Simple equivalent systems for GRIN lenses in inhomogeneous medium

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Simple single-lens equivalent systems for graded-index (GRIN) lenses in inhomogeneous medium obtained using matrix optics are proposed in this letter. Due to its simplicity, the equivalent optical system enables quick analysis of the imaging properties of GRIN lens rod immersed in inhomogeneous medium. This facilitates the optical analysis of complicated optoelectronics systems in inhomogeneous medium utilizing GRIN lens rods.

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Graded-index (GRIN) lens is a special class of optical lens in which the refractive index gradually varies along the radial axis[1−4]. Variations in the refractive index cause bending of light rays from an object, forming images like an ordinary glass lens. However, unlike spherical lens[5], GRIN lens features a planar end surface apart from complex curvature surface. This is one of its major advantages over conventional lenses. Other advantages, including small size and uncomplicated fabrication method, have led to the interest in GRIN lens as a micro-optics component to integrate optoelectronic systems. Consequently, GRIN lens has been extensively utilized in many applications, such as fiber coupling[6], laser diode beam shaping[7], medical endoscope application[8], and optical coherent tomography[9]. GRIN lens has been studied by employing the effective method of ray tracing[10], but this does not facilitate the development of a simple equivalent optical system. A simple optical system equivalent to GRIN lens allows for a quick analysis of its optical properties. In this letter, we use the matrix optics method to obtain a simple single-lens equivalent system for GRIN lens immersed in inhomogeneous medium. Based on this, imaging properties are thoroughly investigated.

When γ and $-\gamma$ type matrices (Eqs. (1) and (2)) are present in front of the matrix denoting optical distance, either the γ or $-\gamma$ type matrix quasi-commutes with optical distance and acts directly on the ray vector of the object.

$$\begin{bmatrix} 1 & 0 \\ 0 & \gamma \end{bmatrix} \begin{bmatrix} 1 & L_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_0 \\ \theta_0 \end{bmatrix} = \begin{bmatrix} 1 & L_1/\gamma \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_0 \\ \gamma\theta_0 \end{bmatrix}, \quad (1)$$

$$\begin{bmatrix} -1 & 0 \\ 0 & -\gamma \end{bmatrix} \begin{bmatrix} 1 & L_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_0 \\ \theta_0 \end{bmatrix} = \begin{bmatrix} 1 & L_1/\gamma \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -y_0 \\ -\gamma\theta_0 \end{bmatrix}, \quad (2)$$

where y_0 is the height of the object, Q_0 is the angle between the incident ray and the optical axis, L_1 stands for the distance between object and the lens, and γ is an arbitrary positive number.

Even if these result in the alteration of (1) incident angle of the ray from objects, (2) optical distance between the object and the imaging system, and (3) the reversion of the object for $-\gamma$ type matrix, they do not affect the imaging properties of the first-order optical system (Fig. 1).

For paraxial approximation, we use the transfer matrix of GRIN lens immersed in inhomogeneous medium,

$$\begin{bmatrix} 1 & 0 \\ 0 & \frac{n_{.0}}{n_2} \end{bmatrix} \begin{bmatrix} \cos(\alpha d) & \frac{\sin(\alpha d)}{\alpha} \\ -\alpha\sin(\alpha d) & \cos(\alpha d) \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_{.1}}{n_0} \end{bmatrix} = \begin{bmatrix} \cos(\alpha d) & \frac{n_1\sin(\alpha d)}{n_0\alpha} \\ \frac{-n_0\alpha\sin(\alpha d)}{n_2} & \frac{n_1\cos(\alpha d)}{n_2} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}, \quad (3)$$

where n_0 is the dielectric constant of GRIN lens along the z axis, n_1 is the refractive index of the material on the left side of GRIN lens, n_2 is the refractive index on the right side of GRIN lens, α is the variation of the refractive index of the GRIN lens along x and y axes, and d is the length of the GRIN lens. In inhomogeneous medium, we find that the two matrix decompositions below led to single-lens equivalent systems for GRIN lens.

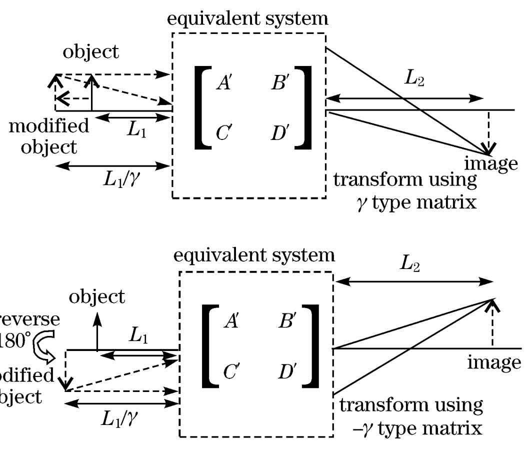

Fig. 1. Transformation of $\pm\gamma$ types of matrix.

$$\begin{bmatrix}1 & L_2\\ 0 & 1\end{bmatrix}\begin{bmatrix}A & B\\ C & D\end{bmatrix}\begin{bmatrix}1 & L_1\\ 0 & 1\end{bmatrix}\begin{bmatrix}y_0\\ \theta_0\end{bmatrix}=\begin{bmatrix}1 & L_2\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & \frac{A-1}{C}\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ C & 1\end{bmatrix}\begin{bmatrix}1 & \frac{D-\lambda^2}{C\lambda^2}\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & \lambda^2\end{bmatrix}\begin{bmatrix}1 & L_1\\ 0 & 1\end{bmatrix}\begin{bmatrix}y_0\\ \theta_0\end{bmatrix}$$

$$=\begin{bmatrix}1 & L_2\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & \frac{A-1}{C}\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ C & 1\end{bmatrix}\begin{bmatrix}1 & \frac{D-\lambda^2}{C\lambda^2}\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & L_1/\lambda^2\\ 0 & 1\end{bmatrix}\begin{bmatrix}y'_0\\ \theta'_0\end{bmatrix}, \quad (4)$$

where $y'_0 = y_0$, $\theta'_0 = \lambda^2\theta_0$.

$$\begin{bmatrix}1 & L_2\\ 0 & 1\end{bmatrix}\begin{bmatrix}A & B\\ C & D\end{bmatrix}\begin{bmatrix}1 & L_1\\ 0 & 1\end{bmatrix}\begin{bmatrix}y_0\\ \theta_0\end{bmatrix}=\begin{bmatrix}1 & L_2\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & \frac{A+1}{C}\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ -C & 1\end{bmatrix}\begin{bmatrix}1 & \frac{D+\lambda^2}{C\lambda^2}\\ 0 & 1\end{bmatrix}\begin{bmatrix}-1 & 0\\ 0 & -\lambda^2\end{bmatrix}\begin{bmatrix}1 & L_1\\ 0 & 1\end{bmatrix}\begin{bmatrix}y_0\\ \theta_0\end{bmatrix}$$

$$=\begin{bmatrix}1 & L_2\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & \frac{A+1}{C}\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ -C & 1\end{bmatrix}\begin{bmatrix}1 & \frac{D+\lambda^2}{C\lambda^2}\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & L_1/\lambda^2\\ 0 & 1\end{bmatrix}\begin{bmatrix}y'_0\\ \theta'_0\end{bmatrix}, \quad (5)$$

where $y'_0 = -y_0$, $\theta'_0 = -\lambda^2\theta_0$.

If $0 < \alpha d < \pi$, then $C < 0$, $-1 < A < 1$, and $-\lambda^2 < D < \lambda^2$. Thus, the term $(A-1)/C$ and $(D-\lambda^2)/C\lambda^2$ in Eq. (4) is always positive. Matrices $\begin{bmatrix}1 & (A-1)/C\\ 0 & 1\end{bmatrix}$ and $\begin{bmatrix}1 & (D-\lambda^2)/C\lambda^2\\ 0 & 1\end{bmatrix}$ represent two optical spacings. Matrix $\begin{bmatrix}1 & 0\\ C & 1\end{bmatrix}$ denotes a single lens. Matrix $\begin{bmatrix}1 & 0\\ 0 & \lambda^2\end{bmatrix}$ could be deduced through factorization and act on the ray of the object according to the previous theorem. GRIN lens at this length could be reduced into a single-lens equivalent system as (see also Fig. 2)

$$\begin{bmatrix}\cos(\alpha d) & \frac{n_1\sin(\alpha d)}{n_0\alpha}\\ \frac{-n_0\alpha\sin(\alpha d)}{n_2} & \frac{n_1\cos(\alpha d)}{n_2}\end{bmatrix}\begin{bmatrix}1 & L_1\\ 0 & 1\end{bmatrix}\begin{bmatrix}y_0\\ \theta_0\end{bmatrix}=\begin{bmatrix}1 & \frac{n_2\tan(\alpha d/2)}{n_0\alpha}\\ 0 & 1\end{bmatrix}$$

$$\times\begin{bmatrix}1 & 0\\ \frac{-n_0\alpha\sin(\alpha d)}{n_2} & 1\end{bmatrix}\begin{bmatrix}1 & \frac{n_2\tan(\alpha d/2)}{n_0\alpha}\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & \frac{n_2}{n_1}L_1\\ 0 & 1\end{bmatrix}\begin{bmatrix}y_0\\ \frac{n_1}{n_2}\theta_0\end{bmatrix}. \quad (6)$$

For GRIN lens with length $\pi < \alpha d < 2\pi$, Eq. (5) is utilized to obtain its single-lens equivalent system (see also Fig. 2).

$$\begin{bmatrix}\cos(\alpha d) & \frac{n_1\sin(\alpha d)}{n_0\alpha}\\ \frac{-n_0\alpha\sin(\alpha d)}{n_2} & \frac{n_1\cos(\alpha d)}{n_2}\end{bmatrix}\begin{bmatrix}1 & L_1\\ 0 & 1\end{bmatrix}\begin{bmatrix}y_0\\ \theta_0\end{bmatrix}=\begin{bmatrix}1 & -\frac{n_2}{n_0\alpha\tan(\frac{\alpha d}{2})}\\ 0 & 1\end{bmatrix}$$

$$\times\begin{bmatrix}1 & 0\\ \frac{n_0\alpha\sin(\alpha d)}{n_2} & 1\end{bmatrix}\begin{bmatrix}1 & -\frac{n_2}{n_0\alpha\tan(\frac{\alpha d}{2})}\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & \frac{n_2}{n_1}L_1\\ 0 & 1\end{bmatrix}\begin{bmatrix}-y_0\\ -\frac{n_1}{n_2}\theta_0\end{bmatrix}. \quad (7)$$

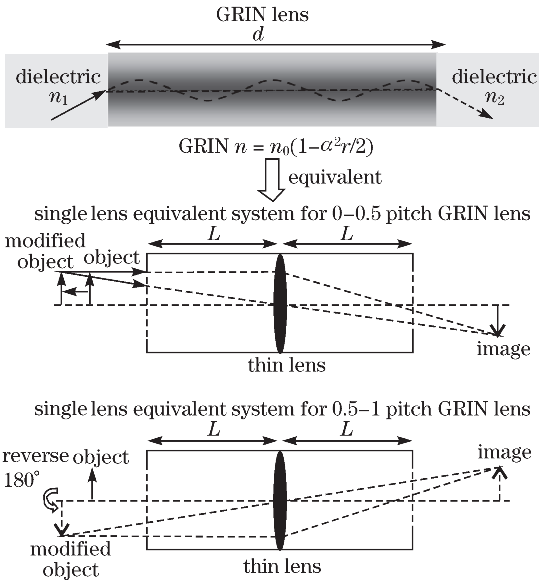

Fig. 2. Equivalent optical system for GRIN rod lens in inhomogeneous medium.

The object is reversed in the process.

Under this condition, GRIN lens is reduced to a single-lens equivalent system. It consists of one positive thin lens at $f = n_2/[n_0\alpha\sin(\alpha d)]$ and two identical optical spacings at $L = n_2\tan(\alpha d/2)/(n_0\alpha)$ (Eq. (6) and Fig. 3). The focal point is located outside the equivalent system at a spacing of $s = n_2/[n_0\alpha\tan(\alpha d)]$. In this system, the object is relocated to $d_1 = n_2L_1/n_1$ and away from the left edge.

We only study the optical properties outside the GRIN lens. Sign convections are defined as follows: 1) The distance from the object to the left of the equivalent system, d_1, is positive; to the right, it is negative. 2) The distance from the image to the right of the equivalent system, d_2, is positive; to the left, it is negative. We use the following object-image equation and magnification:

$$(n_2L_1/n_1 - s)(d_2 - s) = f^2,\ \beta = f/(s - n_2L_1n_1). \quad (8)$$

Full characterization of the object-image relation of GRIN lenses is obtained utilizing this single-lens equivalent system. Evidently, any image inside the equivalent

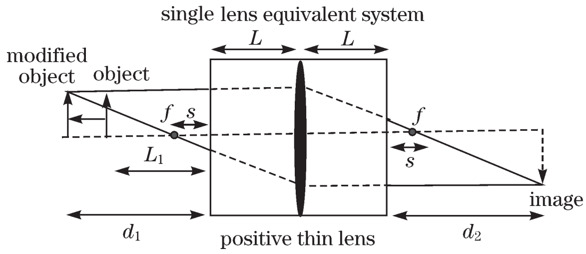

Fig. 3. Equivalent optical system for 0−0.25-pitch GRIN lens in inhomogeneous medium.

Table 1. Object-Image Relation for GRIN Lens Smaller than 0.25 Pitch

Object		Image	
Real	$s' < L_1 < +\infty$	Real, Reversed	$s < d_2 < +\infty$
Real	$0 \le L_1 < s'$	Imaginary, Enlarged, Erect	$-\infty < d_2 \le -D$
Imaginary	$-L' \le L_1 < 0$	Imaginary, Enlarged, Erect	$-D < d_2 \le -L$
Imaginary	$-D' \le L_1 < -L'$	Imaginary, Reduced, Erect	$-L < d_2 \le 0$
Imaginary	$-\infty < L_1 < -D'$	Real, Reduced, Erect	$0 < d_2 <$ s

Note: $D = n_2 \tan(\alpha d)/(n_0\alpha)$, $D' = n_1 \tan(\alpha d)/(n_0\alpha)$, $s' = n_1/[n_0\alpha \tan(\alpha d)]$, $L' = n_1 \tan(\alpha d/2)/(n_0\alpha)$.

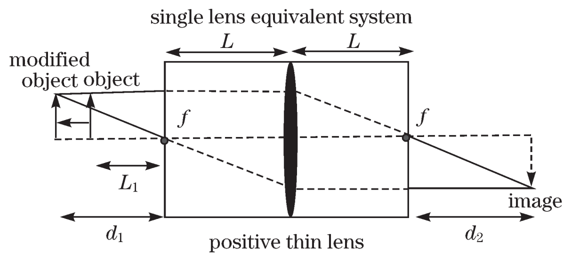

Fig. 4. Equivalent optical system for 0.25-pitch GRIN lens in inhomogeneous medium.

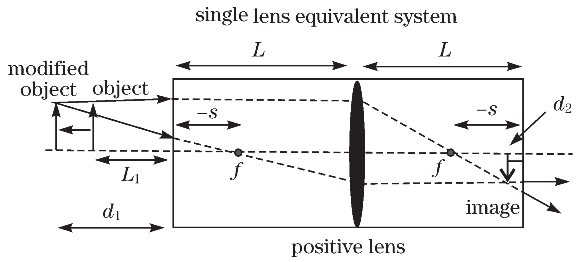

Fig. 5. Equivalent system for 0.25−0.5-pitch GRIN lens in inhomogeneous medium.

Table 2. Object-Image Relation for GRIN Lens with 0.25 Pitch

Object		Image	
Real	$0 \le L_1 < +\infty$	Real, Reversed	$0 < d_2 < +\infty$
Imaginary	$-L' \le L_1 < 0$	Imaginary, Enlarged, Erect	$-\infty < d_2 \le -L$
Imaginary	$-\infty < L_1 < -L'$	Imaginary, Reduced, Erect	$-L < d_2 < 0$

Note: $L' = n_1/(n_0\alpha)$.

Table 3. Object-Image Relation for GRIN Lens with 0.25−0.5 Pitch

Object		Image	
Real	$-D' \le L_1 < +\infty$	Imaginary, Reversed	$-s < d_2 \le 0$
Real	$0 \le L_1 < -D'$	Real, Reversed	$0 < d_2 \le -D$
Imaginary	$s' \le L_1 < 0$	Real, Reduced, Reversed	$-D < d_2 < +\infty$
Imaginary	$-L' \le L_1 < s'$	Imaginary, Enlarged, Erect	$-\infty < d_2 \le -L$
Imaginary	$-\infty < L_1 < -L'$	Imaginary, Reduced, Erect	$-L < d_2 < s$

Note: $D = n_2 \tan(\alpha d)/(n_0\alpha)$, $D' = n_1 \tan(\alpha d)/(n_0\alpha)$, $L' = n_1 \tan(\alpha d/2)/(n_0\alpha)$, $s' = n_1/[n_0\alpha \tan(\alpha d)]$.

system, either real or imaginary, is considered as imaginary; thus, any image with $d_2 < 0$ and object with $d_1 < 0$ are regarded as imaginary. Table 1 summarizes the imaging property of GRIN lens at this length.

Single-lens equivalent system for 0.25-pitch GRIN lens is obtained by Eq. (6), which consists of one positive thin lens at $f = n_2/(n_0\alpha)$ and two equal spatial spacings at $L = n_2/(n_0\alpha)$. The object-image equation and magnification are

$$L_1 d_2 n_2/n_1 = f^2, \ \beta = -n_1 f/L_1 n_2. \tag{9}$$

The focal point is located at the edge of the equivalent system (Fig. 4), which results in a simpler object-image relation (Table 2).

The 0.25−0.5-pitch GRIN lens could be reduced to a single-lens equivalent system with one positive lens at $f = n_2/[n_0\alpha \sin(\alpha d)]$ and two optical spacings at $L = n_2 \tan(\alpha d/2)/(n_0\alpha)$ (Fig. 5). The focal point is located in the equivalent system at a spacing of $s = n_2/[n_0\alpha \tan(\alpha d)]$. The imaging property is listed in Table 3. Object-image equation and magnification are

$$(n_2 L_1/n_1 - s)(d_2 - s) = f^2, \ \beta = f/(s - n_2 L_1/n_1). \tag{10}$$

For the 0.5-pitch GRIN lens in inhomogeneous medium, the transfer matrix is

$$\begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} 1 & L_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -n_1/n_2 \end{bmatrix} \begin{bmatrix} 1 & L_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_0 \\ \theta_0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & L_2 + n_2/n_1 L_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -y_0 \\ -n_1\theta_0/n_2 \end{bmatrix}. \quad (11)$$

In Eq. (11), the 0.5-pitch GRIN lens serves as an optical inverter for the object along z axis (from y to $-y$), changes the angle of the beam from θ to $-\theta$, and repositions the object to n_2L_1/n_1 away from the GRIN lens

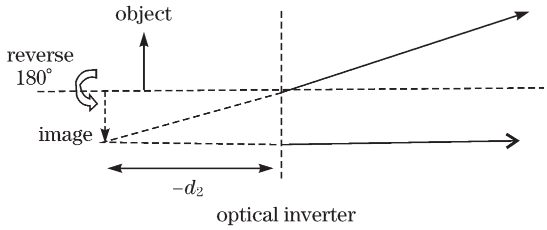

Fig. 6. Equivalent optical system for 0.5-pitch GRIN lens in inhomogeneous medium.

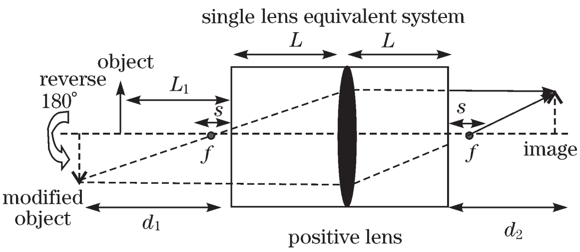

Fig. 7. Equivalent optical system for 0.5−0.75-pitch GRIN lens in inhomogeneous medium.

Table 4. Object-Image Relation for GRIN Lens with 0.5−0.75 Pitch

Object		Image	
Real	$s' < L_1 < +\infty$	Real, Erect	$s < d_2 < +\infty$
Real	$0 \leq L_1 < s'$	Imaginary, Enlarged, Erect	$-\infty < d_2 \leq -D$
Imaginary	$-L' \leq L_1 < 0$	Imaginary, Enlarged, Erect	$-D < d_2 \leq -L$
Imaginary	$-D' \leq L_1 < -L'$	Imaginary, Reduced, Erect	$-L < d_2 \leq 0$
Imaginary	$-\infty < L_1 < -D'$	Real, Reduced, Erect	$0 < d_2 < s$

Note: $s' = n_1/[n_0\alpha\tan(\alpha d)]$.

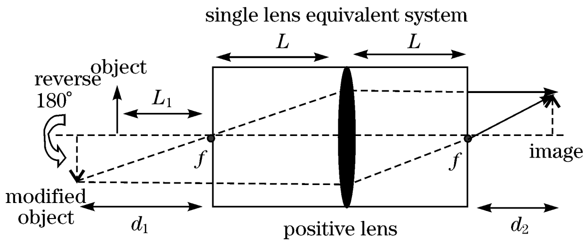

Fig. 8. Equivalent optical system for 0.75-pitch GRIN lens in inhomogeneous medium.

Table 5. Object-Image Relation for GRIN Lens with 0.75 Pitch

Object		Image	
Real	$0 \leq L_1 < +\infty$	Real, Erect	$0 < d_2 < +\infty$
Imaginary	$-L' \leq L_1 < 0$	Imaginary, Enlarged, Reversed	$-\infty < d_2 \leq -L$
Imaginary	$-\infty < L_1 < -L'$	Imaginary, Reduced, Reversed	$-L < d_2 < 0$

Note: $L' = n_1/(n_0\alpha)$.

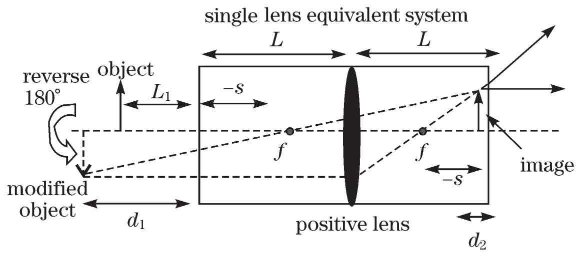

Fig. 9. Equivalent optical system for 0.75−1-pitch GRIN lens in inhomogeneous medium.

(Fig. 6). The object-image relation and magnification are

$$d_2 = n_2L_1/n_1, \beta = -1. \quad (12)$$

Equation (7) could reduce the GRIN lens with a pitch from 0.5 and 0.75 to a single-lens equivalent system consisting of one positive thin lens at $f = -n_2/[n_0\alpha\sin(\alpha d)]$ and two optical spacings at $L = -n_2/[n_0\alpha\tan(\alpha d/2)]$,and with inverted objects (Fig. 7). Its object-image equation and magnification are

$$(n_2L_1/n_1 - s)(d_2 - s) = f^2, \ \beta = f/(n_2L_1/n_1 - s), \quad (13)$$

where $s = n_2/[n_0\alpha\tan(\alpha d)]$. The imaging property is listed in Table 4.

By employing Eq. (7), the 0.75-pitch GRIN lens is reduced to a single-lens equivalent system consisting of one positive lens at $f = n_2/(n_0\alpha)$ and two optical spacings at $L = n_2/(n_0\alpha)$, and with relocated and inverted objects (Fig. 8). The imaging property is summarized in Table 5. The object-image relation and magnification are

$$n_2L_1d_2/n_1 = f^2, \ \beta = n_1f/L_1n_2. \quad (14)$$

Table 6. Object-Image Relation for GRIN Lens with 0.75−1 Pitch

Object		Image	
Real	$-D' \leq L_1 < +\infty$	Imaginary, Erect	$-s < d_2 \leq 0$
Real	$0 \leq L_1 < -D'$	Real, Erect	$0 < d_2 \leq -D$
Imaginary	$s' \leq L_1 < 0$	Real, Reduced Erect	$-D < d_2 < +\infty$
Imaginary	$-L' \leq L_1 < s'$	Imaginary, Enlarged, Reduced	$-\infty < d_2 \leq -L$
Imaginary	$-\infty < L_1 < -L'$	Imaginary, Reduced, Reduced	$-L < d_2 < s$

Note: $D = n_2 \tan(\alpha d)/(n_0\alpha)$, $D' = n_1 \tan(\alpha d)/(n_0\alpha)$, $L' = n_1 \tan(\alpha d/2)/(n_0\alpha)$, $s' = n_1/[n_0\alpha \tan(\alpha d)]$.

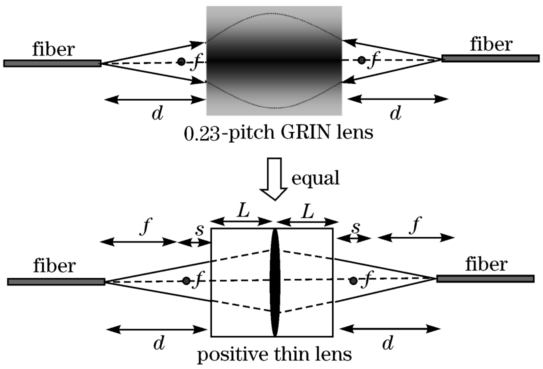

Fig. 10. Expanded beam connector using GRIN lens.

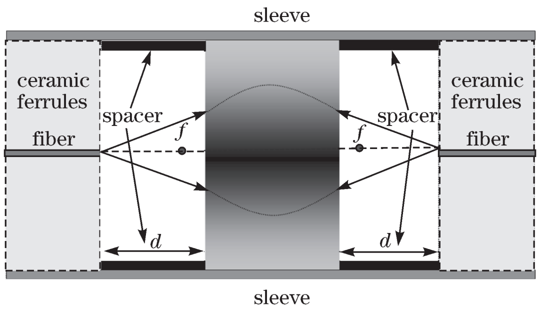

Fig. 11. Structure of an expanded beam connector.

Single-lens equivalent system for the GRIN lens at this range is obtained using Eq. (7), incorporating one positive thin lens at $f = -n_2/[n_0\alpha \sin(\alpha d)]$ and two spatial distances at $L = -n_2/[n_0\alpha \tan(\alpha d/2)]$ (Fig. 9). The imaging property is listed in Table 6. The object-image relation and magnification are

$$(n_2L_1/n_1 - s)(d_2 - s) = f^2,\ \beta = f/(n_2L_1/n_1 - s), \quad (15)$$

where $s = n_2/[n_0\alpha \tan(\alpha d)]$.

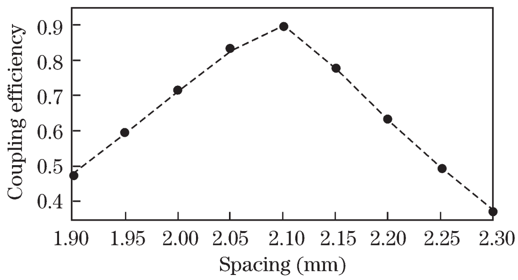

Fig. 12. Coupling efficiency versus spacing.

For the 1-pitch GRIN lens, the transfer matrix is

$$\begin{bmatrix} \cos(\alpha d) & \frac{n_1 \sin(\alpha d)}{n_0\alpha} \\ \frac{-n_0\alpha \sin(\alpha d)}{n_2} & \frac{n_1 \cos(\alpha d)}{n_2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix}. \quad (16)$$

In Eq. (16), the 1-pitch GRIN lens serves as an inhomogeneous interface. Thus, the object-image relation is

$$d_2 = n_2L_1/n_1, \beta = 1. \quad (17)$$

To substantiate the equivalent system for GRIN lens immersed in air, we designed and performed one experiment on its equivalent system consisting of one positive lens and two identical spacings. When utilized as an expanded beam connector for fiber-to-fiber coupling, the single-lens equivalent system for GRIN lens with one fiber symmetrically positioned $2f$ away from the lens on the axis provided for the best coupling efficiency (Fig. 10). This is in accordance with the "symmetrical principle"[10,11].

In the experiment, one fiber coupled laser package was employed as the laser source (810-nm laser; 1-W output power). Each of the two fibers had a numerical aperture (NA) of 0.22 and a silica core of 125 μm. Well-centered fiber ends were assembled into high precision ferrules made of zirconia. The configuration of the fiber-to-fiber connector (Fig. 11) consisted of one GRIN lens, two metal spacers, two fiber optics ferrules, and one sleeve.

A 0.23-pitch GRIN lens (Melles Griot, Inc.) with a length of 4.35 mm, diameter of 1.8 mm, $\alpha = 0.332$ mm^{-1}, and antireflection coat of $\lambda = 810$ nm was used in the experiment. Based on its equivalent system, the best coupling was achieved when the position of each fiber was $d = 2.1$ mm away from the GRIN lens. We measured the coupling efficiency of the connector at various spacings between the fiber and GRIN lens (1.9−2.3 mm). Figure 12 shows the result.

Moreover, Fig. 12 indicates that the maximum coupling efficiency of 89% is reached at the spacing of 2.1 mm, which is in good agreement with the theoretical prediction. Thus, the validity of the single-lens equivalent system for GRIN lens in inhomogeneous medium was proven. However, the maximum coupling efficiency was not 100%. The extra loss was due to Fresnel reflection at fiber ends, aberration of GRIN lens, and the fact that each fiber end had a finite size. In reality, this cannot be regarded as an ideal dimension.

In conclusion, we propose a single-lens equivalent system for GRIN lens with various lengths embedded in inhomogeneous medium. In the paraxial approximation of the matrix optics, this simple optical system is proven equal to the GRIN lens. The object-image relations of GRIN lens under this condition are fully investigated using the substitute system. Consisting of only one lens, the proposed system renders itself a clear and straightforward equivalent system for complex GRIN lenses. Due to its simplicity, this model could facilitate the optical analysis of complicated optical systems employing GRIN lens.

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