High-dimensional sensitivity analysis of complex optronic systems by experimental design: applications to the case of the design and the robustness of optical coatings

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We present the advantages of experimental design in the sensitivity analysis of optical coatings with a high number of layers by limited numbers of runs of the code. This methodology is effective in studying the uncertainties propagation, and to qualify the interactions between the layers. The results are illustrated by various types of filters and by the influence of two monitoring techniques on filter quality. The sensitivity analysis by experimental design of optical coatings is useful to assess the potential robustness of filters and give clues to study complex optronic systems.

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The study of complex optronic systems entails sensitivity analysis with a large number of parameters. Very often the response depends on synergies or interactions between these parameters. Due to interference characteristics of multilayer filters, optical coatings make possible the evaluation of methods that can explore high-dimensional space parameters and the presence of interactions between parts of these parameters. For coatings production with a high number of layers, sensitivity analysis is an efficient way to determine the most critical layers of an optical coating[1]. Refractive index errors or thickness errors during the manufacturing of these layers can induce dramatic consequences on the desired optical properties[2].

We present the advantages of using the method of experimental design[3], which is used for metamodel constructions and high-dimensional code explorations with limited numbers of runs of the code, particularly in the case of coatings with a high number of layers. This methodology is more effective in studying uncertainties propagation (refractive index or thickness values) to determine the influence of errors on the optical properties, and to quantify the interactions between the errors of each layer. The results are illustrated by various types of filters, particularly bandpass filters and multiple half-wave filters. Different designs such as factorial, fractional factorial, and space-filling designs are used to present the results.

Furthermore, we study the influence of two monitoring techniques, and show the most critical coating layers and the dependency of these layers with future manufacturing.

The results show that the study of thin-film filters is very useful in examining the interactions of high-dimensional systems due to the filter's adjustable number of layers, and the existence of interactions between these layers.

Finally, we demonstrate that sensitivity analysis of optical coatings by experimental design is useful in assessing the potential robustness of filters, and gives clues to study complex optronic systems.

The codes to study complex phenomena become more and more realistic with a larger input data set. However, due to the complexity of the mathematical system underlying the computer simulation tools, there are often no explicit input-output formulas. Although computer power has significantly increased in the past years, the evaluation of a particular setting of the design parameters may still be very time-consuming. The simulator is often replaced by a metamodel to approximate the relationship between the code and the design parameters. These metamodels are built using numerical designs of experiments that can indicate interactions between the parameters. The choice of an underlying empirical model (depending on accuracy and interactions level) can be written as

$$Y = C\text{ste} + \sum_i b_i X_i + \sum_{i<j} b_{i,j} X_i X_j$$
$$+ \sum_{i<j<k} b_{i,j,k} X_i X_j X_k + \dots, \quad (1)$$

where Y is the response of the model, X_i is the ith parameter, b_i is the effect of the ith parameter, and $b_{i,j}$, the interaction between the ith and jth parameters. This model is valid for the levels -1 and $+1$ of the undimensional variables (X_i). In our case, parameters X_i can be thickness, refractive index, or optical thickness.

In our study, we use factorial and fractional factorial designs at two levels for each parameter (low level: -1 or $-$; high level: $+1$ or $+$). The number of runs is 2^n with n parameters for a full factorial design, and 2^{n-p} for a fractional factorial design corresponding to a subset that is $1/2^p$ of the full factorial design 2^n where p is the
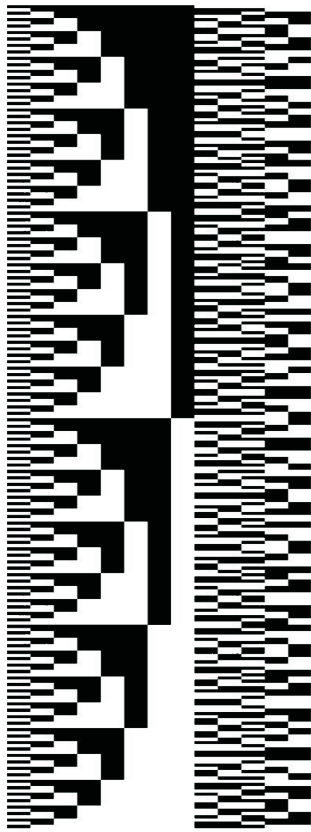
Fig. 1. The 13-layer fractional factorial design $2^{13-5} = 256$ runs. When viewed as pavements, cells with high level ($+1$) and low level (-1) are in black and white, respectively, where each line corresponds to a run and each column corresponds to a parameter. Several parameters are modified simultaneously.
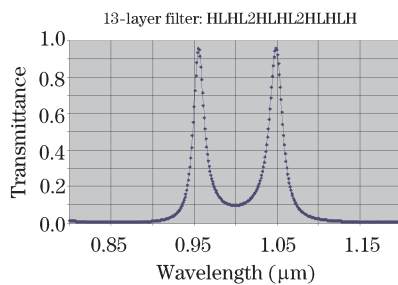

Fig. 2. Transmittance of the perfect 13-layer filter (T_{p}). The high refractive index value is 2.35 and the low refractive index value is 1.3. T_{p} is maximum at λ_1 of about 0.95 μm and λ_2 of about 1.05 μm.

degree of fractionation. Figure 1 presents the 2^{13-5} design to study the sensitivity analysis of a 13-layer filter.

In high-dimensional cases, the number of runs by a full factorial design or a fractional design is too high. There is a need to reduce the number of runs with a good quality of results. The purpose is to use designs that spread the points all over the variation domain of the variables, not only at the limits. These designs are called space-filling designs[4−6].

The study of two monitoring techniques with errors on refractive index values of each layer of the coating are realized with the 13-layer filter (Glass/HLHL2HLHL2HLHLH/air, where H and L are

quarter-wave layers at $\lambda_0 = 1$ μm of high and low refractive indices), presented on Fig. 2, and with a fractional factorial design 2^{13-5}.

We consider for the first monitoring technique (MTA) that the thickness value of each layer is the same value defined for the perfect filter. Only the refractive index values of each layer are modified. For the second monitoring technique (MTB), we consider that an optical monitoring technique is used so that the influence of the error on the refractive index value is reduced by a modified thickness of each layer, in order to obtain the optical thickness at λ_0 ($\lambda_0 = 1$ μm) of the perfectly designed filter[7,8]. The error of refractive index is assumed to be 2% for MTA and 5% for MTB . Figure 3 shows the transmittance responses of two examples of the computer experiments.

The sensitivity analysis is assessed by the following $R1$ and $R2$ (merit function) responses:

$$R1 = \sqrt{\sum_i \left(T(\lambda_i) - T_p(\lambda_i)\right)^2}, \tag{2}$$

$$R2 = \sqrt{\left(\mathrm{T}(\lambda_1) - \mathrm{T_p}(\lambda_1)\right)^2 + \left(\mathrm{T}(\lambda_2) - \mathrm{T_p}(\lambda_2)\right)^2}, \tag{3}$$

where $T(\lambda_i)$ is the transmittance of the filter at the wavelength λ_i in the case of the computer experiment (with error on refractive index values); $T_p(\lambda_i)$ is the transmittance of the perfect filter; and λ_1 and λ_2 are the two wavelengths, corresponding to the maxima of the perfect filter. The merit function $R1$ (respectively $R2$) evaluates the transmittance influence on the domain 0.8−1.2 μm (respectively at λ_1 of about 0.95 μm and λ_2 of about 1.05 μm) due to refractive index errors.

The sensitivity analysis by fractional factorial design gives the major coefficients for the response $R2$ in the case of MTA: $b_{5,9}$ (interaction of layers 5 and 9), $b_{8,9}$, $b_{4,5}$, $b_{9,10}$, and $b_{5,6}$. For the response R1, the difference of each wavelength is modified in the following order: $b_{5,9}$, $b_{8,9}$, $b_{5,6}$, $b_{5,8}$, $b_{6,9}$, $b_{6,8}$, $b_{4,5}$, and $b_{9,10}$. Using this method, we obtain the well-known result for this typical coating; the centers of the bandpass filter are given by the sub-coating L2HL. However, we also obtain the relationship between the layers with errors on refractive index values.

The same analysis for the responses $R2$ and $R1$ in MTB shows that the major coefficients are $b_{6,8}$, $b_{7,8}$, and $b_{6,7}$.
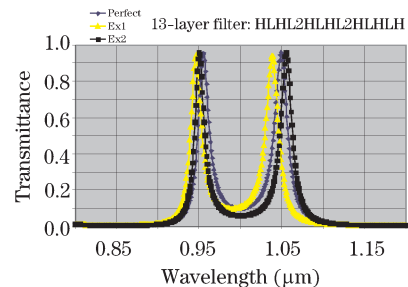

Fig. 3. Transmittance of the perfect filter and two examples, Ex1 and Ex2, of computational runs with error on refractive indices.

Table 1. Critical Layers and Interactions

| Optical Coating	Design, Number of Runs	Major Coefficients
9-layer Filter	Factorial, 512	$R1$: $b_{4,5}$, $b_{5,6}$
		$R2$: $b_{4,5}$, $b_{5,6}$
19-layer Filter	Factorial, 512	$R1$: $b_{5,6}$, $b_{4,5}$, $b_{15,16}$, $b_{14,15}$, $b_{5,15}$
		$R2$: $b_{5,15}$, $b_{4,5}$, $b_{5,6}$, $b_{15,16}$, $b_{14,15}$
29-layer Filter	Factorial, 1024	$R1$: $b_{15,16}$, $b_{14,15}$, $b_{4,5}$, $b_{5,6}$, $b_{5,25}$
		$R2$: $b_{5,15}$, $b_{15,25}$, $b_{15,16}$, $b_{14,15}$, $b_{4,5}$
39-layer Filter	Factorial, 4096	$R1$: $b_{25,26}$, $b_{24,25}$, $b_{15,25}$, $b_{15,16}$, $b_{14,15}$
		$R2$: $b_{15,25}$, $b_{5,15}$, $b_{25,35}$, $b_{24,25}$, $b_{25,26}$
39-layer Filter	SFD, 1485	$R1$: $b_{24,25}$, $b_{15,16}$, $b_{25,26}$, $b_{14,15}$, $b_{15,25}$
		$R2$: $b_{15,25}$, $b_{25,35}$, $b_{5,25}$, $b_{5,15}$
55-layer Filter	SFD, 1100	Interactions between Layers:
		6-7-8-20-21-22-34-35-36-48-49-50
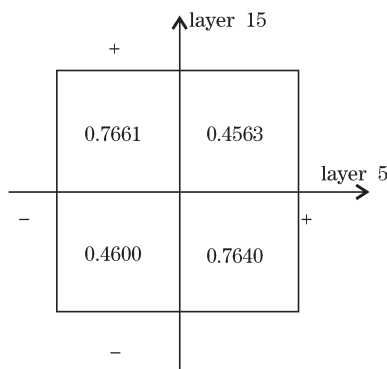
Fig. 4. In the square are given the values of the response $R2$ for each level of layers, 5 and 15 in the case of the 29-layer filter.

To obtain the best filter by this monitoring technique, it is necessary to monitor with high accuracy layers 6, 7, and 8 (LHL) and the influence of the sub-coating L2HL, which is now very low.

Using this example, the results reveal that the most critical layers and the interactions between these layers to obtain the desired optical properties can be achieved by taking into account the characteristics of the monitoring techniques.

Global sensitivity analysis by experimental designs demonstrates the most critical layers of an optical coating and the major interactions between these layers. The criticality of a layer depends on the monitoring technique. Thus, experimental designs are useful in determining the critical difficulties regarding the coating production, with the knowledge of the relationships between layers and monitoring techniques due to stochastic errors. The methodology can be used for other monitoring techniques such as level monitoring[9], optical monitoring of non-quarter-wave layers[10], and broadband optical monitoring[11].

In the sensitivity analysis of optical coatings, the basic filter used is a single Fabry-Pérot filter HLHL4HLHLH with the monitoring technique MTA designed as substrate/HLHL4HLHLH/air.

The high refractive index value is assumed to be 2.35 and the low refractive index value is assumed to be 1.3. The error of refractive index is assumed to be 2.5% for the first monitoring technique and 5% for the second.

The 9-layer filter is studied with a 2-level factorial design ($2^9 = 512$ runs), the 19-layer filter with a two-level fractional factorial design ($2^{19-10} = 512$ runs), and the 29-layer filter with a two-level fractional factorial design ($2^{29-19} = 1024$ runs). The sensitivity analysis of the 39-layer filter is studied with a two-level fractional factorial design ($2^{39-27} = 4096$ runs) and with a space-filling design (SFD), which needs only 1485 runs.

We complete the study by a 55-layer optical coating (four times 13-layer filter) with the MTB.

In Table 1, we present the major coefficients of responses $R1$ and $R2$. These show the most critical interactions between layers and

$$R2 = \sqrt{\left(T(\lambda_0) - T_p(\lambda_0)\right)^2}. \qquad (4)$$

As the number of layer increases, the number of interactions between the two layers becomes more crucial and multiplied. All blocks of three layers (L4HL) interact with the others, and we can observe that the coefficient ranks change when the number of layer is higher; the strongest interactions are always from the filter before last.

The interactions between layers can be very strong. For example, Fig. 4 presents the evolution of the response $R2$ for each level ($-$ and $+$) of layers 5 and 15 in the case of the 29-layer filter; the value modification of $R2$ is around 30%. This interaction graph reveals that the behavior of a layer differs according to the level of the other layer. Moreover, it shows that the response is the same for two different combinations of layers.

The result of MTB with a 55-layer filter extends the previous 13-layer filter result (major influence of the block LHL) in each layer, and reveals the interactions between the layers of different blocks. This is the main perspective of the manufacturing of such filters; it is necessary to monitor each layer during its production, as well as the interactions with the previously deposited layers and the associated errors.

We can observe that the main layers are identified

with an SFD. This result is very interesting because we can obtain good results with a very low number of runs; only 1100 runs are needed to explore a 55-dimension space. The methodology of experimental design adapted to a computer experiment allows us to explore high-dimensional sensitivity analysis of optical coatings, and identify the most critical interactions between layers.

The computational experiments of refractive index errors in optical coatings by experimental design allow us to identify the most critical interactions between layers. The increase of the number of layers in an optical coating is a useful approach to determine the relationship between the parameters in high-dimensional systems. The influence of errors on refractive index values and thickness values should be the next step in the exploration of such high-dimensional spaces.

In conclusion, we proposed the use of experimental design to explore monitoring techniques and determine the most critical layers and interactions in optical coatings. This methodology reveals that the criticality of a set of layers depends on the monitoring technique. In the case of high number of layers, the most critical interactions are determined with a good quality and a low number of computer runs. The method employed is effective in efficiently assessing the potential robustness of filters by taking account of the monitoring strategy. Sensitivity analysis by experimental design of thin-film coatings is very useful to examine high-dimensional systems with interactions resulting from the adjustable number of layers of a filter, as well as the existence of interactions between these layers. Sensitivity analysis of optical coatings is the best way to determine the most effective space-filling designs to explore complex optronic systems.

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