# 3D visualization using pulsed and CW digital holographic tomography techniques 

(Invited Paper)

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#### Abstract

We outline the use of digital holographic tomography to determine the three-dimensional (3D) shapes of falling and static objects, such as lenslets and water droplets. Reconstruction of digitally recorded inline holograms is performed using multiplicative and Radon transform techniques to reveal the exact 3D shapes of the objects.


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Digital holography (DH) with its variations namely digital holographic interferometry (DHI), digital holographic microscopy (DHM), and digital holographic tomography (DHT), has become the method of choice for metrological applications. These applications range from deformation detection, visualization, and quantization as well as surface profilometry in three-dimensional (3D). In this letter, DHT is employed for the tracking of 3D shape of falling objects.

Lenslets, like water droplets, are translucent objects with large curvatures and can scatter light at very large angles. The use of traditional DH to determine the 3D shape results in thousands of fringes per millimeter, which may easily exceed the resolution of chargecoupled device (CCD) cameras. To solve this problem a novel single-beam holographic tomography (SHOT) based technique is developed for the recording and reconstruction of 3D shapes of water droplets and lenslets, and their distribution ${ }^{[1]}$. Since the beam width is larger than several water droplets the light that gets transmitted in between the droplets acts like a reference beam which interferes with the object beam and records the holograms. So, a single beam (in-line) holography reduces system complexity, and for our purposes, we can determine the shape of the droplets without details of interior structure.

Specifically, a non-intrusive technique for the recording and 3D shape reconstruction of water droplets and lenslets using the multiplicative technique (MT), as well as the Radon transform technique (RTT), is described. Both these techniques enable the visualization of the twodimensional (2D) shapes of water droplets along a certain direction. Thus, to visualize the 3D shape multiple projections from multiple directions as in tomographic imaging systems are required. Hence, holographic reconstruction for 2D visualization along a certain projection is used, and the 3D shape is reconstructed using SHOTMT or SHOT-RTT.

The object is assumed to be a 3D object at a distance $d$ from the CCD camera. The diffraction of light is de-
scribed by the Fresnel-Kirchhoff integral and can be approximated by the Fresnel integral ${ }^{[2]}$

$$
\begin{align*}
\Gamma(\xi, \eta)= & \frac{\mathrm{i}}{\lambda d} \mathrm{e}^{\left(-\mathrm{i} \frac{2 \pi}{\lambda} d\right)} \mathrm{e}^{\left[-\mathrm{i} \frac{\pi}{\lambda d}\left(\xi^{2}+\eta^{2}\right)\right]} \times \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x, y) \\
& \cdot E_{R}^{*}(x, y) \mathrm{e}^{\left[-\mathrm{i} \frac{\pi}{\lambda d}\left(x^{2}+y^{2}\right)\right]} \mathrm{e}^{\left[\mathrm{i} \frac{2 \pi}{\lambda d}(x \xi+y \eta)\right]} \mathrm{d} x \mathrm{~d} y \tag{1}
\end{align*}
$$

where $h(x, y)$ is the recorded hologram, $E_{R}^{*}(x, y)=$ $a=$ const. is the conjugate of the reference beam used for digitally reconstructing the hologram. The intensity is calculated by squaring the optical field, i.e., $I(\xi, \eta)=|\Gamma(\xi, \eta)|^{2}$ and the phase is calculated using $\varphi(\xi, \eta)=\operatorname{Im}[\Gamma(\xi, \eta)] / \operatorname{Re}[\Gamma(\xi, \eta)]$. If we perform the following change of variables: $\nu=\xi /(\lambda d), \mu=\eta / \lambda d$, and if the function $\Gamma$ is digitized on a rectangular raster $N_{x} \times N_{y}$ points with steps $\Delta x, \Delta y$ which are pixel to pixel distance on the CCD in horizontal and vertical directions, respectively, Eq. (1) becomes ${ }^{[2,3]}$

$$
\begin{align*}
\Gamma(m, n)= & z(m, n) \times \sum_{k=0}^{N_{x}-1} \sum_{l=0}^{N_{y}-1} E_{R}^{*}(k, l) \\
& \cdot h(k, l) w(k, l) \mathrm{e}^{\left[\mathrm{i} 2 \pi\left(\frac{k m}{N_{x}}+\frac{l n}{N_{y}}\right)\right]} \tag{2}
\end{align*}
$$

where

$$
\begin{align*}
& z(m, n)=\frac{\mathrm{i}}{\lambda d} \mathrm{e}^{\left(-\mathrm{i} \frac{2 \pi d}{\lambda}\right)} \mathrm{e}^{\left[-\mathrm{i} \pi \lambda d\left(\frac{m^{2}}{N_{x}^{2} \Delta x^{2}}+\frac{n^{2}}{N_{y}^{2} \Delta y^{2}}\right)\right]} \\
& w(k, l)=\mathrm{e}^{\left[-\mathrm{i} \frac{\pi}{\lambda d}\left(k^{2} \Delta x^{2}+l^{2} \Delta y^{2}\right)\right]} \tag{3}
\end{align*}
$$

where $N_{x}=\lambda d / \Delta \xi \Delta x=2 X_{\max } / \Delta x, N_{y}=\lambda d / \Delta \eta \Delta y=$ $2 Y_{\max } / \Delta y, \Delta \nu=1 / N_{x} \Delta x, \Delta \mu=1 / N_{y} \Delta y$. Thus, the hologram processing using Fresnel approximation formula is computed using an inverse Fourier transform formula, $\Gamma=z \cdot F^{-1}\left\{E^{*} \cdot h \cdot w\right\}$.

In SHOT-MT, digital holograms $h_{j}(x, y)$ corresponding to each angular orientation $\theta_{j}$ about the $y$ axis coming out of the plane of the paper are recorded, as


Fig. 1. (a) Experimental setup of typical SHOT-MT recording scheme; (b) lab setup showing indigeneously pulsed frequency doubled YAG laser, along with 4 recording high-speed cameras.


Fig. 2. Schematic showing the principle of SHOT-MT reconstruction.
shown in Fig. 1. Thereafter, $h_{j}(x, y)$ are numerically reconstructed and the intensities $I_{j}$ computed on multiple planes around the distance $d$ which corresponds to the middle of the test volume where the droplet is located. The numerical reconstruction involves using the discretized form of the Fresnel diffraction formula. Then after some coordinate transformations, the 3D shape and distribution of this droplet can be reconstructed by multiplying the multiple reconstructed intensities $I=\prod_{1}^{M} I_{j}$, as shown in Fig. 2.
An example of SHOT and reconstruction using SHOTMT is provided below. Figure 3(a) show 4 holograms of a hemispherical lens falling under gravity. The holograms are recorded at angles of $0,45,90$, and 135 degrees, respectively. The typical terminal speed of fall of the lens/droplet with typical size in the order of a few millimeters is approximately $10 \mathrm{~m} / \mathrm{s}$. Holograms are captured using an indigeneously built frequency doubled YAG laser of wavelength of 532 nm with a pulse width of 10 ns , pulse energy of 100 mJ , and repetition rate of 1 Hz . It is easily estimated that the displacement of the lens/droplet during this time is in the order of 100 nm , thereby eliminating "blurring" of the hologram during capture. Figure 3(b) shows the reconstructed 3D picture.

In SHOT-RTT, the projection 3D matrix of all the numerically reconstructed holograms from different angles $\theta_{j}=[0: j \pi / M: \pi], j=1,2, \cdots, M$ is formed, and then the inverse 3D Radon transform is calculated by computing the inverse 2D Radon transform of each slice using the Fourier Slice theorem ${ }^{[4]}$. Finally, some morphological image processing techniques are applied to the inverted 3D matrix to get the final 3D shape. Specifically, for each orientation we record an inline single-beam holo$\operatorname{gram} h_{j}(x, y)$, for $\theta_{j}=\left[0^{\circ}: 360^{\circ} / M: 360^{\circ}\right]$ where $j=1,2,3,4, \cdots, M$, respectively. Also, for each orientation, we reconstruct the corresponding hologram and compute its intensity $I_{j}$ according to Eq. (2) on a single plane at the distance $d$ which corresponds to the middle of the test volume where the droplets are located. The algorithm works as follows.

1) Construct the 3D projection matrix of all the reconstructed holograms according to Eq. (2) from $M$ different angles as shown in Fig. 4(a).
2) Compute the inverse Radon transform of the 3D matrix as shown in Fig. 4(b) by computing the inverse 2D Radon transform of each slice using the Fourier slice theorem which states that the Fourier transform of a projection is a slice of the 2D FT of the region from which the projection was obtained (Figs. 4(a) and (b)).
3) Perform some morphological image processing techniques on the 3D matrix obtained from step 3 to get the


Fig. 3. (a) Recorded inline holograms of a free-falling 5 mm hemisphere lens; (b) 3D reconstruction.


Fig. 4. (a) Projection matrix; (b) inverse Radon transform matrix; (c) 3D shape; (d) illustration of the Fourier slice theorem. The 1D Fourier transform of a projection is a slice of the 2 D Fourier transform of the region from which the projection is obtained ${ }^{[4]}$; (e) definition of coordinates used for Radon transform ${ }^{[4]}$.


Fig. 5. (a) Recorded inline holograms of water droplet suspended from a syringe along several orientations with $M=10$ and (b) the 3D reconstruction ${ }^{[5]}$.
3D shape as shown in Fig. 4(c).
The inverse RT is computed according to the following formula:

$$
\begin{align*}
& G(\omega, \theta)=\int_{-\infty}^{\infty} g(\rho, \theta) \mathrm{e}^{-\mathrm{j} 2 \pi \omega \rho} \mathrm{~d} \rho= \\
& {[F(u, v)]_{u=\omega \cos \theta, v=\omega \sin \theta}=F(\omega \cos \theta, \omega \sin \theta),} \tag{4}
\end{align*}
$$

where $\Re\{f(x, y)\}=g(\rho, \theta)=\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(x \cos \theta+$
$y \sin \theta-\rho) \mathrm{d} x \mathrm{~d} y$ is the Radon transform of a function $f(x, y)$ and $x \cos (\theta)+y \sin (\theta)=\rho$ (see Figs. 4(d) and (e)).

A second example of SHOT, and reconstruction using SHOT-RTT is provided below. Figure 5 (a) show multiple inline holograms of a water droplet hanging from a syringe needle, recorded using $M=10$. A continuons wave (CW) Ar laser operating at 514 nm has been used for illumination. Figure 5(b) shows the 3D reconstructed image using SHOT-RTT.
In conclusion, we show examples of pulsed and CW inline digital holographic tomography using MT and RTT for reconstruction. While SHOT-MT is easier to set up and use, SHOT-RTT is usually more accurate. Applications of pulsed SHOT techniques are likely to find applications for high-speed moving objects. The effect of noise, which may comprise multiple objects, fog, atmospheric effects etc., is currently under investigation.

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