Construction of optimal 2D variable-weight optical orthogonal codes for high-speed OCDMA networks

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A new construction method of two-dimensional (2D) variable-weight optical orthogonal codes (VWOOCs) is proposed for high-speed optical code-division multiple-access (OCDMA) networks supporting multiple qualities of services (QoS). The proposed codes have at most one-pulse per wavelength (AM-OPPW) property. An upper bound of the codeword cardinality of the 2D VWOOCs with AM-OPPW property is derived. It is then shown that the constructed codes have ideal correlation properties and optimal cardinality. Moreover, the code length and the bit-error-rate (BER) performance of the proposed codes are compared with those of the codes proposed previously.

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Optical code-division multiple-access (OCDMA) technique has gained significant attention in optical communication networks due to its potential for simplicity in all-optical implementation, inherent security against interception, and asynchronous access[1,2]. The user address code with better performance, such as optical orthogonal code (OOC)[1,3,4], is the basis for the implementation of an OCDMA network.

Recently, there are more requirements on the high-speed OCDMA system supporting multiple qualities of services (QoS). One-dimensional (1D) variable- weight (VW) OOCs were proposed by Yang[5] for this purpose. Due to the unipolar characteristic of optical signals, 1D optical codes[5−7] are always very long so that they reduce multiple-access interference (MAI) and increase the codeword cardinality. To overcome this drawback, much attention has been paid to two-dimensional (2D) VWOOCs recently[8−11]. The 2D optical encoder/decoder can be realized by fiber Bragg gratings (FBGs)[12,13]. However, due to the use of 1D VWOOCs as time spreading patterns in the 2D VWOOCs[8−11], the code length will also be large and thus the data rate will decrease. Therefore, codes with shorter lengths than that of previous codes[8−11] need to be constructed to meet the requirements of high-speed multimedia transmissions.

To simplify practical implementation and provide scalability to OCDMA networks[3,11], 2D VWOOCs with at most one-pulse per wavelength (AM-OPPW)[3] property were proposed recently by Piao *et al.*[11] based on the combinatorial method and computer searching. The codes have ideal correlation properties, i.e., maximum out-of-phase autocorrelation equal to 0 and cross-correlation equal to 1. An upper bound of the codeword cardinality was also derived[11]. However, the set of codeword cardinality distributions was not considered[11].

In this letter, a new construction method of 2D VWOOCs with AM-OPPW property is proposed for high-speed OCDMA networks. An upper bound of the codeword cardinality is derived to show the optimality of the presented construction. It is also shown that the constructed codes have ideal correlation properties. Moreover, the code length and the bit-error-rate (BER) performance of the proposed codes are compared with those of the codes proposed previously.

Throughout this letter, let W, Λ, and Q denote the sets $\{w_1, w_2, \cdots, w_l\}$, $\{\lambda_a^1, \lambda_a^2, \cdots, \lambda_a^l\}$, and $\{q_1, q_2, \cdots, q_l\}$, respectively. Without loss of generality, we assume that $w_1 \le w_2 \le \cdots \le w_l$. Let $Z_n = \{0, 1, \cdots, n-1\}$ denote the group of residues modulo n. $|A|$ indicates the cardinality of a set A.

A 2D $(u \times v, W, \Lambda, \lambda_c, Q)$-VWOOC, C, is a collection of binary (0,1) $u \times v$ code matrices (codewords) such that the following three properties hold.

1) Weight distribution: each codeword in C has a Hamming weight contained in the set W; furthermore, there are exactly $q_k \cdot |C|$ codewords of weight w_k, i.e., q_k indicates the fraction of codewords of weight w_k.

2) Auto-correlation: for any $X = (x_{i,j}) \in C$ with Hamming weight $w_k \in W$ and any integer τ, $0 < \tau \le v-1$,

$$R_{X,X}(\tau) = \sum_{i=0}^{u-1}\sum_{j=0}^{v-1} x_{i,j}x_{i,j+\tau} \le \lambda_a^k. \quad (1)$$

3) Cross-correlation: for any $X = (x_{i,j})$, $Y = (y_{i,j}) \in C$ such that $X \ne Y$ and any integer τ,

$$R_{X,Y}(\tau) = \sum_{i=0}^{u-1}\sum_{j=0}^{v-1} x_{i,j}y_{i,j+\tau} \le \lambda_c. \quad (2)$$

All subscripts here are reduced modulo v so that periodic correlations are considered. In the following, the notation $(u \times v, W, 0, 1, Q)$-VWOOC is used to denote an $(u \times v, W, \Lambda, \lambda_c, Q)$-VWOOC with ideal correlation properties, i.e., $\lambda_a^1 = \lambda_a^2 = \cdots = \lambda_a^l = 0$ and $\lambda_c = 1$.

Let X be a code matrix of a $(u \times v, W, \Lambda, \lambda_c, Q)$-VWOOC with weight w and some integers $r_i \in Z_u$, $c_i \in Z_v$ such that $x_{r_0,c_0} = x_{r_1,c_1} = \cdots = x_{r_{w-1},c_{w-1}} = 1$, where $0 \le r_0 \le r_1 \le \cdots \le r_{w-1} \le u-1$. The fact that $x_{r_i,c_i} = 1$ means an optical pulse of wavelength r_i at the time chip c_i. The set $B_X = \{(r_0, c_0), (r_1, c_1), \cdots, (r_{w-1}, c_{w-1})\}$ associated with X is called the position block of X.

A cyclic $(n, W, 1, Q)$ difference family (DF)[7] is a collection of subsets (called blocks) of Z_n, $A = \{A_0, A_1, \cdots, A_{t-1}\}$, where the block size $|A_i| \in W$, $i = 0, 1, \cdots, t-1$, such that the multiset union $\bigcup_{i=0}^{t-1} \{x - y | x, y \in A_i, x \neq y\} = Z_n \backslash \{0\}$.

The $(n, W, 1, Q)$-DF is used here to construct 2D VWOOC. The construction steps are described as follows.

1) Wavelength hopping patterns. Let $\mathbf{A} = \{A_h = \{a_{h,0}, a_{h,1}, \cdots, a_{h,w_k-1}\}, h = 0, 1, \cdots, s-1\}$ be an $(n, W, 1, Q)$-DF over Z_n with cardinality s, s_k blocks of size w_k, and $q_k = s_k/s$. The wavelength hopping patterns are $A_h^r = \{a_{h,0} + r, a_{h,1} + r, \cdots, a_{h,w_k-1} + r\}$, $h = 0, 1, \cdots, s-1$, $r = 0, 1, \cdots, n-1$, where "+" is modulo-n addition.

2) Time spreading patterns. Take a prime $p \geq w_l$ and let the time spreading patterns be $T_j = \{j \cdot 0, j \cdot 1, \cdots, j \cdot (w_k - 1)\}$, $j = 0, 1, \cdots, p-1$, where "$\cdot$" is modulo-p multiplication.

3) 2D VWOOC codewords. The position block of an $n \times p$ codeword based on A_h^r and T_j is $B_h^{r,j} = \{(a_{h,0} + r, j \cdot 0), (a_{h,1} + r, j \cdot 1), \cdots, (a_{h,w_k-1} + r, j \cdot (w_k - 1))\}$. The set of position blocks of the 2D VWOOC is $F = \bigcup_{h=0}^{s-1} \bigcup_{r=0}^{n-1} \bigcup_{j=0}^{p-1} B_h^{r,j}$.

In the following, an upper bound of the codeword cardinality of a $(u \times v, W, 0, 1, Q)$-VWOOC is derived to show the optimality of the presented construction. Let F be the set of position blocks of a $(u \times v, W, 0, 1, Q)$-VWOOC. For any $B_X \in F$, the list of differences from B_X is defined as

$$\Delta B_X = \{(c_i - c_j)_{r_i, r_j} | (r_i, c_i), (r_j, c_j) \in B_X, i \neq j\}, \quad (3)$$

where the subtraction "$-$" is performed in Z_v. It is obvious that ΔB_X contains exactly $w(w-1)$ distinct elements, i.e.,

$$|\Delta B_X| = w(w-1). \quad (4)$$

Further, let $\Delta F = \bigcup_{X \in F} \Delta B_X$ be the differences from F.

Example 1 A codeword X with $u = 4$, $v = 3$, and weight 3 is shown in Fig. 1, where $B_X = \{(0,0), (1,1), (3,1)\}$ and $\Delta B_X = \{1_{1,0}, 2_{0,1}, 1_{3,0}, 2_{0,3}, 0_{3,1}, 0_{1,3}\}$.

The following result provides an equivalent condition to keep the maximum cross-correlation value between any two codewords at most one.

Lemma 1 Let X and Y be two distinct $u \times v$ codewords. Then, the cross-correlation property in expression (2) will hold, i.e., $R_{X,Y}(\tau) \leq 1$ for all $0 \leq \tau \leq v-1$, if and only if ΔB_X and ΔB_Y are disjoint.

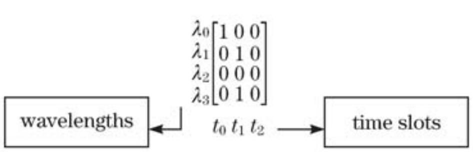

Fig. 1. Illustration of a codeword with u=4, v=3, and weight 3.

Proof Let $B_X = \{(r_0, c_0), (r_1, c_1), \cdots, (r_{w_k-1}, c_{w_k-1})\}$ and $B_Y = \{(s_0, t_0), (s_1, t_1), \cdots, (s_{w_l-1}, t_{w_l-1})\}$ be the position blocks of any two distinct codewords X and Y, respectively. First, we prove that if $\Delta B_X \cap \Delta B_Y = \phi$ (empty set), then $R_{X,Y}(\tau) \leq 1$ for all $0 \leq \tau \leq v-1$. If there are at least two coincidences of nonzero elements in any relative cyclic time shift τ of these two codewords, then $(r_i, c_i + \tau) = (s_{i'}, t_{i'})$ and $(r_j, c_j + \tau) = (s_{j'}, t_{j'})$ must hold simultaneously, where $i \neq j$, $i' \neq j'$, $i, j \in \{0, 1, \cdots, w_k - 1\}$, and $i', j' \in \{0, 1, \cdots, w_l - 1\}$. However, these conditions cannot be both true at the same time because they require that $(c_i - c_j)_{r_i, r_j} = (t_{i'} - t_{j'})_{s_{i'}, s_{j'}}$, where $(c_i - c_j)_{r_i, r_j} \in \Delta B_X$ and $(t_{i'} - t_{j'})_{s_{i'}, s_{j'}} \in \Delta B_Y$. This violates the condition $\Delta B_X \cap \Delta B_Y = \phi$.

Next, we show that if $R_{X,Y}(\tau) \leq 1$ for all $0 \leq \tau \leq v-1$, then $\Delta B_X \cap \Delta B_Y = \phi$. Suppose that $\Delta B_X \cap \Delta B_Y \neq \phi$, then there exist two pairs of elements $(r_i, c_i), (r_j, c_j) \in B_X$ and $(s_{i'}, t_{i'}), (s_{j'}, t_{j'}) \in B_Y$, respectively, such that $(c_i - c_j)_{r_i, r_j} = (t_{i'} - t_{j'})_{s_{i'}, s_{j'}}$, where $(c_i - c_j)_{r_i, r_j} \in \Delta B_X$ and $(t_{i'} - t_{j'})_{s_{i'}, s_{j'}} \in \Delta B_Y$. The equality $(c_i - c_j)_{r_i, r_j} = (t_{i'} - t_{j'})_{s_{i'}, s_{j'}}$ implies $r_i = s_{i'}$, $r_j = s_{j'}$, and $t_{i'} - c_i = t_{j'} - c_j$. If we let $\tau = t_{i'} - c_i = t_{j'} - c_j$, then it is obtained that $(r_i, c_i + \tau) = (s_{i'}, t_{i'})$ and $(r_j, c_j + \tau) = (s_{j'}, t_{j'})$. In other words, there are two coincidences of nonzero elements in a cyclic time shift τ of these two codewords. This contradicts with the condition $R_{X,Y}(\tau) \leq 1$ for all $0 \leq \tau \leq v-1$. The proof is completed.

Let $\Phi(u \times v, W, 0, 1, Q) = \max\{|C| : C$ is a $(u \times v, W, 0, 1, Q)$-VWOOC$\}$ and let $[x]$ denote the integer part of x. An upper bound of $\Phi(u \times v, W, 0, 1, Q)$ is obtained.

Lemma 2 $\Phi(u \times v, W, 0, 1, Q) \leq \left[\frac{uv(u-1)}{\sum_{k=1}^{l} q_k w_k (w_k - 1)}\right]$.

Proof Suppose F is the corresponding set of position blocks of the code C. For $d_{\alpha,\beta} \in \Delta F$, since d can be taken from Z_v and $\alpha, \beta \in Z_u \times Z_u$, $\alpha \neq \beta$, the total number of distinct differences $d_{\alpha,\beta}$ in ΔF is at most $v \cdot u(u-1)$. From Eq. (4) and Lemma 1, we have $\sum_{k=1}^{l} q_k \Phi w_k (w_k - 1) \leq uv(u-1)$, i.e.,

$$\Phi(u \times v, W, 0, 1, Q) \leq \frac{uv(u-1)}{\sum_{k=1}^{l} q_k w_k (w_k - 1)}. \quad (5)$$

A $(u \times v, W, 0, 1, Q)$-VWOOC is optimal if $\Phi(u \times v, W, 0, 1, Q)$ meets the upper bound in Lemma 2.

From the presented construction, we can see that the number of the codewords with weight w_k in the proposed code is $s_k np$, the codeword cardinality is snp, and the fraction of the codewords with weight w_k is $q_k = s_k/s$. Since there is at most one pulse per wavelength in each codeword, it is obvious that each codeword with weight w_k has the maximum out-of-phase auto-correlation $\lambda_a^k = 0, k = 1, 2, \cdots, l$. Further, we have the following result.

Lemma 3 The proposed code is an optimal $(n \times p, W, 0, 1, Q)$-VWOOC.

Proof From the above discussions, we only need to prove that $\lambda_c = 1$. From Lemma 1, it is sufficient to show that any $d_{\alpha,\beta}$, where $d \in Z_p$, $\alpha, \beta \in Z_n \times Z_n$, and $\alpha \neq \beta$, occurs at most once in ΔF. Since $\mathbf{A}$ is a DF and p is a prime, there exists a block $B_h^{r,j} \in F$ such that $\alpha = a_{h,t_1} + r, \beta = a_{h,t_2} + r$, and $d = ja_{h,t_1} - ja_{h,t_2}$. In

Table 1. Codewords of a (9×3,{2,3},0,1,{1/2,1/2})-VWOOC Based on A_0=[0,4], A_1=[0,1,3], and r=0, j=0,1,2

$B_0^{0,0}$ ={(0, 0), (4, 0)}	$B_1^{0,0}$ ={(0, 0), (1, 0), (3, 0)}
$B_0^{0,1}$ ={(0, 0), (4, 1)}	$B_1^{0,1}$ ={(0, 0), (1, 1), (3, 2)}
$B_0^{0,2}$ ={(0, 0), (4, 2)}	$B_1^{0,2}$ ={(0, 0), (1, 2), (3, 1)}

Table 2. Comparison of the Code Length Between Proposed Codes and Known Codes

W	Q	Minimum Length of the Proposed Codes	Minimum Length of Liang's Codes
{3,4}	{1/2,1/2}	5	19
{3,4}	{1/3,2/3}	5	31
{3,6}	{1/2,1/2}	7	37
{3,6}	{1/3,2/3}	7	67
{4,6,7}	{1/3,1/3,1/3}	7	85
{4,6,7}	{1/4,1/4,2/4}	7	127

addition, it is not difficult to see that the total number of differences in ΔF is at most $n(n-1)p$. Therefore, each difference occurs exactly once in ΔF. This completes the proof.

Example 2 Let $p = 3$ and $\mathbf{A} = \{A_0, A_1\}$ be a $(9, \{2, 3\}, 1, \{1/2, 1/2\})$-DF, where $A_0 = \{0, 4\}, A_1 = \{0, 1, 3\}$. The partial position blocks of a $(9 \times 3, \{2, 3\}, 0, 1, \{1/2, 1/2\})$-VWOOC from A_0, A_1 with $r = 0, j = 0, 1, 2$ are shown in Table 1. This code has two Hamming weights $w_1 = 2$, $w_2 = 3$, and the codeword cardinality is 54.

Now, we compare the code length of the proposed codes with that of the previous codes. We first recall some notations introduced by Wu *et al*[7]. For each $q_k \in Q$, write $q_k = b_k/a_k$, where a_k, b_k are integers and gcd $(a_k, b_k) = 1$, $1 \le k \le l$. Let $f(Q)$ be the least common multiple of $a_1, a_2, \cdots, a_l$, and $q_k = f_k(Q)/f(Q)$. Let $w = \sum_{k=1}^{l} f_k(Q)w_k(w_k-1)$. For an $(n, W, 1, Q)$-VWOOC, in order to make the codewords set nonempty, the code length n is at least $w+1$, i.e., $n \ge w+1$.

Without loss of generality, we choose the codes presented by Liang *et al.*[8] for comparison. The numerical results are shown in Table 2, where the minimum code lengths for the different sets W and Q are listed. On the whole, the proposed codes have shorter code lengths than Liang's codes. With the code weight in W increasing, the length of Liang's codes increases rapidly while the length of the proposed codes increases slowly. Therefore, the proposed codes are more suitable for high-speed OCDMA networks because of their shorter code length.

The performance of the OCDMA system depends on the code weights of the local address matrix and the arriving address matrix. We now analyze the performance of an OCDMA system using the proposed $(n \times p, W, 0, 1, Q)$-VWOOC. Let q_k, $k = 1, 2, \cdots, l$, denote the probability of getting one hit between a local address matrix and any arriving address matrix with the same code weight w_k. Then

$$q_k = \frac{(p-1)w_k + pw_k(w_k-1) + pw_k^2(s_k-1)}{2p(nps_k-1)}, \quad (6)$$

where $(p-1)w_k$ represents the number of hits which come from the codewords with the same h and r but a different j, $pw_k(w_k-1)$ is the number of hits which come from the codewords with the same h but different r and j, $pw_k^2(s_k-1)$ denotes the number of hits which originate from the codewords with same weight w_k but different h. The factor 1/2 comes from the assumption of equiprobable on-off data bit transmission, the factor p is the code length, and the factor $nps_k - 1$ is the number of the address matrices with the same weight w_k except the local address matrix. Briefly,

$$q_k = \frac{pw_k^2 s_k - w_k}{2p(nps_k-1)}. \quad (7)$$

Since the number of hits in every time slot between two address matrices is no greater than one, the chip collision probability between a local address matrix with weight w_k and an arriving address matrix with weight $w_{k'}$ is given by

$$q_{k,k'} = \frac{pw_k w_{k'} s_{k'}}{2p \cdot nps_{k'}} = \frac{w_k w_{k'}}{2np}. \quad (8)$$

Suppose there are N_k, $k = 1, 2, \cdots, l$, active users with weight w_k that coexist in the system. The error probability $P_{e,w_k}, k = 1, 2, \cdots, l$, of the user with address matrices of weight w_k is given by[9]

$$P_{e,w_k} = \frac{1}{2} - \frac{1}{2} \sum_{e_1+\cdots+e_l=0}^{w_k-1} \binom{N_k}{e_k} (q_k)^{e_k} \cdot (1-q_k)^{N_k-1-e_k} \cdot \prod_{k'=1,k'\neq k}^{l} \binom{N_{k'}}{e_{k'}} (q_{k,k'})^{e_{k'}} \cdot (1-q_{k,k'})^{N_{k'}-e_{k'}}. \quad (9)$$

For a fair comparison, we assume that the codewords are with approximately the same code size[14], which is defined as the product of the number of wavelengths and the number of time slots. Figure 2 shows the BER performance versus the number of active users of the (16×73, {3, 6}, {1, 1}, 1, {1/2, 1/2})-VWOOC[8] and the proposed (73×17, {3, 6}, 0, 1, {1/2, 1/2})-VWOOC. Here, we consider different types of active users. Figure 2 shows that the BER performance decreases as the total number of active users increases and the users with larger-weight codewords perform better than those with smaller-weight codewords. In addition, we observe that the BER performances of the two codes are nearly identical because both of the codes have the same cross-correlation constraint and approximately the same code size.

Figure 3 shows the BER performance versus the number of active users of the (16×73, {3,4,6}, {1,1,2}, 1, {1/3, 1/3, 1/3})-VWOOC[10] and the proposed (97×13, {3,4,6}, 0, 1, {1/3, 1/3, 1/3})-VWOOC. Numerical results show that the proposed codes have better BER performance. However, compared with the proposed codes, the number of the wavelengths of the codes[10] is smaller and thus the channel-spacing may be larger. Therefore,

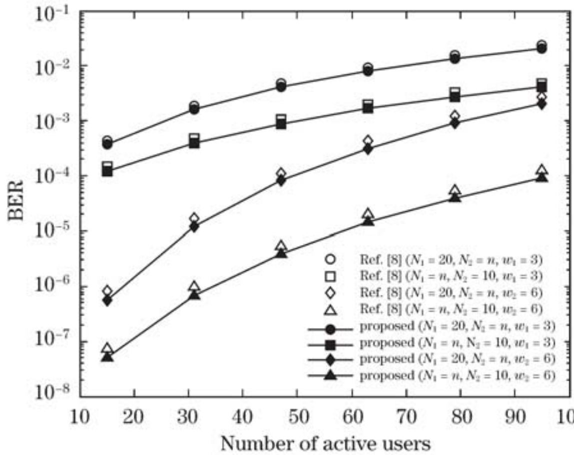

Fig. 2. BER performance versus the number of active users of the (16×73, {3,6}, {1, 1}, 1, {1/2,1/2})-VWOOC[8] and the proposed (73×17, {3,6}, 0, 1, {1/2,1/2})-VWOOC.

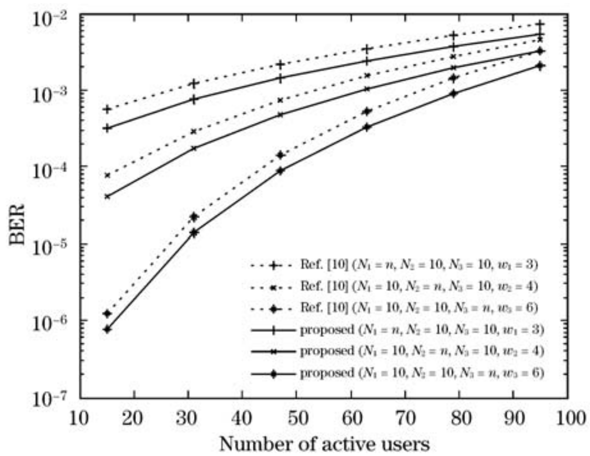

Fig. 3. BER performance versus the number of active users of the (16×73, {3,4,6}, {1,1,2}, 1, {1/3, 1/3, 1/3})-VWOOC[10] and the proposed (97×13, {3,4,6}, 0, 1, {1/3, 1/3, 1/3})-VWOOC.

the adverse impact of the wavelength drift[13] in the system may be smaller. In addition, the proposed codes exist only for prime length.

In conclusion, a new construction method of optimal 2D VWOOCs with AM-OPPW property has been proposed based on DFs. An OCDMA system employing the proposed codes can support multiple QoS requirements. It has been shown that the proposed codes have similar or better BER performance compared with the codes proposed previously. In particular, since the constructed codes have shorter code length, it is well suitable for the high-speed OCDMA networks.

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