Image design for normal viewing image-plane disk-type multiplex hologram

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In this letter, the method of direct object-image relationship in normal-view disk-type multiplex holography is adopted for theoretical analysis. The corresponding parameters in hologram fabrication and reconstruction process for both virtual-image and real-image generation are also introduced through numerical simulation and experimental results.

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Since the concept of wavefront reconstruction was first presented by Gabor^[1], the holographic technology has become one of the most important branches of optics. In 1962, Denisyuk developed a new type of hologram called reflection hologram^[2]. The reconstructed image ray generated from a reflection hologram is monochromatic due to its distinguishing feature of high wavelength selectivity, making it suitable for full-color image display^[3,4]. Rainbow hologram, the reconstructed images of which can be demonstrated under white-light illumination, was proposed by Benton in 1969^[5]. Recently, multiplex hologram^[6,7] proposed by DeBitetto has been applied into several formats, including planar-type, cylindricaltype^[8], conical-type^[9], and disk-type^[10] holograms.

Conventional multiplex holograms are composed of a series of long thin individual holograms, which inevitably cause the reconstructed images overlaid with a fence structure^[10]. To eliminate the fence structure overlaid on the reconstructed image generated from conventional multiplex hologram, the image-plane method is utilized. The image-plane method has been designed for various formats of holograms, including cylindrical-type, conical-type, and disk-type holograms^[11-13]. Due to the lower diffraction efficiency of multiple-exposure holograms, the two-step recording technique was developed to overcome this limitation^[14,15]. Reflection holograms have been widely used for display; thus, we developed reflection image-plane disk-type multiplex hologram (IPDTMH) through both one-step and two-step recordings^[16,17].

The disk-type multiplex hologram has the advantage of commercial mass production owing to the utilization of the well-developed CD technology. Recently, we developed a 360-degree-viewable IPDTMH by one-step recording^[18]. In the current letter, we describe how to make a normal-view IPDTMH using a one-step process. The 3D-display conditions for virtual-image and realimage generation are discussed. Suitable parameters for holographic recording are obtained by numerical analysis, and experimental results are also demonstrated.

The holographic processes for making an IPDTMH are introduced by Figs. 1 and 2. Figure 1 shows our optical system for IPDTMH fabrication by one-step recording while Fig. 2 shows the reconstruction condition for a transmission-type IPDTMH. In the hologram fabrication process, the recording film is illuminated by a normalincidence object beam $W_{\rm O}$ and a reference beam $W_{\rm C}$. On the holographic film plane, a series of image-plane holograms is recorded. For viewers at the observation plane during the reconstruction process (Fig. 2), the hologram disk can display 3D real or virtual images under white-light source illumination ($P_{\rm w}$).

As shown in Fig. 1, the corresponding parameters for creating a hologram include the numerical relationship



Fig. 1. Schematic for making a normal-view image-plane disktype hologram.



Fig. 2. Geometry of simultaneous two kinds of reconstruction processes for a finished disk-type multiplex hologram.



between the object information on the input plane (LCTV) and the image on the recording film plane in the object beam $W_{\rm O}$. A suitable interfering angle θ_r between the object beam $W_{\rm O}$ and the reference beam $W_{\rm C}$ is also needed. The key parameter $\theta_{\rm r}$, one of the main factors for our numerical analysis, is obtained by computer simulation in the following.

Referring to Fig. 3, we need to design the imaging condition for the object beam. This optical setup is designed to image the 2D object information on the LCTV (P_i) onto the holographic film (P_o) . The illuminating source (P_l) is focused on a point (P_v) at distance d_{fe} behind the recording film plane. We note that P_v is the best viewing position for the observer in the reconstruction process, as shown in Fig. 2.

In the object beam $W_{\rm O}$, the light beam originating from the beam splitter (BS) is first spatially expanded by a spatial filter SF₁ before passing through the three-lens optical system. It is designed to be focused by the lens L_1 onto the second lens L_2 , where zeroth-order filtering is performed. On its way to L_2 , it acquires a 2D image taken by a CCD camera aiming at a suitable inclined angle θ_a at an original 3D object (Fig. 4) from the LCTV plane (P_i). The object information is enlarged and imaged by the lens pair L_2 and L_3 onto the recording film (P_o). After interfering with a diverging spherical wave $W_{\rm C}$, an image-plane hologram is created. We then rotate the original 3D object and the recording film by the same small angle. This process continues until both the object and the film are rotated a full round.

In order to achieve the observation condition (Fig. 2), the parameter design for the interfering angle $\theta_{\rm r}$ in our one-step holographic recording and the geometrical relationship in the viewing coordinate is needed. We first describe the reconstruction condition, and then obtain the parameters for hologram production by numerical simulation. Referring to the observation coordinate shown in Fig. 2, an observer, with a separation $D_{\rm e}$ between his two eyes, can see a 3D image provided by a set of holograms separated by a distance $D_{\rm h}$ and spanned by an angle $\theta_{\rm h}$ on the hologram disk. The parallax θ_{vr} (θ_{vv}) of the 3D real (virtual) image seen by this observer and the distance $d_{\rm if}$ from the 3D image to the hologram plane can be determined. We define the parallax $\theta_{\rm v}$ ($\theta_{\rm vr}$ or $\theta_{\rm vv}$) of the observed image by adopting the distance from the image point $(P_{\rm cr} \text{ or } P_{\rm cv})$ to the eye and the separation between two eyes $(D_{\rm e})$ of the observer.

The parallax of 3D image from the finished hologram can be expressed as

$$\theta_{\rm v} = 2 \tan^{-1} \left[\frac{A}{d_{\rm if}} \sin\left(\frac{\theta_{\rm h}}{2}\right) \right],$$
(1)

where A is the distance between the center of any individual hologram and the center of disk and d_{if} is the distance of the observed image from the hologram plane. Figure 2 shows two reconstruction conditions for 3D display in normal-view disk-type multiplex holography. The corresponding parameter $d_{\rm if}$ can be obtained from

$$d_{\rm if} = \frac{2Ad_{\rm fe}\sin\left(\frac{\theta_{\rm h}}{2}\right)}{D_{\rm e} - 2A\sin\left(\frac{\theta_{\rm h}}{2}\right)},\tag{2}$$

and

$$d_{\rm if} = \frac{2Ad_{\rm fe}\sin\left(\frac{\theta_{\rm h}}{2}\right)}{D_{\rm e} + 2A\sin\left(\frac{\theta_{\rm h}}{2}\right)},\tag{3}$$

for virtual-image and real-image generation, respectively. Finally, the suitable inclination angle (θ_a) of the CCD camera (Fig. 4), i.e., the angle between the CCD axis and the vertical axis Z_o , for obtaining 2D information from an original 3D object, can be determined using $\theta_v = \theta_h \sin \theta_a$.

Next, based on coordinate transformation, the relationship between image points ($P_{\rm cr}$ and $P_{\rm cv}$ in Fig. 2) and the original object point in object coordinates ($P_{\rm c}$ in Fig. 4) can be determined. An object point $P_{\rm o}$ (in Fig. 3; $P_{\rm or}$ and $P_{\rm ov}$ in Fig. 2) on the film can be determined from the corresponding original object point $P_{\rm c}$ in Fig. 4. By numerical simulation with computer, we can determine whether or not the reconstructed image ray can reach the eye of the observer.

Under white-light source illumination $(P_w \text{ in Fig. } 2)$ and on the condition that both eyes of the observer can see the respective object point belonging to the same original object point (P_c) , the real (virtual) image point $P_{\rm cr}$ ($P_{\rm cv}$) can be viewed by the observer through the intersection of the lines of sight of the two eyes. Again, utilizing computer simulation, we can determine the effects on the reconstructed image by changing the holographic parameters. The optimized parameters for hologram fabrication can be obtained after this analysis process. By combining the key holographic factors $\theta_{\rm h}$ and $\theta_{\rm r}$, a series of parameters can be demonstrated. Here we introduce another key factor, $N_{\rm h}$, indicating the hologram number, for hologram production, which can be estimated by the factor θ_h , the spanning angle on the plane of hologram disk. In Fig. 5, an $N_{\rm h}$ value corresponding to any specified angle of the reference source point $\theta_{\rm r}$, shows the number of the individual hologram seen by one eye of an observer at the designated observation position, shown in Fig. 2. The other eye of the observer would perceive another individual hologram with number- $N_{\rm h}$ due to symmetry. Thus, a final 3D image can be observed by the brain of the observer through the image combination from this pair of holograms belonging to the same



Fig. 4. Parameters showing 2D images captured from an original 3D object.



Fig. 5. Hologram numbers $(N_{\rm h})$, as seen by one eye of the observer situated at the designated viewing position, as a function of the reference beam angle $(\theta_{\rm r})$ for the conditions of real-image and virtual-image generation. Here, $N_{\rm h}$ is defined as a positive number in the half plane X > 0 in Fig. 2, as counted from Y-axis while it is negative in the half plane X < 0.

original object. We note that, in Fig. 5, the positive (negative) value for hologram number $N_{\rm h}$ represents the viewing condition for virtual-image (real-image) display.

Figure 5 shows our simulated results for the hologram number $N_{\rm h}$ as seen by one eye of an observer as a function of the reference beam angle $\theta_{\rm r}$. On the condition that a suitable reference beam angle is chosen, the corresponding hologram number can be obtained. The larger the angle $|\theta_{\rm r}|$, the lower $|N_{\rm h}|$ of the individual hologram. When the reference source point θ_r is set as $\theta_{\rm r} > 50^{\circ} \ (\theta_{\rm r} < -10^{\circ})$, the value of $N_{\rm h}$ stays nearly the same, which is more suitable for holographic display because the finite angular bandwidth of the reconstruction reference line source would generate a closely similar individual hologram for any eye of the observer. Hence, the reconstructed image can stay relatively clear. In contrast, if θ_r is chosen to be near 20°, one eye of the observer would perceive too many individual holograms simultaneously, thereby causing the image to be blurred. We thus can theoretically identify the suitable parameters for fabricating hologram based on our design.

According to the above mentioned model for holographic recording and reconstruction, we can combine two reconstruction conditions to form one IPDTMH experimentally. The finished hologram can generate both virtual image and real image simultaneously with only one white light point source illumination. One of the corresponding experimentally reconstructed images is shown in Fig. 6. The holographic parameters are chosen as follows: $d_{\rm fe} = 80$ cm, A = 7 cm, $\lambda = 633$ nm, $d_{\rm r} = 50$ cm, and $\theta_{\rm r} = 40^{\circ}$ for real-image display, and $d_{\rm fe} = 80$ cm,



Fig. 6. Typical experimentally reconstructed images simultaneously generated from a disk-type multiplex hologram. (a) Real image and (b) virtual image.

A=7 cm, $\lambda=633$ nm, $d_{\rm r}=42$ cm, and $\theta_{\rm r}=-25^\circ$ for virtual-image display.

In conclusion, we use a direct object-image relationship to build the model for disk-type multiplex hologram. By theoretical analysis of the holographic processes in normal viewing disk-type multiplex holography, we can calculate an optimized set of parameters for holographic recording. We demonstrate both theoretically and experimentally that fabrication of a disk-type multiplex hologram as an image-plane hologram is possible and have analyzed some of the characteristics of such a hologram. This hologram is suitable for white-light line-source reconstruction, and it is very suitable for 3D display. This kind of hologram can generate a virtual image behind the hologram. When the recording and the reconstruction reference sources are moved to the other side of the axis of the hologram disk, a real image is generated in front of the hologram disk. The information for real image is generated from the upper area of the hologram disk, whereas that for virtual image is from the lower area of the hologram disk. The reconstructed image is free from the picket-fence effect that occurs in the conventional multiplex hologram. The most important advantage of the IPDTMH is the capability of mass production using the well-developed CD technology. This technique also has the capability of generating both virtual image and real images simultaneously with only one whitelight point source illumination. However, the diffraction efficiency is an important issue that needs further study.

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