process of iterations. We note that all the coefficients α_k in the iteration process tend to unity in the case when the iterative process converges.

In this letter, we study the potentialities of the WIFT algorithm at the synthesis of the off-axis kinoforms of binary objects with regard for the quantization of a phase registered with a spatial light modulator (SLM). In the course of modeling and optical experiments, we compared the WIFT algorithm with the kinoform version of the Fienup input-output (IO) algorithm[4]. The latter is most convenient for the sake of comparison, because it is well known and, like the weighting algorithm, differs from the GS-algorithm only by means of processing of a field in the object plane. The programming of the IO algorithm is realized by Eqs. (8)−(10) from Ref. [4].

A number of model experiments with various binary objects (with sizes of 64×64 and 128×128) were realized with the purpose to compare the potentialities of the WIFT and IO algorithms at the synthesis of quantized kinoforms. In all the cases, the same phase starting diffuser with a uniform distribution of phases in the interval (0–2π) is used. In the experiments involving the IO algorithm, we used the optimum value of the object-depended coefficient $\beta = \beta_{\rm opt}$ in Eqs. (8) and (9) of Ref. [4], which was determined by means of the cyclic repetition of the procedure of synthesis for various values of β (from the interval 0.1−5.0 with a step of 0.1). In Figs. 2 and 3 the results of model experiments on the kinoforms' synthesis for one of the typical binary objects (Fig. 1(a)) with a dimension of 128×128 counts are presented. The variance of the intensities of images reconstructed in the process of iterations was evaluated by the standard formula[3]. The presented plots for the ranges and the dispersions of the output intensity clearly demonstrate the advantages of the stepwise quantization[5] as compared with the direct quantization of the kinoform phases. As seen from Figs. 2 and 3 and Tables 1 and 2, the advantage of the WIFT algorithm over the IO algorithm as for the quality of reconstructed images of binary objects at the continuous representation of the phase of a kinoform[3] is lost at the transition to the quantized representation. If the spread of the intensities of bits $\Delta I^{\rm WIFT}_{\rm one-bits}$ for bit array-object is less than $\Delta I^{\rm IO}_{\rm one-bits}$ for any number of quantization levels (NQL),

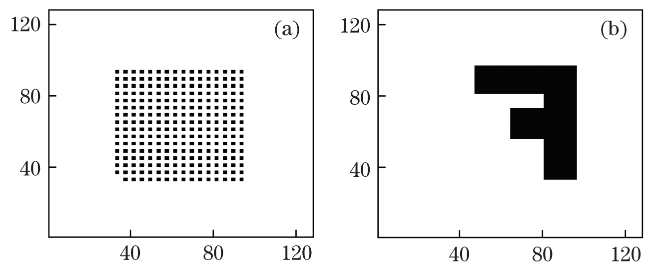

Fig. 1. Binary objects of 128×128.

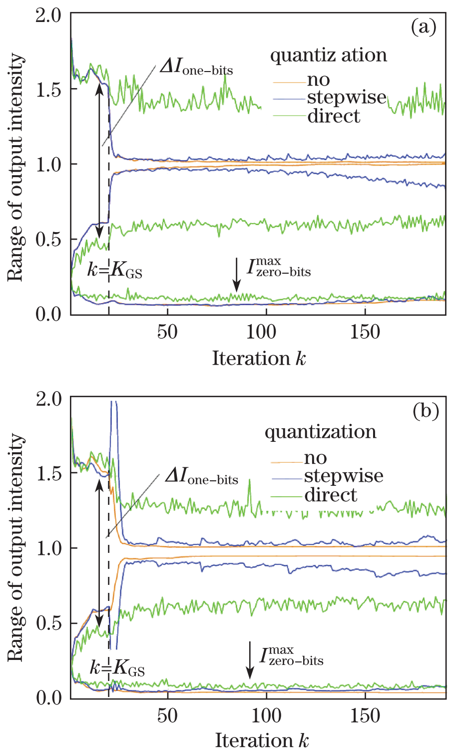

Fig. 2. Ranges of output intensity without quantizing as well as with stepwise and direct quantizing of the kinoform phase onto seven levels: (a) WIFT algorithm; (b) IO algorithm.

Table 1. Dependence of the Ration $I^{\rm WIFT}_{\rm one\text{-}bits}/\ I^{\rm IO}_{\rm one\text{-}bits}$ from the Number of Quantization Levels

Number of Quant.	Ratio $\Delta I^{\rm WIFT}_{\rm one-bits}/\Delta I^{\rm IO}_{\rm one-bits}$	
Levels	Bit-array Object	Letter-F Object
3	0.693	1.011
7	0.858	1.121
15	0.639	1.112
31	0.631	1.103
63	0.523	0.651
127	0.371	0.72
255	0.384	0.45

Table 2. Dependence of the Ration $I^{\rm min}_{\rm one\text{-}bits}/\ I^{\rm max}_{\rm zero\text{-}bits}$ from the Number of Quantization Levels

Number of Quant.	Ratio $I^{\rm min}_{\rm one-bits}/I^{\rm max}_{\rm zero-bits}$			
	Bit-array Object		Letter-F Object	
Levels	WIFT	IO	WIFT	IO
3	1.20	1.31	0.65	0.58
7	8.50	7.84	3.95	3.42
15	11.70	12.01	6.83	4.76
31	10.62	14.10	6.20	8.83
63	17.50	14.95	6.23	7.07
127	13.01	14.14	8.18	8.26
255	14.03	12.80	8.61	9.59

then the IO algorithm is preferable for letter-objects at NQL$\leqslant$31. At the same time, by the ratio $I^{\rm min}_{\rm one-bits}/I^{\rm max}_{\rm zero-bits}$(which can be regarded as a stronger version of the ordinary signal/noise ratio), we have the inverse pattern for letter-objects: at small NQL, the WIFT algorithm is preferable.

The data in the Tables 1 and 2 were calculated using the stepwise quantization of the kinoform phases. Note that the values in Tables 1 and 2 can vary by depending on the chosen start-diffuser. However, the total pattern is conserved, that is none of the algorithms has the obvious advantage. But the WIFT algorithm conserves some advantage, because it contains no parameters requiring the optimization (contrary to the IO algorithm), which essentially accelerates the counting rate.

In the synthesis of an axial Fourier-kinoform which reconstructs the image in the zero order of diffraction, the inaccuracy of the representation of the kinoform's phase leads to the appearance of a bright spot surrounded by noises at the center of the image. This effect can be eliminated in the single way due to a displacement of the image, as a whole, to the side from the optical axis of the system of reconstruction. This can be achieved by the synthesis of an off-axis kinoform which reconstructs the image in the nonzero order. We proposed a simple method of construction of off-axis kinoforms[6] by means of the introduction of a space carrier (linear phase) in the axis kinoform. The method has certain advantages over the known methods. In particular, it admits a greater shift of the reconstructed image from the optical axis of the Fourier-system. In this method, the linear phase $2\pi(x_{\rm o}u+y_{\rm o}v)$ is supplemented to the phase $\psi(u,v)$ of an on-axis kinoform at the last iteration. As a result, in the ideal case, the calculated kinoform will reconstruct the image as

$$\begin{aligned} & f_{\rm off}(x,y) = \\ & \Im^{-1}\{\exp[{\rm i}(\psi(u,v)+2\pi(x_{\rm o}u+y_{\rm o}v)]\} = \\ & f(x,y)\otimes\delta(x-x_{\rm o},y-y_{\rm o}) = \\ & f(x-x_{\rm o},y-y_{\rm o}) \ . \end{aligned} \quad (1)$$

Here, $\Im^{-1}$ is the operator of the inverse Fourier-transformation, $f_{\rm off}(x,y)$ is the off-axis image, $f(x,y)$ is the axial image, δ is the delta-function, the symbol $\otimes$ stands for the operation of convolution, and $x_{\rm o}, y_{\rm o}$ are the shifts of the image along the axes x, y. In practice[7], in order to eliminate the influence of a finite size of the pixel-active window $\tau_1\times\tau_2$ of the SLM on the reconstructed image, the input object $f(x,y)$ must be predistorted in the start of iterations by

$$\widetilde{f}(x,y) = f(x,y)[{\rm sinc_1(x)sinc_2(y)}]^{-1}, \quad (2)$$

where

$$\begin{aligned} {\rm sinc_1(x)} &= {\rm sinc}[\tau_1({\rm x+x_o})/\Delta{\rm x}], \\ {\rm sinc_2(y)} &= {\rm sinc}[\tau_2({\rm y+y_o})/\Delta{\rm y}], \\ {\rm sinc(z)} &= {\rm sin}\pi z/\pi z, \end{aligned} \quad (3)$$

where $\Delta x\times\Delta y$ is the size of the image. Under conditions of $|x|\leqslant\Delta x/2$, $|y|\leqslant\Delta y/2$, and $\tau_1,\tau_2\leqslant 1$, assuming that $\Delta x=\Delta y$ and $\tau_1=\tau_2$, in order that the denominator in Eq. (2) do not become zero in the region $|x,y|\leqslant\Delta x/2$, it

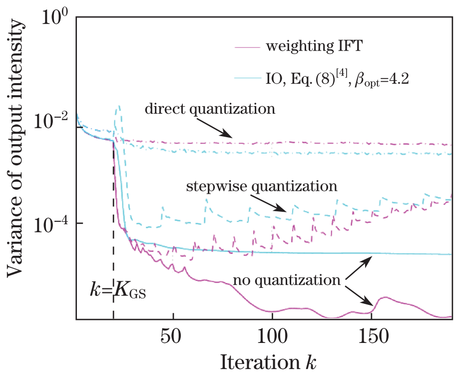

Fig. 3. Variance of the intensity of reconstructed images versus the iteration number.

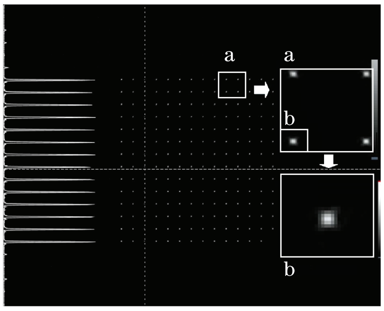

Fig. 4. Reconstructed image of the bit array with the intensity profile (third column) and the structure (a, b) of spots.

is necessary to satisfy the condition $|x_o, y_o| \leqslant \Delta x/2$. This implies that the maximally admissible shift of the reconstructed image is equal to a half of the linear size of the diffraction order. Illuminating by a plane wave, the kinoform calculated for the object $\tilde{f}(x, y)$ in the fractional order $P_x = x_o/\Delta x$, $P_y = y_o/\Delta y$ will reconstruct the nondistorted image of the object $f(x, y)$.

We note that, at the reconstruction into the orders $P_x = 0.25$ and P_x=0.50 ($P_y = 0$) the lattice period will be formed by the values of phases 0, $\pi/2$, π, $3\pi/2$ and 0, π, respectively; for the rest values of P_x, the carrier acquires a more complicated shape. It is worth noting that the diffraction efficiency (DE) of off-axis kinoforms decreases, as the shift of an image increases. We shall present below the quantitative results of experimantal measurements of the DE of off-axis kinoforms.

A typical optical-digital Fourier system with a He-Ne laser (543 nm) is used to investigate the kinoform reconstruction characteristics. Here, we use a reflection-type phase-only SLM (HEO 1080 Pluto, HOLOEYE, Germany). The spatial calibration of the SLM was performed by the method proposed in Ref. [8]. The reconstructed images were recorded and carried out with the use of a CCD-camera (SP620-USB, Spiricon, USA) with a high dynamic range. The size of objects and kinoforms was 1000×1000 (pixels) format. As the objects, we took a 14 × 14 two-dimensional bit array occupying 200 × 200 counts of the input plane and letter F occupying 250 × 150 counts of the input plane. We have studied the output intensities, diffraction efficiency, as well as the effects of quantization of the kinoform phase on the reconstructed image[9].

In Fig. 4, we present the results of experiments with a bit array. In calculations, we applied the stepwise quantization with the number of quantization levels M=256 and the ratio of iterations GS/weighting = 25/200. The measured intensity profile demonstrates a high homogeneity of light spots. Each spot of the image was recorded by an area consisting of 9×9 pixels of a CCD-chamber, which allows us to control the regularity of the position of intensity maxima of a spot in the output plane. No deviations from the regularity were observed.

In Fig. 5, the plots characterizing the quality of the image (Fig. 4) reconstructed by an off-axis kinoform are presented. As distinct from the model experiments in which the WIFT-method was preferable over the IO-method at NQL=256 (see Tables 1 and 2), both methods in this experiment gave practically the same results. Image reconstruction was carried out in order of $P_x = 0.4$, $P_y = 0.4$. The error of variance of intensity obtained in the experiment was ~ 6 % .

Figure 6 shows the image of letter F reconstructed into the order P_x=0.50, P_y=0. In Fig. 7, we demonstrate how the DE of a kinoform varies at the subsequent shift

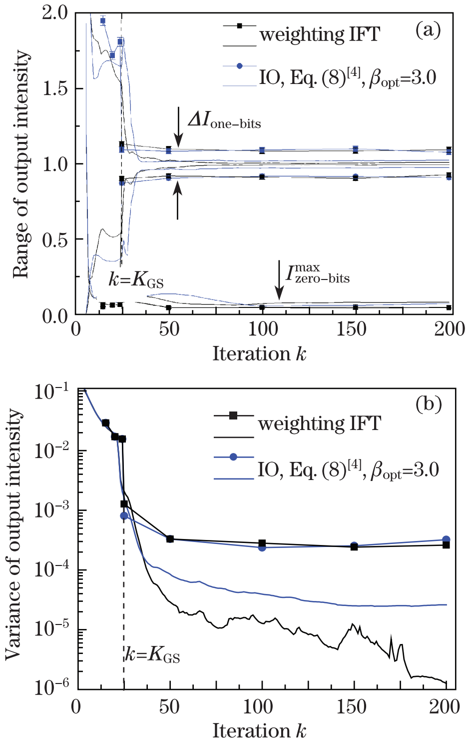

Fig. 5. (a) Ranges of output intensities; (b) variance of spots-intensities versus iteration number. Curve"—"stands for the computing results; Curves —■—, —●— stand for the experimental results.

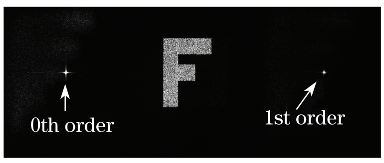

Fig. 6. Reconstruction into the order of $P_x = 0.5$, P_y =0.

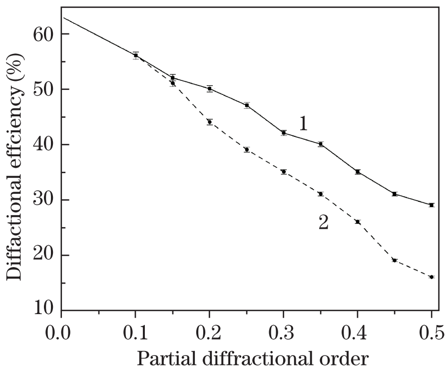

Fig. 7. Diffraction efficiency of the off-axis kinoform vs the partial diffraction order (for the object-letter F). Curve 1: shift along the axis x; Curve 2: shift along the diagonal in the x,y-plane.

of the image. Curves 1 and 2 correspond to a shift along the axis x and along the diagonal in the x,y-plane (P_x, $P_y = 0$, 0.1, 0.15, $\cdots$, 0.50), respectively. DE was defined as the ratio of the intensity of the diffracted light to the light intensity available for diffraction pattern. Accuracy of the calculation DE was $\sim$ 2.3%. In these cases, the synthesis of kinoforms was realized with the help of the WIFT algorithm at the ratio of the numbers of iterations GS/weighting = 25/200.

In conclusion, we present the results of numerical and optical experiments studying the quality of the reconstructed images of binary objects for off-axis quantized kinoforms calculated with the help of the WIFT and IO algorithms. To obtain off-axis kinoforms from on-axis ones, we propose a simple procedure of introduction of a spatial carrier to an on-axis kinoform, which admits a greater shift of the reconstructed image from the optical axis of the Fourier-system as compared with the available ones. During the experiments with the WIFT and IO algorithms, we find that the physico-technical parameters of available SLMs do not allow one to completely realize the potentialities of high-precision algorithms; both ones, in this case. Apparently, we may assert that a kinoform calculated with the help of any of the algorithms ensuring the rated variance of the intensity of reconstructed images $\leqslant 1\times10^{-4}$ will give the image of approximately identical quality under the realization with an SLM of the type used by us.

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