Guided-mode resonances in multimode planar periodic waveguides

Tianyu Sun (孙天玉)^{1,2*}, Yunxia Jin (晋云震)¹, Jianda Shao (邵建达)¹, and Zhengxiu Fan (范正修)¹

¹Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai 201800, China

²Graduate University of Chinese Academy of Sciences, Beijing 100049, China

*E-mail: siomsun@gmail.com

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Using the effective medium theory and the waveguide eigenvalue equation, we design a multimode planar periodic waveguide. When a plane wave illuminates the grating at the designed angle and wavelength, more than one leaky modes are excited coincidentally. Then the reflection efficiency around this designed angle and wavelength is investigated using a rigorous coupled wave analysis formulation and a gap of reflection peaks is found. Electric field distributions reveal that this high reflectivity gap is due to the coupling between these two coincidentally excited leaky modes.

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Light propagation in resonant grating waveguide structure is of practical interest in many areas of physics and engineering^[1]. Resonant phenomena associated with these structures have been investigated for some time both theoretically and experimentally beginning with the work of $Wood^{[2]}$. Incident plane waves can induce leaky waveguide modes, thereby generating guided-mode resonance (GMR) field responses in a spectrum. This resonance effect leads to the redistribution of diffracted energy and manifests as reflection or transmission peaks arising from the background, which is provided by the effective-medium thin-film characteristics of the structure. Normally, the resonant grating waveguide structure comprises a substrate, a waveguide layer, and a grating layer. Many research works have focused on this structure from different aspects, for example, to broaden the angular tolerance using a bi-atom $grating^{[3,4]}$. Another configuration is the planar periodic waveguide or photonic-crystal waveguide, in which the grating layer also serves as the waveguide layer. Since the structure is very simple, increasing attention has been paid to this single-layer sub-wavelength dielectric grating, and many distinguished structures have been proposed and realized by previous experiments^[5-10]. However, all these studies</sup> concentrated on single-mode planar periodic waveguides wherein only a leaky mode is excited. With the thickness of the grating increasing, multiple leaky modes can be excited and the multimode planar periodic waveguide becomes universal. To our knowledge, little attention has been paid to multimode planar periodic waveguides and reflectivity when more leaky modes are excited simultaneously. In this letter, a multimode planar periodic waveguide is designed with emphasis placed on investigating the reflectivity when more than one leaky mode is excited coincidentally.

Figure 1 shows a general planar periodic waveguide. The structure is periodical along x direction and infinite along y direction. Herein, d is the thickness of the equivalent waveguide (grating), Λ is the period of the planar waveguide, and f is the ratio of the width of the high refractive index material $(n_{\rm h})$ to the period. Other representations are marked in the figure. For the sake of convenience, we only consider transverse electric (TE) waves.

According to the effective medium theory (EMT), a periodic structure may be replaced by an anisotropic homogeneous medium only if the zero order diffraction propagates and higher diffraction orders are evanescent^[11]. Under the TE-waves illumination, the effective index can be written as

$$n_{\rm e} = \left[\bar{\varepsilon} + \frac{\pi^2}{3} f^2 (1-f)^2 (n_{\rm h}^2 - n_{\rm l}^2)^2 \left(\frac{\Lambda}{\lambda}\right)^2\right]^{1/2}, \quad (1)$$

where $\bar{\varepsilon} = n_{\rm h}^2 f + n_{\rm l}^2 (1 - f)$ and λ is the free-space wavelength. With this effective index away from resonance, the waveguide eigenvalue equation can be written as

$$\tan(\kappa d) = \kappa(\gamma + \delta) / (\kappa^2 - \delta^2), \qquad (2)$$

where $\kappa = (n_{\rm e}^2 k^2 - \beta^2)^{1/2}$, $\delta = (\beta^2 - n_{\rm c}^2 k^2)^{1/2}$, and $\gamma = (\beta^2 - n_{\rm s}^2 k^2)^{1/2}$ are the wave numbers of the grating layer, cover, and substrate in z direction, respectively; $k = 2\pi/\lambda$, and β is the waveguide propagation constant of a certain diffracted wave in x direction^[12,13]. To achieve similar resonance between the first diffracted order of the illumination and the leaky mode, the resonant wavelength should be first obtained by the root of Eq. (2) with $\beta = k \sin \theta \pm 2\pi/\Lambda^{[14]}$.



Fig. 1. Structure of a general planar periodic waveguide.

In the succeeding analysis, we assume the following values of the dielectric constants with $n_c = 1.0$, $n_s = 1.46$, $n_l = 1.46$ (as appropriate for SiO₂ or other oxides), and $n_h = 2.0$ (as appropriate for Ti₂O₅ or other oxides). f = 0.75, d = 700 nm, and $\Lambda = 700$ nm are also specified. This structure can be realized on an etched BK7 glass substrate deposited by a layer of Ta₂O₅ with a magnetron sputtering system subsequently. To excite two different counter-propagating modes simultaneously, we can choose free-space wavelength λ_0 and the incident angle θ_0 to satisfy the following conditions:

$$\frac{2\pi}{\lambda_0}\sin\theta_0 + \frac{2\pi}{\Lambda} = \beta_{\mathrm{TE}_0}(\lambda_0), \qquad (3)$$

$$\frac{2\pi}{\lambda_0}\sin\theta_0 - \frac{2\pi}{\Lambda} = -\beta_{\mathrm{TE}_1}(\lambda_0),\tag{4}$$

where $\beta_{\text{TE}_0}(\lambda_0)$ and $\beta_{\text{TE}_1}(\lambda_0)$ are the propagation constants of the fundamental TE₀ and first TE₁ eigenmodes of Eq. (2). The desired values deduced from the above-mentioned equations are about $\lambda_0 = 1145$ nm and $\theta_0 = 7.02^{\circ}$. That is to say, if we illuminate the grating using TE plane waves with wavelength of 1145 nm and incident angle of 7.02°, TE₀ and TE₁ leaky modes should be excited simultaneously by the +1st diffracted order and -1st diffracted order, respectively. The succeeding segments shall expound on reflectivity.

Figure 2(a) shows reflection contour as a function of wavelength and incident angle around $\lambda_0 = 1145$ nm and $\theta_0 = 7.02^{\circ}$ using the rigorous coupled wave analysis (RCWA) formulation^[15]. High reflectivity corresponds to the excitation of GMR in the periodic waveguide and the GMR locations closely track the dispersion curves of the leaky modes. A wide gap of high reflectivity can be seen in Fig. 2(a). As wavelength is within this gap, no reflection peaks are found,



Fig. 2. (a) Reflectivity contour calculated using a RCWA formulation and (b) reflection spectra at the wavelengths of 1138 and 1145 nm.



Fig. 3. Normalized amplitude of the electric field profile in two periods of the structure at the corresponding resonant states. (a) $\lambda = 1138$ nm and $\theta = 7.25^{\circ}$; (b) $\lambda = 1138$ nm and $\theta = 5.19^{\circ}$; (c) $\lambda = 1150.34$ nm and $\theta = 6.42^{\circ}$.

and thus, no leaky modes are excited. Constrained by the resolution of the picture, high reflections at the lower (upper) branch are not clearly profiled when the incident angle is larger (smaller) than 6.5° . Figure 2(b) gives a reflection spectrum at the specific wavelengths of 1138 and 1145 nm. The wavelength of 1138 nm is below this gap and so the reflection spectrum has two peaks, one with larger angular tolerance and the other with smaller angular tolerance. At 1145 nm, which is within the gap, no reflection peaks are found and so no leaky modes are excited.

Electric field in resonant planar periodic waveguides can help us explain resonance processes and may contribute significantly to the design of GMR structures^[13,16,17]</sup>. To investigate the origin of the high</sup>reflectivity gap, rigorous calculations were conducted on electric field distributions at several resonant states using the Fourier modal method^[18]. Figures 3(a) and (b) demonstrate that the reflection peaks with larger angular tolerance is caused by the excitation of the TE_1 leaky mode, while the reflection peaks with smaller angular tolerance is caused by that of the TE_0 leaky mode. In multilayer homogeneous planar waveguide, all eigenmodes are orthogonal and propagate independently. Meanwhile, coupling between the two counter-propagating leaky modes is inevitable in the planar periodic waveguide due to periodical modulation^[19]. Mode coupling lifts the degeneracy of TE_0 and TE_1 leaky modes and results in high reflectivity gap. Figure 3(c) shows the electric field distributions at the resonant state corresponding to the upper edge of this gap. It has the characteristics of TE_0 mode (no node along the vertical direction) and TE_1 mode (one node along the vertical direction). Coupling between TE_0 and TE_1 leaky modes is markedly observed.

In conclusion, we design a multimode planar periodic waveguide with the EMT and waveguide eigenvalue equation, and obtain more than one leaky modes that are excited simultaneously in the structure. Using a RCWA formulation, we investigate the reflectivity around this designed incident angle and wavelength wherein a gap of reflection peaks has appeared. This high reflectivity gap is caused by the coupling between the two simultaneously excited counter-propagating TE₀ and TE₁ leaky modes. This work was supported by the National Natural Science Foundation of China under Grant No. 10704079.

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